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## EXTENSION OF MAINTENANCE OPPORTUNITY WINDOWS TO GENERAL MANUFACTURING SYSTEMS

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### ABSTRACT

*In manufacturing systems, many maintenance tasks require equipment to be stopped in order to safely perform them. However, such tasks cannot last for a long time since the stoppage of machines might directly result in production losses. In this paper, we investigate how long we can perform maintenance during scheduled operations by strategically shutting down equipment without bringing extra production losses to the system. Using the concept of maintenance opportunity window (MOW) we calculate such time intervals with given information of manufacturing systems. MOWs are analytically derived with various system configurations. Simulations are used to deal with uncertainties in production lines such as random machine failures, starvations, blockages, etc. Moreover, the proposed MOW algorithms are demonstrated through numerical case studies.*

### 1. INTRODUCTION

Manufacturing systems usually consist of machines and material handling systems that are connected in a combination of serial or parallel lines. Since manufacturing systems in today's world are highly complicated and interconnected, reliability is becoming more and more important in operating manufacturing systems which are subject to deterioration with age and usage. In order to achieve a satisfactory level of reliability, maintenance is conducted on the system or its units.

There are large bodies of literature [1-6] discussing maintenance policies, which can be generally classified into reactive maintenance (RM), preventive maintenance (PM), and condition-based maintenance (CBM). RM happens when the

units are down, and generally will result in longer time and cost. Therefore, it is necessary to perform PM tasks to reduce the random breakdown of the machines. In other words, machines should be strategically shut down for PM before their failure occurs. In the literature, the relationship between PM and the manufacturing system performance is explored [6, 7], and a number of maintenance models are developed to find the optimal policies [1, 3, 5]. Van der Duyn Schouten *et al.* [8] formulated the problem as a semi-Markov decision process, and found the optimal policy. Yao *et al.* [9] used a Markov decision process to find a joint PM and production policy by minimizing total costs (inventory holding, backlog, and maintenance cost). Riane *et al.* [10] formulated it as a linear programming problem to obtain an optimal policy.

Most PM policies developed are based on statistical parameters of a system (mean time between failure, mean time to repair, availability, etc.), which are collected for long-term, and thus the PM planning is predetermined to improve the system performance in the long run. On the other hand, CBM policies pay more attention to not only machine conditions but also real-time information of current system, concentrating more on the transient behavior of a system rather than production performance in steady state [11, 12].

In manufacturing systems, the buffers are used to mitigate short-term negative effects of the failure of the machine, protect stations from blockages and starvations, thus can have immediate impact on system performance. Gershwin *et al.* [13] used a continuous-flow model to analyze the manufacturing system with a buffer of finite capacity. Peters *et al.* [14] built hybrid models with restricted buffer sizes.

Although many researchers aimed to design lean buffers to keep inventory level low, extra space in buffers can protect the system from production losses for short time even if certain machines are down for maintenance. Then one question arises: how long can a station be idle while keeping throughput rate constant? To answer this question, the maintenance opportunity windows (MOWs) of a specific machine were defined by Chang *et al.* [15] as the maximum time duration that allows maintenance without bringing extra production loss to a system. Chang *et al.* analytically calculated MOWs in serial lines in [15], and accounted for the slowest station in [16]. These papers showed that if the station is idle for time less than or equal to its MOW, the production line will be running without throughput losses.

However, in this work, we consider the MOW as the maximum time window we can shut down one machine without causing the bottleneck machine(s) starved or blocked. The short-term bottleneck machines in manufacturing systems are the one whose sensitivity of the system production rate to a machine's production rate is the highest of all machines in the system. In other words, the production interruption on these short-term bottleneck machines will most likely result in the severest system production losses. Thus we do not want maintenance actions on non-bottleneck machines to introduce unnecessary stoppage of bottleneck machines. The data-driven method to detect such short-term bottleneck machines has been developed in [17, 18]. Furthermore, we extend the MOW calculation from serial lines to assembly and disassembly system, and finally to a general manufacturing system which can be a combination of serial and parallel lines.

The remainder of this paper is organized as follows. In Section 2, we analyze MOWs of machines in serial and assembly/disassembly systems, and then extend them to a general manufacturing system. In Section 3, a simulation-based algorithm is used and two case studies are performed. Finally, Section 4 summarizes our conclusions and future research.

## 2. MAINTENANCE OPPORTUNITY WINDOWS

### 2.1. MOW for a two-machine one-buffer system (2M1B)

A 2M1B system shown in Figure 1 is discussed to understand basic ideas of an MOW. This 2M1B model can be used as a building block for longer lines and more complex systems.

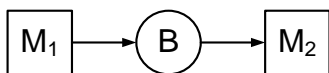


Figure 1: A two-machine one-buffer line (2M1B)

The assumptions and notations for this model are:

- The production rates for machines  $M_1$  and  $M_2$  are denoted as  $m_1$  and  $m_2$ , respectively.

- A machine can process only one part at a time.  $I_i = 1$  if there is initially a part in machine  $M_i$  and  $I_i = 0$  if there is no part in machine  $M_i$ .
- The buffer level  $N$  is continuous and  $N(t)$  is a buffer level at time  $t$ .
- No machine failure occurs.
- No material travelling time at the buffer is considered.

Assume that at time  $t = 0$ , machine  $M_1$  is strategically shut down for preventive maintenance. Then the system will be eventually empty at time  $T$  as shown in Figure 2. The time duration  $T$  depends on the total number of parts in buffer  $B$  and machine  $M_2$ , and is given by

$$N(0) + I_2 = m_2 T \quad (1)$$

However, after time  $T$ , machine  $M_2$  will be starved and cause production losses to the system. Therefore, we can shut down machine  $M_1$  only until buffer  $B$  has at least one part at time  $T$ ,

$$N(T) = 1. \quad (2)$$

Assuming machine  $M_1$  will be down for  $MOW_1$ , we have

$$N(T) - (N(0) + I_2) = -m_2 MOW_1 + (m_1 - m_2)(T - MOW_1). \quad (3)$$

Then,  $MOW_1$  can be calculated as

$$MOW_1 = \frac{N(0) + I_2}{m_2} - \frac{1}{m_1}. \quad (4)$$

The change in a buffer level over time is shown in Figure 2 to illustrate how to calculate  $MOW_1$  for a 2M1B line. Note that we calculate  $MOW_1$  with the bottleneck machine  $M_2$  ( $m_1 \geq m_2$ ).

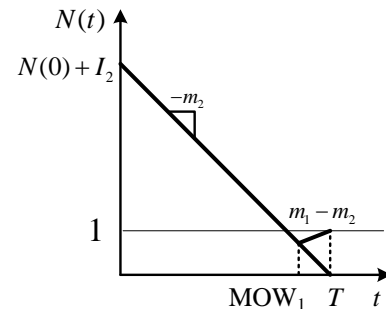


Figure 2: Illustration of  $MOW_1$  calculation of a 2M1B line

On the other hand, if  $m_1 < m_2$ , the bottleneck is machine  $M_1$  and we can shut down machine  $M_2$  for  $MOW_2$  without bringing system production losses. The illustration is shown in Figure 3, where  $T$  is the time that buffer  $B$  will become full (i.e., it reaches its capacity,  $C$ ) if machine  $M_2$  is down during  $(0, T]$ . After time  $T$ , machine  $M_1$  will be blocked and cause production losses to the system. Therefore, we are allowed to shut down machine  $M_2$  only until buffer  $B$  has an empty space for at least one part at time  $T$ . Similar with Equations from (1) to (3), we have

$$C - (N(0) - I_1) = m_1 T \quad (5)$$

$$N(T) = C - 1 \quad (6)$$

$$N(T) - (N(0) - I_1) = m_1 \text{MOW}_2 + (m_1 - m_2)(T - \text{MOW}_2) \quad (7)$$

Then,  $\text{MOW}_2$  is expressed by

$$\text{MOW}_2 = \frac{(C - N(0)) + I_1}{m_1} - \frac{1}{m_2}. \quad (8)$$

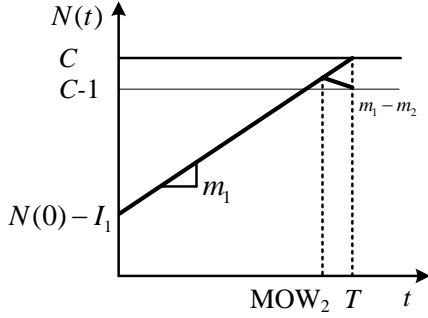


Figure 3: The  $\text{MOW}_2$  calculation when  $m_1 < m_2$

In summary, the MOWs for each machine are given by:

$$\begin{cases} \text{MOW}_1 = \frac{N(0) + I_2}{m_2} - \frac{1}{m_1} & \text{if } m_1 \geq m_2 \\ \text{MOW}_2 = 0 & \\ \text{MOW}_1 = 0 & \\ \text{MOW}_2 = \frac{(C - N(0)) + I_1}{m_1} - \frac{1}{m_2} & \text{if } m_1 < m_2 \end{cases} \quad (9)$$

From Equation (9), we can observe that MOWs depend on the production rates of two machines, initial buffer contents, and part in machines. Furthermore, Equation (9) provides us with insights of a reversibility property.

$$N(0) \leftrightarrow C - N(0)$$

$$I_2 \leftrightarrow I_1$$

$$m_2 \leftrightarrow m_1$$

The reversed 2M1B line of the original line in Figure 1 can virtually be imagined and given by the following line in Figure 4. Then machine  $M_2$  becomes an upstream machine and machine  $M_1$  is a downstream one with the same production rates.

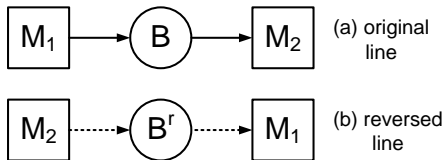


Figure 4: Reversed 2M1B line

Suppose  $m_1 < m_2$  and an initial buffer content is  $N^r(0)$ , then  $\text{MOW}_2$  for machine  $M_2$  in the reversed line can be calculated by Equation (4) as

$$\text{MOW}_2 = \frac{N^r(0) + I_1}{m_1} - \frac{1}{m_2} \text{ in the reversed line.} \quad (10)$$

On the other hand,  $\text{MOW}_2$  in the original line in Equation (8) is given by

$$\text{MOW}_2 = \frac{(C - N(0)) + I_1}{m_1} - \frac{1}{m_2} \text{ in the original line.} \quad (11)$$

Compared with Equations (10) and (11), MOWs are identical if we set  $N^r(t) = C - N(t)$ . If the bottleneck machine is located upstream, we can reverse a line to have the bottleneck machine downstream with changes of buffer contents from  $N(0)$  to  $N^r(0)$ . Then equivalent MOWs can be achieved. We call it as the reversibility property and use it to calculate MOWs for more complicated manufacturing systems.

## 2.2. MOW for a Serial Line

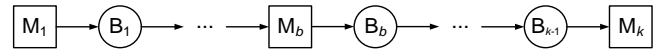


Figure 5: A  $k$ -machine ( $k-1$ )-buffer serial line

We consider MOWs of machines in a  $k$ -machine ( $k-1$ )-buffer balanced serial line in Figure 5. The assumptions are similar to what we have made for the 2M1B model.

- The buffer level  $N$  is continuous and  $N(t)$  is a buffer level at time  $t$ .
- $I_i = 1$  if there is an initial part in machine  $M_i$ ,  $I_i = 0$  if there is no initial part in machine  $M_i$ .
- Production flow is continuous and the production rate for machine  $M_i$  at time  $t$  is equal to  $m_i(t)$ .
- No machine failures.
- No traveling time at buffers is considered.

### 2.2.1. MOW for a Balanced Serial Line

Suppose that all production rates are the same (i.e.,  $m_1 = m_2 = \dots = m_k$ ) in Figure 5. We want to calculate the  $\text{MOW}_1$  of machine  $M_1$ . By the definition of MOW, the throughput of the system in time interval  $(0, \text{MOW}_1]$  will be equal to  $m_k \times \text{MOW}_1$ . The dynamics of the system satisfies:

$$\int_0^T (m_i(t) - m_{i-1}(t)) dt = N_{i-1}(0) + I_i - I_{i-1}^L \quad i = 2, \dots, k \quad (12)$$

where  $I_i^L$  is denoted as the minimum number of parts we should maintain in machine  $M_i$  to prevent machine  $M_k$  from starvation.

Set  $T = \text{MOW}_1$ , and add all  $i$ 's for both sides of Equation (12),

$$\begin{aligned} m_k \text{MOW}_1 &= \sum_{i=2}^k (N_{i-1}(0) + I_i - I_{i-1}^L) \\ \therefore \text{MOW}_1 &= \frac{\sum_{i=2}^k (N_{i-1}(0) + I_i) - \sum_{i=1}^{k-1} I_i^L}{m_k} \end{aligned} \quad (13)$$

In the continuous-flow model, the value of  $I_i^L$  can be calculated as

$$I_i^L = CT_i \cdot m \quad (14)$$

where  $CT_i$  is the cycle time of machine  $M_i$  and  $m$  is the system flow rate determined by the production rate of the bottleneck machine. Therefore,

$$\frac{I_i^L}{I_k^L} = \frac{CT_i \cdot m}{CT_k \cdot m} = \frac{m_k}{m_i}. \quad (15)$$

Since machine  $M_k$  has been running during  $(0, \text{MOW}_1]$ , there is always one part in machine  $M_k$  (i.e.,  $I_k^L = 1$ ). Then,

$I_i^L = \frac{m_k}{m_i} = 1$  due to the same production rates. Substituting it

into Equation (13), we have

$$\text{MOW}_1 = \frac{\sum_{i=2}^k (N_{i-1}(0) + I_i) - (k-1)}{m_k} \quad (16)$$

### 2.2.2 MOW for an Unbalanced Serial Line

Consider an unbalanced serial line whose machine-level production rates  $m_i$  ( $i = 1, 2, \dots, k$ ) are different in Figure 5. Assume that machine  $M_b$  is the bottleneck machine in the system. The  $\text{MOW}_1$  of machine  $M_1$ , which is located upstream of machine  $M_b$ , can be computed by Equation (17):

$$\begin{aligned} \text{MOW}_1 &= \frac{\sum_{i=2}^b (N_{i-1}(0) + I_i) - \sum_{i=1}^{b-1} I_i^L}{m_b} \\ &= \frac{\sum_{i=2}^b (N_{i-1}(0) + I_i) - \sum_{i=1}^{b-1} \frac{m_b}{m_i}}{m_b} \\ &= \frac{\sum_{i=2}^b (N_{i-1}(0) + I_i)}{m_b} - \sum_{i=1}^{b-1} \left( \frac{1}{m_i} \right) \end{aligned} \quad (17)$$

On the other hand, machine  $M_k$  is located downstream of the bottleneck machine  $M_b$ . We can use the reversibility property to derive the  $\text{MOW}_k$ :

$$\begin{aligned} \text{MOW}_k &= \frac{\sum_{i=k-1}^b (N_i^r(0) + I_i)}{m_b} - \sum_{i=k}^{b+1} \frac{1}{m_i} \\ &= \frac{\sum_{i=b}^{k-1} (N_i^r(0) + I_i)}{m_b} - \sum_{i=b+1}^k \frac{1}{m_i} \\ &= \frac{\sum_{i=b}^{k-1} ((C_i - N_i(0)) + I_i)}{m_b} - \sum_{i=b+1}^k \frac{1}{m_i} \end{aligned} \quad (18)$$

Of course, Equation (18) can be directly derived from Equation (12).

$$\begin{aligned} \int_0^{\text{MOW}_k} (m_i(t) - m_{i+1}(t)) dt &= (C_i - N_i(0)) + I_i - I_{i+1}^L \quad i = b, \dots, k-1 \\ \therefore \text{MOW}_k &= \frac{\sum_{i=b}^{k-1} ((C_i - N_i(0)) + I_i - I_{i+1}^L)}{m_b} \\ &= \frac{\sum_{i=b}^{k-1} ((C_i - N_i(0)) + I_i) - \sum_{i=b+1}^k \frac{m_b}{m_i}}{m_b} \end{aligned} \quad (19)$$

## 2.3. MOW for Assembly/Disassembly Systems

### 2.3.1. Assembly Systems

Real manufacturing systems often consist of serial and assembly systems. Therefore, we also investigate MOWs for assembly systems in addition to the MOW in the serial lines. Suppose that machine  $M_1$  produces part  $p_1$  and machine  $M_2$  makes part  $p_2$ . Parts  $p_1$  and  $p_2$  are assembled in machine  $M_3$  as shown in Figure 6.

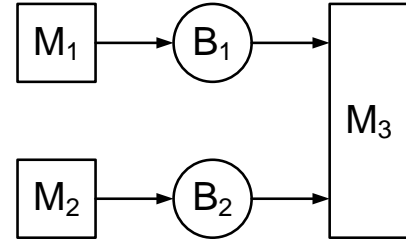


Figure 6: A model for an assembly system

We analyze the MOWs of machine  $M_1$  in two different cases:

#### Case 1: machine $M_3$ is the bottleneck machine

Machine  $M_2$  has little influence on machine  $M_1$  so that we can use a 2M1B model with  $M_1$ - $B_1$ - $M_3$  for  $\text{MOW}_1$  and with  $M_2$ - $B_2$ - $M_3$  for  $\text{MOW}_2$ . By Equation (4), the  $\text{MOW}_1$  and  $\text{MOW}_2$  would be given as

$$\begin{aligned} \text{MOW}_1 &= \frac{N_1(0) + I_1 - I_1^L}{m_3} - \frac{I_1^L}{m_1} \\ \text{MOW}_2 &= \frac{N_2(0) + I_2 - I_2^L}{m_3} - \frac{I_2^L}{m_2} \end{aligned} \quad (20)$$

### Case 2: machine $M_2$ is the bottleneck machine

The dynamics for buffers  $B_1$  and  $B_2$  can be written as Equations (21) and (22), respectively.

$$\int_0^{\text{MOW}_1} m_1(t) dt - \int_0^{\text{MOW}_1} m_3(t) dt = N_1(0) + I_3 - I_1^L, \quad (21)$$

$$\int_0^{\text{MOW}_1} m_2(t) dt - \int_0^{\text{MOW}_1} m_3(t) dt = C_2 - N_2(0) + I_2 - I_3^L. \quad (22)$$

Therefore,

$$\begin{aligned} \text{MOW}_1 &= \frac{N_1(0) + I_3 - I_1^L + (C_2 - N_2(0)) + I_2 - I_3^L}{m_2} \\ &= \frac{N_1(0) + I_3 + (C_2 - N_2(0)) + I_2 - I_1^L + I_3^L}{m_2} \end{aligned} \quad (23)$$

The MOW in an assembly system can be also calculated using the reversibility property which is discussed in Section 2.1. The assembly system in Figure 6 can be redrawn like the one in Figure 7 (a), which can be further transformed equivalently to a serial line in Figure 7 (b).

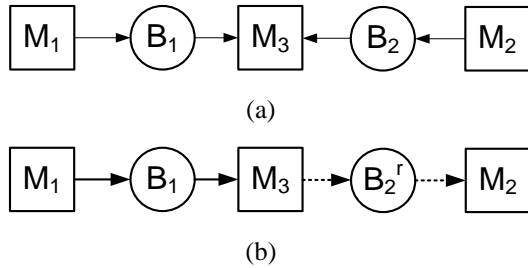


Figure 7: Equivalent serial lines of an assembly system

By using Equation (13) which is developed for MOWs of serial lines, MOWs for machines  $M_1$  and  $M_3$  are calculated as

$$\begin{aligned} \text{MOW}_3 &= \frac{N_2^r(0) + I_2 - I_3^L}{m_2} \\ \text{MOW}_1 &= \frac{N_1(0) + I_2 + N_2^r(0) + I_3 - I_1^L - I_3^L}{m_2} \\ &= \frac{N_1(0) + I_2 + (C_2 - N_2(0)) + I_3 - I_1^L + I_3^L}{m_2} \end{aligned} \quad (24)$$

which is exactly the same with Equation (23).

### 2.3.2. Disassembly Systems

A basic disassembly system is shown in Figure 8 and its equivalent serial lines are presented in Figure 9 (a) and (b) for the cases that the split machine is the bottleneck and a non-split machine is the bottleneck one, respectively.

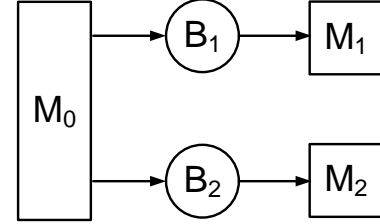
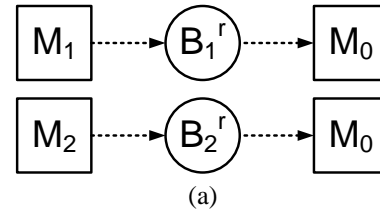
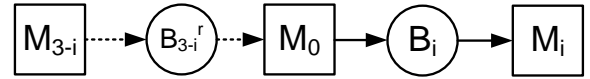


Figure 8: Disassembly system



(a)



(b)

### Figure 9: Equivalent serial lines for the disassembly system

Their MOWs for the non-bottleneck machines in both cases are

given by

$$\begin{cases} \text{MOW}_0 = 0 \\ \text{MOW}_i = \frac{C_i - N_i(0) + I_0 - I_i^L}{m_0} \quad \text{if } M_b = M_0, i = 1, 2 \end{cases} \quad (25)$$

$$\begin{cases} \text{MOW}_0 = \frac{N_i(0) + I_i - I_0^L}{m_i} \\ \text{MOW}_i = 0 \\ \text{MOW}_{3-i} = \frac{C_{3-i} - N_{3-i}(0) + I_0 + N_i(0) + I_i - (I_{3-i}^L + I_0^L)}{m_i} \end{cases} \quad (26)$$

if  $M_b = M_i, i = 1, 2$

### 2.4. MOW for General Manufacturing Systems

Using all the results shown above, we are able to compute the MOW for machines in a general manufacturing system shown in Figure 10, including assembly/disassembly and serial/parallel lines.

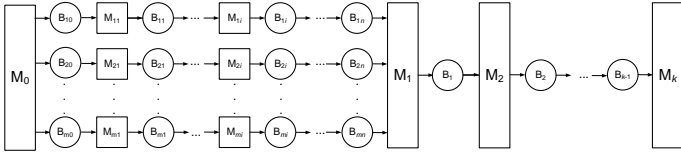


Figure 10: MOW in a combined system

We define  $l^{ij}$  as a line (or path) from machine  $M_i$  to machine  $M_j$  that satisfies the following:

- The structure of the line is M-B-M-B-...-B-M.
- All the connecting pairs in  $l^{ij}$  can be found in the original system either with the forward or backward order.

Let  $L^{ib} = \{l_1^{ib}, l_2^{ib}, \dots\}$  be the set of all the possible lines (or paths) from machine  $M_i$  to the bottleneck machine  $M_b$ . Then  $MOW_i$  of machine  $M_i$  in a general system is given by

$$MOW_i = \min_{\text{all } n} MOW_i(l_n^{ib}) = \min_{\text{all } n} (MOW_i(l_n^{ib})), \quad (27)$$

where  $MOW_i(l_n^{ib})$  means the MOW of machine  $M_i$  in line  $l_n^{ib}$ .

### 3. CASE STUDY IN SIMULATION

#### 3.1. Simulation-based MOW calculation

For transient behavior of a system with many uncertainties such as random machine failures, it is difficult to analyze them even if the behavior of machines obeys some probability models. Hence, it might not be feasible to obtain analytical MOWs of such systems. Uncertainties (random machine failure and its consequences) and disturbance (maintenance and raw material readiness, etc.) will affect production line smoothness. In order to have a model as close to a real production line as possible, a simulation is often used. Chang *et al.* [15] proposed an algorithm to estimate the MOW via a simulation and is given by

$$MOW_i = T_i^{\text{empty}} - T_i^{\text{resume}}, \quad (28)$$

where  $T_i^{\text{empty}}$  is the time until the buffers and all machines between machine  $M_i$  and machine  $M_b$  (excluding machine  $M_i$  but including the bottleneck machine  $M_b$ ) are empty, and  $T_i^{\text{resume}}$  is the time duration from the time when the job enters machine  $M_i$  to the time when that job is ready to enter machine  $M_b$ .

If the simulation model for a production line is built, it is easy to obtain values of  $T_i^{\text{empty}}$  and  $T_i^{\text{resume}}$  from the simulation model for the MOW calculation. Moreover, we can show that this simulation-based MOW algorithm in Equation (28) agrees with the analytical results in Equation (17). By definition,  $T_i^{\text{empty}}$  and  $T_i^{\text{resume}}$  are given by

$$T_1^{\text{empty}} = \frac{\sum_{i=2}^b (N_{i-1}(0) + I_i)}{m_b}, \quad T_1^{\text{resume}} = \sum_{i=1}^{b-1} \frac{1}{m_i}.$$

Hence,  $MOW_1$  for machine  $M_1$  can be calculated as

$$MOW_1 = T_1^{\text{empty}} - T_1^{\text{resume}} = \frac{\sum_{i=2}^b (N_{i-1}(0) + I_i)}{m_b} - \sum_{i=1}^{b-1} \left( \frac{1}{m_i} \right) \quad (29)$$

which is identical with Equation (17).

#### 3.2. MOW Validation via Simulation

MOW can be validated through a simulation using the MOW definition. Since MOW is defined as the maximum time interval that will not influence the bottleneck machine despite shutting down a specific machine  $M_i$ , stopping machine  $M_i$  for shorter than its  $MOW_i$  will not induce any work loss on the bottleneck machine. On the other hand, if machine  $M_i$  is shut down for longer than its  $MOW_i$ , it will cause the bottleneck machine  $M_b$  to be either starved or blocked, resulting in a system production loss. Using this idea, the validation steps via a simulation can be summarized:

- 1) Calculate  $MOW_i$  for machine  $M_i$
- 2) Force machine  $M_i$  to be idle for a given time duration  $T$
- 3) Collect completion times for each completed job on the bottleneck machine  $M_b$  via a simulation
- 4) Repeat Steps 2 and 3 with different  $T$ s
- 5) Compare the results in Step 4 under different  $T$ s to see if the completion time difference between consecutive jobs is larger than the cycle time of machine  $M_b$

#### 3.3. Numerical Case Studies

We perform the simulation-based MOW calculations and validate them with the following two manufacturing systems. This simulation work is done using the discrete event simulation of SIMUL8.

##### 3.3.1. Case Study 1: Serial Line

Consider a system whose layout, characteristics, and initial levels are shown in Figure 11 and Table 1.



Figure 11: The layout of a serial line for case study 1

Table 1: System characteristics for a system in Figure 11

Machine	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>	M <sub>5</sub>	M <sub>6</sub>	M <sub>7</sub>
Avg. Cycle Time	60	60	60	66	60	60	60
Initial Content, $I_i$	1	1	1	1	1	1	1
Buffer	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	B <sub>6</sub>	
Capacity, $C_i$	5	5	5	5	5	5	
Initial Contents, $N(0)$	3	3	4	1	2	2	

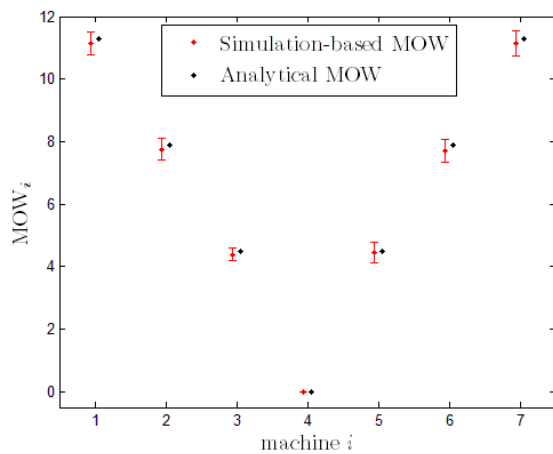
With the one hour warm-up period, the percentages of each machine's working, starvation, and blockage time are recorded via a simulation and shown in Table 2. It reveals that machine  $M_4$  operates all the time and can be regarded as the short-term bottleneck machine of this serial production line. MOWs can be calculated such that their stoppages will not influence the working time of machine  $M_4$ . MOW for each machine is shown in Table 3 and plotted in Figure 12 after running the simulation model 200 times.

**Table 2: Percentage of working/starvation/blockage time**

Machine	Working Time (%)	Starvation Time (%)	Blockage Time (%)
$M_1$	99.29( $\pm 0.63$ )	0	0.71( $\pm 0.63$ )
$M_2$	96.84( $\pm 1.47$ )	2.02( $\pm 1.36$ )	1.14( $\pm 0.82$ )
$M_3$	93.44( $\pm 2.05$ )	1.19( $\pm 1.02$ )	5.37( $\pm 2.15$ )
$M_4$	100	0	0
$M_5$	91.74( $\pm 1.57$ )	8.26( $\pm 1.57$ )	0
$M_6$	91.85( $\pm 1.97$ )	8.15( $\pm 1.97$ )	0
$M_7$	91.89( $\pm 1.71$ )	8.11( $\pm 1.71$ )	0

**Table 3: MOWs for system in Figure 11**

Machine	$M_1$	$M_2$	$M_3$	$M_4$	$M_5$	$M_6$	$M_7$
Analytical $MOW_i$	11.3	7.9	4.5	0	4.5	7.9	11.3
Simulation-based $MOW_i$ (95% C.I.)	11.14 ( $\pm 0.37$ )	7.76 ( $\pm 0.36$ )	4.39 ( $\pm 0.21$ )	0	4.46 ( $\pm 0.32$ )	7.70 ( $\pm 0.36$ )	11.15 ( $\pm 0.41$ )

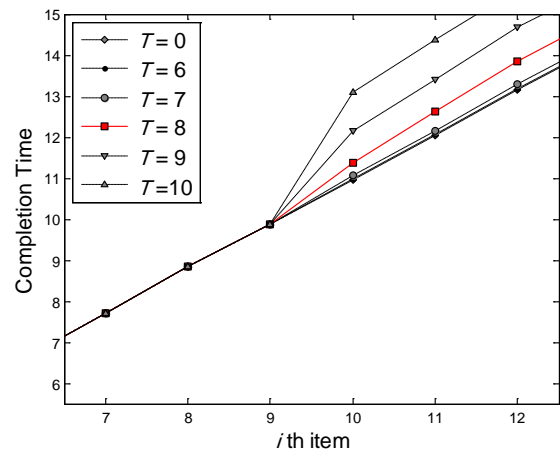


**Figure 12: MOWs for system in Figure 11**

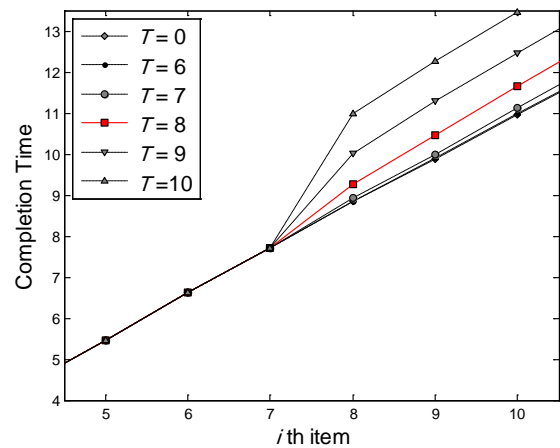
We set up the initial buffer levels  $N_i(0)$  in a way that the reversed line of  $M_4-B_4-M_5-B_5-M_6-B_6-M_7$  (equal to  $M_7-B_6^r-M_6-B_5^r-M_5-B_4^r-M_4$ ) is exactly identical with the forward line of  $M_1-B_1-M_2-B_2-M_3-B_3-M_4$ . As results, we can observe symmetric MOWs values about machine  $M_4$  as shown in Figure 12.

The validation is performed for machines  $M_2$  and  $M_6$  which are symmetric around  $M_4$ . For machine  $M_2$ , we set  $T$  as the time for which we stop machine  $M_2$ , and record the completion time of consecutive parts that go through the bottleneck machine  $M_4$  under different  $T$ s. The simulation is run 200 repetitions and the average completion time is plotted in Figure 13.

If we stop machine  $M_2$  first, there are still 9 parts remaining to be processed by machine  $M_4$  (3 parts in buffer  $B_2$ , 4 parts in buffer  $B_3$ , 2 parts in both machines  $M_3$  and  $M_4$ ). Therefore, no matter what  $T$  value is, there is no difference in completion time for the first 9 parts. However, starting from the 10th part, the discontinuity appears. As shown in Figure 13, if  $T = 6$  or 7 minutes, there are almost no differences comparing with the case of  $T = 0$  (i.e., no machine downs). On the other hand, if we shut down machine  $M_2$  for 8 minutes, the discontinuity will occur in several repetitions. It indicates that  $MOW_2$  ( $7.76 \pm 0.36$ ) lies between 7 and 8 minutes.



**Figure 13: MOW validation for machine  $M_2$**



**Figure 14: MOW validation for machine  $M_6$**

Similarly, if we shut machine  $M_6$  down, there will be 7 spaces left until machine  $M_4$  gets blocked (3 spaces in buffer  $B_5$  and 4 spaces in buffer  $B_4$ ) and the discontinuity may occur starting

from the 8th parts. Figure 14 shows the completion times under different  $T_s$  for which we shut down machine  $M_6$ , and the fact that the behavior is similar with that in Figure 13. Again, Figure 14 provides an evident that the actual  $MOW_6$  ( $7.70 \pm 0.36$ ) lies between 7 and 8 minutes.

### 3.3.2. Case Study 2: Combined System

Next, we consider a system combined with serial lines and assembly/disassembly system. The machine cycle times and buffer capacities are shown in Table 4.

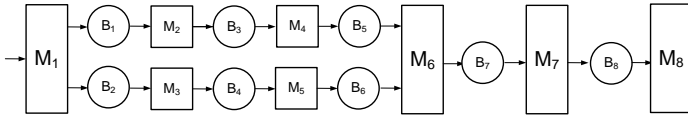


Figure 15: The layout of the system in case study 2

Table 4: Characteristics of the system

Machine	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>	M <sub>5</sub>	M <sub>6</sub>	M <sub>7</sub>	M <sub>8</sub>
Cycle Time	60	60	60	60	60	62	60	65
Buffer	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	B <sub>6</sub>	B <sub>7</sub>	B <sub>8</sub>
Capacity, C	3	3	3	3	3	3	3	3

Table 5: Percentage of working/starvation/blockage time

Machine	Working Time (%)	Starvation Time (%)	Blockage Time (%)
M <sub>1</sub>	97.31(±1.39)	0	2.69(±1.39)
M <sub>2</sub>	94.63(±1.60)	1.64(±1.23)	3.73(±1.57)
M <sub>3</sub>	96.73(±1.10)	0.57(±0.54)	2.70(±1.06)
M <sub>4</sub>	93.94(±1.99)	0.69(±0.74)	5.37(±2.02)
M <sub>5</sub>	95.30(±1.96)	1.07(±0.87)	3.63(±1.92)
M <sub>6</sub>	96.41(±1.56)	1.61(±1.18)	1.98(±1.59)
M <sub>7</sub>	93.13(±1.52)	2.84(±1.49)	4.04(±1.80)
M <sub>8</sub>	98.86(±1.16)	1.14(±1.16)	0

The starvation and blockage time for all machines during the collection period (1 hour) are provided in Table 5, showing that machine  $M_8$  is the short-term bottleneck machine of the system. For machines  $M_6$  and  $M_7$ , there is only one path to machine  $M_8$ . On the other hand, for machines  $M_1$ ,  $M_2$ ,  $M_3$ ,  $M_4$ , and  $M_5$ , there are two paths to reach machine  $M_8$ . For example, machine  $M_2$  can reach machine  $M_8$  via either a forward path of  $M_2$ - $B_3$ - $M_4$ - $B_5$ - $M_6$ - $B_7$ - $M_7$ - $B_8$ - $M_8$  or a backward of  $M_2$ - $B_1$ - $M_1$ - $B_2$ - $M_3$ - $B_4$ - $M_5$ - $B_6$ - $M_6$ - $B_7$ - $M_7$ - $B_8$ - $M_8$ .

Table 6: Initial buffer levels for system

Machine	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>	M <sub>5</sub>	M <sub>6</sub>	M <sub>7</sub>	M <sub>8</sub>
Initial Contents, $I_i$	1	1	1	1	1	0	1	1
Buffer	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	B <sub>6</sub>	B <sub>7</sub>	B <sub>8</sub>
Initial Contents, $N(0)$	3	1	2	1	2	1	2	2

Here we consider the case that the initial values of the buffers are chosen as shown in Table 6, and MOWs are calculated and displayed in Table 7 and Figure 16.  $MOW_2$  is calculated from the reversed line  $M_2$ - $B_1$ - $M_1$ - $B_2$ - $M_3$ - $B_4$ - $M_5$ - $B_6$ - $M_6$ - $B_7$ - $M_7$ - $B_8$ - $M_8$  which is a disassembly system.

Table 7: MOWs with the initial buffer level in Table 6

Machine	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>	M <sub>5</sub>	M <sub>6</sub>	M <sub>7</sub>	M <sub>8</sub>
Analytical $MOW_i$	6.90	6.99	5.75	5.67	4.58	4.5	2.25	0
Simulated $MOW_i$ (95% C.I.)	6.79 (±0.68)	6.89 (±0.94)	5.59 (±1.19)	5.53 (±0.88)	4.45 (±1.11)	4.47 (±0.88)	2.18 (±0.42)	0

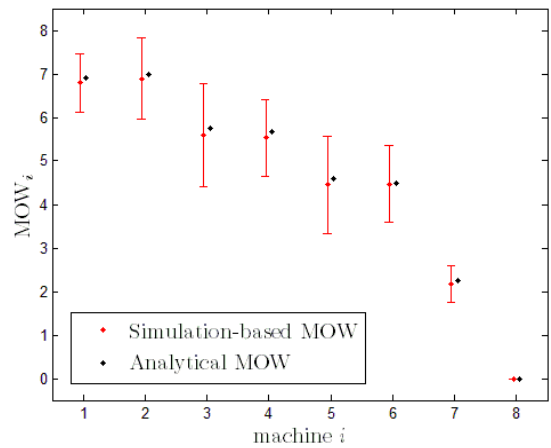


Figure 16: MOWs calculation

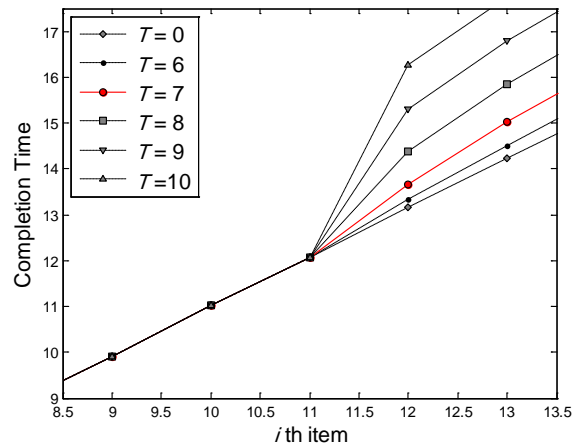


Figure 17: MOW validation for machine  $M_2$

Buffer  $B_1$  is initially full, indicating that machine  $M_1$  will be blocked until machine  $M_2$  resumes working. Hence  $MOW_1$  and



MOW<sub>2</sub> are similar with each other. Additionally, MOW<sub>3</sub> and MOW<sub>4</sub> are similar since  $N_4(0) + N_6(0) = N_5(0)$ . Moreover, when we calculate MOW<sub>5</sub>, the case of  $N_6(0) = 1$  and  $I_6 = 0$  is the same with the case of  $N_6(0) = 0$  and  $I_6 = 1$ . Therefore, MOW<sub>5</sub> and MOW<sub>6</sub> are similar as shown in Figure 16. Its validation of MOW<sub>2</sub> is performed in Figure 17, which shows that discontinuity on machine M<sub>8</sub> begins when machine M<sub>2</sub> is down for 7 minutes, evidenced the analytical and simulated MOW<sub>2</sub> ( $6.89 \pm 0.94$ ).

#### 4. CONCLUSIONS AND FUTURE WORK

In this paper, we have analyzed the MOW for serial and assembly/disassembly systems, and extended its results to a more general manufacturing system. A simulation-based MOW algorithm is studied to calculate the MOW via a simulation. Numerical case studies are conducted to demonstrate how to compute MOWs with different configurations of manufacturing systems. This MOW will provide the maximum maintenance time window before starting to lose production throughputs. Future work regarding the theoretical MOW and its application includes: 1) effective way to detect a short-term bottleneck machine of complex systems, 2) analytical models for the stochastic lines, and 3) decision-making policies for maintenance opportunities based on the real-time data and its MOW.

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