ABSTRACT
Battery applications (computer, cell phones or even in cars) have been extensively used in our daily life. The reasons for their success and extensive usage in the real world applications are their light weight, smaller sizes and greater energy densities. These unique characteristics render this class of battery an ideal candidate for powering electrical vehicles. However, due to lack of battery information, often time we will observe machine down time, operation malfunctioning, and even some catastrophic failure due to fast battery degradation and depletion. Thus, much of the attention has been focused on prognostics and health management of battery technologies for the stated purpose. In this paper, we will present two main algorithms that cannot only estimate a one-step-ahead prediction of the battery state but also can estimate the battery remaining useful life. The first method is the linear prediction error method. The second approach is the neural network algorithms. Both methods can predict the battery information accurately. However, particular algorithm specializes in different area of interest.

1. INTRODUCTION
Batteries form a core component of many machines and are often times critical to the well being and functional capabilities of the overall system. Failure of a battery could lead to reduced performance, operational impairment and even catastrophic failure, especially in aerospace systems. The ability to quickly evaluate the remaining charge in a fielded battery and to reliably identify batteries that have reduced capacity with the potential for failure are the hallmarks of battery prognostics applications. Modern war fighters depend heavily on the availability of reliable battery power for their communications, night vision, fire control, and perimeter security systems. Any uncertainties concerning the present energy capacity or the state of charge in a battery adds to the risk of any missions. The uncertainty of a battery's state-of-charge (SOC), its state-of-health (SOH), or its remaining useful life (RUL) can lead to premature disposal by the war fighter, causing a cost and a supply logistics nightmare.

The phrase “battery health monitoring” has a wide variety of connotations, ranging from intermittent manual measurements of voltage and electrolyte specific gravity to fully automated online supervision of various measured and estimated battery parameters. Researchers have looked at the various failure modes of the battery subsystems. Different diagnostic methods have been evaluated, like discharge to a fixed cut-off voltage, open circuit voltage, voltage under load and electrochemical impedance spectrometry (EIS) and combining conductance technology with other measured parameters like battery temperature/differential information and the amount of float charge.

While the subject of electric vehicles continues to grab the headlines around the world with the introduction of more and more popular electric cars it seems that many people have forgotten about electric vehicle batteries and the fact that the technology to power these cars would appear on the surface to be way behind the actual car technology. Electric vehicle batteries are nowhere near the finished article but thankfully there has been significant progress over the last few years and there is hope that the catch-up between car technology and electric car battery technology is gathering pace.

Lithium ion batteries are by far and away the most popular in the worldwide today as they are commonplace in consumer electronics such as laptops and mobile phones. The weight ratio
and the number of batteries required compared to the overall weight of the electric vehicle today is very favorable and the fact they will discharge around 80% to 90% of their power before they need to be recharged is also useful. We have seen developments within the lithium ion battery sector and many believe this is where the future of electric vehicle power lies. However, at this point in time there are issues with regards to the short number of cycles which any lithium ion battery can accommodate before it needs to be replaced. This significant reduction in efficiency in a relatively small space of time has led to increased cost issues with regards to the long-term running of any electric vehicle.

This paper presents a novel battery health management system based on two prognostic frameworks. The aim is to be able to predict the one-step-ahead prediction that indicates that the state of battery at some future arbitrary point so drivers or others can detect the state of batteries for any given journey. This paper explores the techniques that fit the context of predicting remaining useful Li-ion battery life for electrical vehicles. These techniques are also applicable to other batteries as well.

2. METHODS
This section will outline two main techniques that have been analyzed for future battery capacity data prediction. These two methods are linear prediction error method and neural network respectively. We will discuss each technique theoretically orderly in the following.

Method 1: Linear Prediction Error Methods
Prediction Error Method (PEM) is a broad family of parameter estimation methods that can be applied to quite arbitrary model parameterizations [1]. The advantage of this method is that it can be applied to a wide spectrum of model parameterizations.

For example, given \( x(t-1), x(t-2), \ldots, x(t-M) \), and we are asked to predict the value of \( x(t) \). In general, this predicted value can be expressed as a function of the given M past samples:

\[
\hat{x}(t | t-1, \ldots, t-M) = \Phi(x(t-1), \ldots, x(t-M))
\]  

(1)

In Equation (1), the left side of the equation denotes the one-step ahead prediction of the output, and \( \Phi \) is an arbitrary function of past and observed data. If we assume \( \Phi \) is a linear function, we can say that the prediction is a linear PEM. And this can be expressed in the following:

\[
\hat{x}(t | t-1, \ldots, t-M) = \sum_{k=1}^{M} \theta_k x(t-k)
\]  

(2)

We are assuming that the last data set is a linear combination of the previous M ones. And \( \theta_k \) in Equation (2) is vector coefficient. Then, our goal is to find these vector coefficient \( \theta_k \).

Once the parameterized function has been defined along with the measured model data, \( \theta_k \) can be found by minimizing the distance between the predicted outputs \( \hat{x}(t) \) and the measured outputs \( x(t) \). In Equation (3), everything inside the summation is called a distance measure. Equation (3) gives the result of the maximum likelihood estimate.

\[
\theta_M = \arg \min_{\theta} \sum_{k=1}^{M} ||\hat{x}(t-k) - x(t-k)||
\]  

(3)

The family of PEMs has the advantage of being applicable to a wide variety of model structures so we want to apply this method to establish a linear model for the battery health prognostic and predict RUL from its model since a good understanding of battery performance degradation can aid immensely in improving user satisfaction and overall reliability.

If you have a data set that represents the system behavior containing a large number of data with data size \( n \), then window width could be understood as a specific data range or subset of the vector that has the size of \( \omega \) such that \( \omega \) is usually less than or equal to \( n \). The reason why we need a window width range is that we first assumed the model behaves linearly. In order to predict the system behavior at time \( t \), we would use the assumption that the data at time \( t \) has linear combination relationship with its past data. In reality, the system would not be linear. However, if we choose a system window width at a proper range, then each subsystem with specific window range could be treated as a linear system. By selecting different spectrum of the data set with this specific window width, we will then be able to analyze a series of window width prediction along the entire spectrum of the data set. Thus we call it moving window width.

It is possible to predict the future behavior of the time sequence of the battery degradation features, such as impedance and internal resistance, using the estimated stochastic dynamic model from Equation (2). The goal of the prediction is to determine the distribution of the times that the feature value will exceed the critical value. In other words, we want to predict the distribution of the RUL of battery by calculating the distribution of time when the system reaches a critical state. This distribution of the RUL is estimated based on the first passage time (FPT) of the selected feature value. The FPT of a random process is determined as the time at which a random process reaches a specific value for the first time. In this paper, the distribution of the FPT is calculated through a Monte Carlo simulation of the stochastic model in the following way. At time \( t \), at which we are making a prediction, the simulation starts with the state vector value \( x(t) \). The initial value is then propagated through the stochastic model, with a certain realization of the stochastic processes in the model. This is repeated for as long as the system output reaches the critical value. If the simulation is repeated many times, the distribution of the times at which the simulated output reaches the critical value can approximate the distribution of the RUL in terms of the FPT.
The purpose of PEMs mentioned previously is to construct the model structure for the system. We then utilize this model structure to further predict what will happen to the model system behavior after time \( t \). If we would have possibly predicted the system future behavior, we then could prevent potential failure of malfunctioning of the device due to low battery health. Now the question is how does our prediction work? The prediction technique uses past data set of the system and gives prediction of the present or later time assuming that the predict data point is a linear combination of the previous. Suppose we have a data set ‘\( D \)’ containing 50 data ranging “\( d_1,d_2,\ldots,d_{50} \)” and what prediction can do is that it utilizes the past data points from \( d_1,d_2,\ldots,d_{50} \) assuming they are linear and calculate the potential predicted value of \( d_{50} \). And the accuracy of the prediction depends on many things. But one can clearly tell that it definitely depends on the window width range that’s been selected. Based on our intuition, a large window width will result in inaccuracy since the system behavior for such a long time period may not be linear, whereas a narrower window width can give you relatively accurate result since the system behavior in a short period is approximately linear.

![Figure 1: Process Chart](image)

We provide the underlying theory behind the PEM, which is the “Kalman filter.” A Kalman filter is simply an optimal recursive data processing algorithm [2-4]. It incorporates all information given and processes all available measurements to estimate the current value of the variables of interest, with use of

1) Knowledge of the system and measurement device dynamics,
2) Statistical description of the system noise, measurement errors, and uncertainty in the dynamic models, and
3) Any available information about initial conditions of the variables of interest.

Kalman filter can only be shown to be the best filter of any form under the major assumptions/restrictions: system can be described through a linear model and measurement noises are white and Gaussian.

A linear system can be usually modeled as:

\[
y = Ax + v
\]

where \( x \) and \( v \) are independent, \( x \sim N(\bar{x}, \Sigma_x): \) prior distribution of \( x \), \( v \sim N(\bar{v}, \Sigma_v): \) (noise bias, noise covariance).

Then we can represent Equation (4) in the state space form with its mean and variance:

\[
\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} I & 0 \\ A & I \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix}
\]

\[
\begin{bmatrix} x \\ y \end{bmatrix} \sim N\left( \begin{bmatrix} \bar{x} \\ A\bar{x} + \bar{v} \end{bmatrix}, \begin{bmatrix} \Sigma_x & \Sigma_x A^T \\ A\Sigma_x & A\Sigma_x A^T + \Sigma_v \end{bmatrix} \right)
\]

Our goal is to predict values of interest at time \( t \), so we will rewrite our linear model by the stochastic process. This could be described as follows:

\[
x_{t+1} = Ax_t + Bu_t
\]

By using the same methodology used in Equation (6), we obtained the mean and variance:

\[
\begin{bmatrix} \bar{x}_{t+1} \\ \Sigma_{x_{t+1}} \end{bmatrix} = A\bar{x}_t + B\bar{u}_t
\]

\[
\Sigma_x(t+1) = A\Sigma_x(t)A^T + B\Sigma_u(t)B^T
\]

Note that in Equation (9), we assume that the covariance is 0 since \( x \) and \( u \) are independent. Equations (8) and (9) tell that by finding the mean and variance in Equation (6) of a linear dynamical system at time \( t \), we will be able to find the mean and variance at time \( t+1 \). So far in our analysis, we have not mentioned our error distribution in our linear system. And error terms are definitely important in real physical applications. This allows us to introduce the “Gauss-Markov” theory into our linear model. The Gauss-Markov linear statistical model satisfies all conditions of general linear model except for the normality of error terms. Let’s consider a linear dynamical system:

\[
x_{t+1} = Ax_t + \omega_t
\]

\[
y_{t+1} = Cx_t + v_t
\]

where \( x_t \) is the state, \( y_{t+1} \) is the observed output, \( \omega_t \) is the process noise or state noise, and \( v_t \) is the measured noise.

Using the Markov assumptions, we have:

\[
x_t : \text{Markov properties for the process,}
\]

\[
P\{x_t \mid x_{t-1}, \ldots, x_1\} = P\{x_t \mid x_{t-1}\}
\]

\[
\omega_t : \text{Independent and identically distributed,}
\]

\[
E[\omega_t] = 0, \quad \sum \omega_t = W
\]

\[
v_t : \text{Independent and identically distributed,}
\]

\[
E[v_t] = 0, \quad \sum v_t = V
\]

With Markov assumptions, we are able to model our system mean and variance as:

\[
\bar{x}_{t+1} = A\bar{x}_t
\]

\[
\Sigma_x(t+1) = A\Sigma_x(t)A^T + W
\]

So far we are able to calculate the state using the stochastic process. It is great yet accurate. For a real system, often times, we know the states of the system and also known measurements read from either the sensors or radars etc. Thus intuitively, we knew that given those output data measurements at a previous time would definitely help improve prediction accuracy of the current system values of interest. So we start analyzing the
mean and variance of the state variables given the conditions of previous output measurements. This could be modeled as:

\[
\hat{x}_{ts} = E(x_t \mid y_1, \ldots, y_s)
\]

(14)

\[
\Sigma_{ts} = E((x_t - \hat{x}_{ts})(x_t - \hat{x}_{ts})^T)
\]

(15)

where \(\hat{x}_{ts}\) is the estimated mean of the state variable \(x\) at time \(t\) given the past outputs up till \(s\), and \(\Sigma_{ts}\) is the covariance of the error of the estimate \(\hat{x}_{ts}\).

Since \(x_t, Y_t\) are jointly Gaussian distributions, we can use the standard formula to find \(\hat{x}_{ts}\):

\[
\hat{x}_{ts} = E[X \mid Y] = \bar{x}_t + \Sigma_{s,t}\Sigma_{t}^{-1}(Y_t - \bar{Y}_t)
\]

(16)

where \(\bar{x}_t\) is the system mean at time \(t\), \(\Sigma_{t}\) is the error of the measurements, \(\Sigma_{s,t}\Sigma_{t}^{-1}\) is the system gain, or coefficients, which is also known as the Kalman filter, usually called “K.”

**Method 2: Neural Networks Method**

A neural network is inspired by the structure and/or functional aspects of biological neural networks [5]. The neural networks approach is non-linear statistical data modeling tools. They are usually used to model complex relationships between inputs and outputs or to find patterns in data. This input-output relationship is shown in the figure below. Between the inputs and the outputs, there are layers called hidden layers. There may occur several hidden layers. The input layer distributes the values to each of the neurons in the hidden layer and the value from each input neuron is multiplied by a weight and the resulting weighted values are added together producing a combined value. The weighted sum is fed into a transfer function, which outputs a value. The outputs from the hidden layer are finally distributed to the output layer. The mathematical weights assignment to the inputs can be modeled in Equation (23).

\[
a = x_1\omega_1 + x_2\omega_2 + \cdots + x_n\omega_n
\]

(23)

**Figure 2: General Structure of Neural networks**

In our diagnostics of battery health management, neural networks method plays an important role in estimating and predicting the battery capacitance [6-10]. For example, our capacitance data is given from \(C_1\) to \(C_m\). By selecting the window size of 5 data points, in this case, as an appropriate linear approximation, we could have summarized our logic thinking in the following table. We will have our input variables and we will use neural networks to train our input data to give an estimate of our output data and then compare the estimated results with our accurate measured target values. The difference between the comparisons is called the mean square error (MSE). This term represents the performance of the estimation. The lower the error, the better the prediction.

<table>
<thead>
<tr>
<th>(C_1)</th>
<th>(C_2)</th>
<th>(C_3)</th>
<th>(C_4)</th>
<th>(C_{m-5})</th>
<th>(C_{m-4})</th>
<th>(C_{m-3})</th>
<th>(C_{m-2})</th>
<th>(C_{m-1})</th>
<th>(\cdots)</th>
<th>(C_m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>INPUT Values</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(C_6)</td>
<td>(C_{11})</td>
<td>(C_{16})</td>
<td>(\cdots)</td>
<td>(C_{m})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TARGET Values</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

**3. RESULTS**

The data used in this paper is for a set of Li-ion batteries tested at the Idaho National Lab and available online for public download on the NASA Prognostic Center of Excellence Data Repository (http://ti.arc.nasa.gov/tech/dash/pcoe/prognostic-data-repository/) [11]. A set of four Li-ion batteries (B5, B6, B7 and B18) were run through three different operational profiles (charge, discharge and impedance) at room temperature. Battery cell B18 will not be used as the data acquired is too limited and insufficient for analysis.

Charging was carried out in a constant current mode at 1.5A until the battery voltage reached 4.2V and then continued in a constant voltage mode until the charge current dropped to 20mA. Discharge was carried out at a constant current level of 2A until the battery voltage fell to 2.7V, 2.5V and 2.2V for batteries B5, B6, B7, and B18, respectively. Impedance measurement was carried out through an electrochemical impedance spectroscopy (EIS) frequency sweep from 0.1Hz to 5 kHz. Repeated charge and discharge cycles result in accelerated aging of the batteries while impedance measurements provide insight into the internal battery parameters that change as aging progresses. The experiments were stopped when the batteries reached end-of-life (EOL) criteria, which was a 30% fade in rated capacity (from 2 Ahr to 1.4 Ahr).

This dataset is used in this paper for two purposes: (1) the estimation of battery health condition, State of Health (SoH) for a given discharge cycle, and (2) Remaining Useful Life (RUL) of the battery in terms of capacity.

**3.1. One-step-ahead prediction**

Often times, we would like to know the battery short-term behaviors at near future time. Thus our one-step-ahead prediction is advantageous in achieving this goal. Figures 3 and 4 capture both PEMs and NNs methods for this short-term prediction. By just inspection, we could only say both methods seem accurate in one-step-ahead prediction after certain training.
cycles. But if we take a look at Table 1 quantitatively, we will soon notice that NNs provides slightly better accuracy on one-step-ahead prediction in given data sets. As can be seen in the table, the accuracy using NNs for all three batteries are significantly less than using the PEM method. Notice that all the data presented in Table 1 are all in terms of the mean square error (MSE).

<table>
<thead>
<tr>
<th>Battery</th>
<th>MSE (NN)</th>
<th>MSE (PEM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0005</td>
<td>$1.003 \times 10^{-4}$</td>
<td>$1.764 \times 10^{-4}$</td>
</tr>
<tr>
<td>B0006</td>
<td>$1.454 \times 10^{-4}$</td>
<td>$2.692 \times 10^{-4}$</td>
</tr>
<tr>
<td>B0007</td>
<td>$6.649 \times 10^{-5}$</td>
<td>$1.202 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

3.2. Battery Remaining Useful Life Prediction

One-step-ahead prediction is useful in short-term prediction. However, people care more about how much time this battery could last from now on until its functionality is totally lost [12]. Thus, much of the attention has been focused on a remaining useful battery life since it provides insightful meaning of decision makings. This RUL prediction could potentially prevent machine breakdown, operation impairment and even some catastrophic failure of machines or structures. In this section, we will use the estimated stochastic dynamic model to determine the distribution of the times that the capacity value will cross the critical value. This random process can be determined when a random process reaches a specific value for the first time and we will use a Monte Carlo simulation of the stochastic model to predict the RUL by knowing the initial predicting time and its state vector $x(t)$ and let the initial value propagated through the stochastic model, with a certain realization of the stochastic processes in the model. This is repeated for as long as the system output reaches the critical value. Figure 5 illustrates the approximated probability density function of RUL after the 324th cycles from the histogram.

As you can see from Figure 5, the cut-off dotted line is at 324th cycle, and the FPT distributions are shown on top of the graph. The MC and FPT techniques are run for at least 10 iterations and the red lines are the predicted trend. Later in this section, a comparative RUL prediction study is presented using PEMs and NNs on the B0005 experimental battery data as shown in Figures 6 and 7. We have run repeated tests at 236th, 324th, and 371st cycles respectively. As the battery usage cycles reaches End of Life (EOL), the prediction accuracy increases and prediction variance gradually decreases in both PEMs and NNs. By inspecting Figures 6 and 7 and several quantitative data shown in Table 2, three conclusions could be made out of those information. First, it can be observed that the error of PEM is less than that of NNs given lower training cycles. Secondly as we increase number of training cycles, neural network uncertainty becomes much lower than PEM with narrower prediction bounds (ex. Figure 6(c)). Lastly, by overall...
Looking, adNNs gives an overall high MSE value compared to PEMs except for the 324th cycle and this resulting fluctuation may be attributed to the weight adjustment of the neural network as it initializes with random number and the weight adjustment technique needs to be further developed.

Table 2: Benchmark NNs and PEMs in terms of MSE

<table>
<thead>
<tr>
<th>Battery</th>
<th>MSE (NN)</th>
<th>MSE (PEM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>After 236th Cycle</td>
<td>1.221925</td>
<td>0.010442</td>
</tr>
<tr>
<td>After 324th Cycle</td>
<td>0.004287</td>
<td>0.006024</td>
</tr>
<tr>
<td>After 371st Cycle</td>
<td>0.009941</td>
<td>0.003564</td>
</tr>
</tbody>
</table>

Figure 6: RUL Prediction using NN Model

4. Conclusions and Recommendations

In this paper, we have successfully implemented two prognostic methods in predicting battery future behavior. Both linear prediction error method and neural networks approach work well in predicting the future behavior of battery capacitance. However, as we discussed in our result section, the neural network approach is much preferred for short term (one-step-ahead) prediction since it requires less training data. And the linear prediction method is more useful in terms of knowing the battery remaining useful life. For future study, we could actually expand the analysis from a single cell to a whole electric vehicle.
battery pack in order to observe whether the algorithms are scalable for deployment in real-time operational settings. Also we could explore more algorithms that fit the areas of study. In such a way, we will open to more opportunities in exploring what algorithm is the best for the needs.

ACKNOWLEDGEMENTS
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