Math 215 Winter 2022
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Syllabus on the Canvas page.

12.1. Coordinate systems

Course theme: We study multivariable functions and 3-dimensional coordinate systems using our knowledge on single-variable calculus and 2-dimensional coordinate systems.

Def The standard (rectangular) coordinate systems are defined by perpendicular axes.

\[ \begin{align*}
\text{2-dim' system } & \mathbb{R}^2 \\
\text{3-dim' system } & \mathbb{R}^3
\end{align*} \]
**Thm (Distance formula) = Pythagorean theorem**

1. **Distance between \((x_1, y_1)\) and \((x_2, y_2)\) on \(\mathbb{R}^2\)**
   \[
   \text{is } \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}
   \]

2. **Distance between \((x_1, y_1, z_1)\) and \((x_2, y_2, z_2)\) on \(\mathbb{R}^3\)**
   \[
   \text{is } \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}
   \]

**Ex (1) Find an equation of the circle in \(\mathbb{R}^2\)**

- **of radius** \(r\) **and center** \((a, b)\).

**Sol**

- **radius = distance from center**
  \[
  r = \sqrt{(x-a)^2 + (y-b)^2}
  \]

- **Eq:** \(r^2 = (x-a)^2 + (y-b)^2\)

- **(2) Find an equation of the sphere in \(\mathbb{R}^3\)**
  - **of radius** \(r\) **and center** \((a, b, c)\).

**Sol**

- **radius = distance from center**
  \[
  r = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}
  \]

- **Eq:** \(r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2\)
**Ex** Sketch the graph of \( z = x^2 + y^2 \)

**Sol**

Tips for sketching graphs:

1. Look at "cross sections" by setting one of the variables to be constant.
2. It's often convenient to put the output variable on the vertical axis.

Output variable in this case is \( z \)

\( z \) is given by a function of \( x \) and \( y \)

Set \( x = 0 \) : \( z = y^2 \) \( \sim \) a parabola in the \( yz \)-plane

Set \( y = 0 \) : \( 0 = x^2 + y^2 \) \( \sim \) \( x = y = 0 \) \( \sim \) a point.

Set \( z = 1 \) : \( 1 = x^2 + y^2 \) \( \sim \) a circle of radius 1.

Set \( z = 4 \) : \( 4 = x^2 + y^2 \) \( \sim \) a circle of radius 2.

* This surface is called a (circular) paraboloid.
(1) Sketch the surface \( x^2 + y^2 = 9 \).

**Sol**
- \( z = 0 : x^2 + y^2 = 9 \) is a circle of radius 3.
- \( z = 1 : x^2 + y^2 = 9 \) is another circle of radius 3.

![Graph](image)

This surface is a circular cylinder of radius 3 along the \( z \)-axis.

(2) Describe the region \( x^2 + y^2 \leq 9 \).

**Sol**
- For each \( z \), the cross section is given by \( x^2 + y^2 \leq 9 \), which represents the disk of radius 3 and center \((0,0)\).
- \( x^2 + y^2 \leq 9 \) implies \( \sqrt{x^2 + y^2} \leq 3 \).
- \( \Rightarrow \) distance from \((0,0)\) is at most 3.

So the region \( x^2 + y^2 \leq 9 \) in \( \mathbb{R}^3 \) is the solid cylinder of radius 3 along the \( z \)-axis (surface + inside).