

Review Sheet for 2nd Midterm Exam – Part II

1. True or False. Determine whether the following statements are true or false and provide an explanation for your answer (5 points each).

(a) If a particle's trajectory is described by the vector curve $\mathbf{r}(t)$ satisfying $|\mathbf{r}'(t)| = c$, for all values of t , where c is a constant, then its acceleration vector satisfies, $\mathbf{a}(t) = \mathbf{0}$.

(b) If the position vector $\mathbf{r}(t)$ of a moving particle satisfies $|\mathbf{r}(t)| = c$ for all values of t , where c is constant, then the particle path is a straight line.

(c) If a function $f(x, y)$ satisfies $f(0, 0) = \frac{1}{2}$ and

$$\lim_{(x,y) \rightarrow (0,0)} f(x, y) = \frac{1}{2}$$

along the paths $L_1 : y = x$ and $L_2 : y = x^2$, then the function is continuous at $(0, 0)$.

(d) The function $f(x, y) = \frac{\cos x}{x^2 + y^2 + 2}$ is continuous everywhere in \mathbb{R}^2 .

(e) The function $f(x, y) = \frac{x^4 + 2y^2 + 3}{x^2 + y^2 - 1}$ is continuous everywhere in \mathbb{R}^2 .

(f) The function

$$f(x, y) = \begin{cases} \frac{x^2 + \sin^2 y}{x^2 + 2y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

is continuous at $(0, 0)$.

(g)

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dy \, dx = 4\pi$$

(h)

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{4-x^2-y^2} \, dy \, dx = \frac{32}{3}\pi$$

2. (15 points) Consider the solid bounded by the elliptic paraboloid $z = 2x^2 + y^2$ and the plane $z = 4$. Find its volume.

3. (10 points) Consider the region D of the xy -plane bounded by the curves $x = 2$, $y = 0$, and $y = x^2$. (a) Sketch D , and express the double integral

$$\iint_D f(x, y) \, dA$$

as an iterated integral of the form: (before you proceed with (b) and (c), read the note on the back)

(b) $\int_{x_1}^{x_2} \int_{y_1}^{y_2} f(x, y) \, dy \, dx$

(c) $\int_{y_1}^{y_2} \int_{x_1}^{x_2} f(x, y) dx dy$

CAUTION: the limits of integration x_1 , x_2 , y_1 , and y_2 may or may not be constants.

4. (10 points) Find the center of mass of a thin plate whose shape is the region of the xy -plane bounded by the curves $y = 2 - \sqrt{4 - x^2}$ and $y = \sqrt{4 - (x - 2)^2}$ and has density $\rho(x, y) = k(x^2 + y^2)$

5. (15 points) Evaluate the integrals:

(a)

$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^1 dz dy dx.$$

(b)

$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^1 (x^2 + y^2)^{3/2} dz dy dx.$$