MATH 255 **Applied Honors Calculus III** **Winter 2008**

Review Sheet for 2nd Midterm Exam – Part I

1. True or False. Determine whether the following statements are true or false and provide an explanation for your answer (5 points each).

(a) If a particle's trajectory is described by the vector curve $\mathbf{r}(t) = \langle 1.5\cos t, 1.5\sin t\rangle$, then its acceletation vector satisfies, $\mathbf{a}(t) = \mathbf{0}$.

(b) If a vector $\mathbf{r}(t)$ satisfies $|\mathbf{r}(t)| = c$ for all values of t, where c is constant, then $\mathbf{r}(\mathbf{t})$ and $\mathbf{r}'(t)$ are orthogonal.

(c) If the maximum value of the function $f(x, y)$ along the line $L : ax + by = c$ occurs at the point (x_0, y_0), then $\nabla f(x_0, y_0)$ is perpendicular to L.

(d) The region D for which the value of the double integral

$$\int\int_D (1 - x^2 - y^2)\ dA$$

is maximized is the unit circle

(e) The function $u(x, t) = e^{(x-t)^2}$ satisfies the partial differential equation $u_t + u_x = 0$

(f) The function $f(x, y) = xe^{-y}$ is a joint probability function of two random variables taking values in the semi-infinite rectangle $[0, 1] \times [0, \infty)$.

2. (10 points) Evaluate the integral

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1 + x^2 + y^2} dx\ dy$$

3. (10 points)
(a) Find the equation of the level curves

$$f(x, y) = k,$$

of the function $f(x, y)$ whose gradient at the point (x, y) is equal to $\langle 2x, 2y\rangle$.

(b) Sketch the level curves.

(c) For what values of k do the level curves $f(x, y) = k$ exist?

4. (10 points) Evaluatge the integral

$$\int_0^1 \int_x^1 e^{x/y} dy\ dx$$

5. (15 points) The surface of a volcanic mountain range is given by

$$z = f(x, y) = \begin{cases} 40 - x^2y^2 & \text{if } x^2y^2 < 40, \\ 0 & \text{if } x^2y^2 \geq 40 \end{cases}$$

If lava flows from an orifice located at the point $(1, 2, 36)$, then, assuming it travels down the range always flowing in the direction of steepest descent, find the coordinates $(x, y, 0)$ of the point where the lava first reaches the ground level.

6. (10 points) Let

$$w(r, s) = f(x(r, s), y(r, s), z(r, s)),$$

where

$$x(r, s) = \frac{r}{s}, \quad y(r, s) = s - r, \quad z(r, s) = s^2.$$

Use the chain rule to estimate the value of $w(2.1, 0.9)$ given that

$$f(2, -1, 1) = 5 \quad \text{and} \quad \nabla f(2, -1, 1) = (1, 2, 3).$$