

OPTIMAL TAXATION WITH VOLATILITY
A THEORETICAL AND EMPIRICAL DECOMPOSITION
JOB MARKET PAPER

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ABSTRACT. I demonstrate U.S. state tax revenue volatility increased by 500 percent in the 2000s relative to previous decades. This increased volatility magnified U.S. state budget crises because of their inability to smooth volatile revenue streams due to self-imposed balanced budget rules. The theoretical model in this paper demonstrates three possible causes of the increase in volatility: changes in tax rates (which change the tax bases states rely on), economic conditions, or tax bases (e.g., what types of consumption are taxable). I find that changes in tax rates alone explain seventy percent of the increase in tax revenue volatility, despite important tax base changes, such as the rise of e-commerce, and amplified business cycles in the 2000s. Motivated by this result, I create a normative model of taxation to consider how governments should tax different volatile tax bases. Typically, taxes are set optimally by minimizing deadweight loss, which in a special case reduces to setting tax rates that are inversely proportional to their elasticities of demand. This model demonstrates that when tax bases are volatile the optimally set tax rates must consider costs from volatile tax revenues (e.g. costs from volatile public consumption) and trade these costs off with deadweight loss. I estimate this optimal condition and find thirty-six states in 2005 relied too heavily on either the income or sales tax, up from twenty-six states in 1965. This increase in inefficiency is due to an increased reliance on the income tax; twelve more states relied too heavily on the income tax in 2005 than did in 1965. This paper finds strong evidence the increase in tax-revenue volatility state governments recently experienced is due to changes in the tax bases on which governments rely, causing states to expose their revenues to unnecessary levels of risk.

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1. INTRODUCTION

Governments around the world are experiencing budget crises. The severity of these budget crises may be magnified by the recent increase in the volatility of tax revenue. For example, U.S. state governments experienced a 500 percent increase in volatility in the 2000s relative to previous decades, according to my metrics explained below. With limited opportunities to borrow, often by statute, sudden dramatic declines in revenues have caused state governments to make large cuts in expenditures.¹ Year-over-year shocks to U.S. state tax revenues increased by nearly \$20 billion in the 2000s. The increase in uncertainty in state finances led to increased uncertainty in the economy and increased uncertainty of individuals' tax payments.² Not only has tax revenue volatility increased dramatically in the last decade but its negative impact has also increased for governments around the world because the cost of borrowing has increased, especially for countries in Europe. This paper analyzes the increase in tax revenue volatility experienced by U.S. state governments and the policy mechanisms that exist to stabilize tax revenues.

The empirical analysis quantifies the three possible causes of the increased volatility identified by the theoretical model: changes in tax rates (which change the tax bases states rely on), economic conditions, or tax bases (e.g., what types of consumption are taxable). I collect data on tax rates and economic conditions from numerous sources to create a panel of all fifty states from 1951-2010. I adapt empirical decomposition methods by Oaxaca (1973), Blinder (1973), and DiNardo, Fortin, and Lemieux (1996) to quantify the contribution of changes in economic conditions, tax rates, and tax bases in explaining the increase in tax revenue volatility. These methods, appropriately adapted, allow me to quantify tax base changes, which are otherwise nearly impossible to quantify because they are nearly impossible to observe completely.³ I find that changes in tax rates alone explain seventy percent of the increase in tax revenue volatility, despite important tax base changes, such as the rise of e-commerce, and amplified business cycles in the 2000s. This means the increase in

¹State governments' inability to smooth volatile revenue is a consequence of self-imposed balanced budget rules which 49 states impose explicitly, though with varying strictness.

²The second of four maxims described by Adam Smith with regards to taxation is certainty. He claims, "The certainty of what each individual ought to pay is, in taxation, a matter of so great importance, that a very considerable degree of inequality, it appears, I believe, from the experience of all nations, is not near so great an evil as a very small degree of uncertainty." *The Wealth of Nations* p. 778.

³The change I quantify as tax base changes is the structural change or "treatment effect" in the empirical decomposition.

tax revenue volatility can be reversed by appropriate tax reform, motivating the question, “how should governments set tax rates when the volatility of tax revenue is considered.”

I develop a normative model of taxation to determine the optimal tax policy when economic production, tax revenues, and therefore public and private consumption are volatile. Standard optimal taxation models consider deterministic economic environments. In these environments, lump sum taxation is optimal because it eliminates deadweight loss. Remarkably, introducing uncertainty about tax revenue collections - a salient feature of the real-world decision facing state governments - can overturn this result. I show that a government facing volatile economic conditions from aggregate production risk should choose to tax state-dependent tax bases instead of using lump sum taxes. By taxing state-dependent tax bases the government is able to distribute the aggregate production risk between public and private consumption. Lump sum taxes are suboptimal because they concentrate all economic volatility in private consumption. I derive the volatility-adjusted Ramsey rule which characterizes the optimal tax policy with volatile economic conditions. This optimality condition nests the traditional Ramsey rule and generalizes it to account for uncertainty.

Empirically, the tax bases state governments rely on have changed over the last 60 years, with an increased reliance on income taxes. For example, between 1952 and 2008 the reliance on income tax increased from 5 percent to 23 percent as a percentage of total state and local tax revenue. In comparison, the reliance on the sales tax remained steady, accounting for 33 percent in 1952 and 34 percent in 2008.⁴ To determine if the empirical shift toward the income tax is optimal I estimate a sufficient condition derived from the volatility-adjusted Ramsey rule. I find twenty-six states relied too heavily on the income tax in 2005, an increase of twelve states from 1965. In contrast, only ten states relied too heavily on the sales tax in 2005, a decrease of two states from 1965. In total thirty-six states in 2005 exposed their tax revenues to unnecessary levels of risk by inefficiently relying on the income or sales tax.

This paper makes four contributions to the literature. First, I document a large increase in volatility in tax revenue at the state level. Second, I show this increased tax revenue volatility is

⁴This paper focuses on the income, sales, and corporate taxes because they are the main tax bases relied on by state and local governments. Property taxes are also important, but data for these tax rates do not exist because the property tax is typically administered at a local level. The reliance on the property tax decreased from 45 percent in 1952 to 31 percent in 2008.

mostly due to changes in tax rates. Third, I derive a novel condition for optimal tax policy, and fourth, I test whether states meet this condition. Through these contributions, this paper finds strong evidence that the 500 percent increase in tax revenue volatility state government's recently experienced is due to changes in tax rates, causing states to expose their revenues to unnecessary levels of risk.

2. LITERATURE REVIEW

Two recent papers discuss the dramatic increase in tax revenue volatility in the 2000s at the state level. Boyd and Dadayan (2009), discussing this fact, claim "Tax revenue is highly related to economic growth, but there also is significant volatility in tax revenue that is not explained solely by one broad measure of the economy." They conclude by quoting the National Conference of State Legislatures (NCSL) on the fiscal situation at the state level, "The fiscal challenges are enormous, widespread and, unfortunately, far from over."⁵ McGranahan and Mattoon's 2012 research lead them to conclude, "State governments are facing a period of fiscal turbulence," and suggest understanding the dynamics of state tax revenue collections is imperative to keeping the boat from capsizing.

This paper offers a structural framework to analyze the increase in shocks to uncertainty (2nd moments) of tax revenue volatility. Bloom's 2009 paper studies the impact to the business cycle of shocks to uncertainty due to events such as the Cuban missile crisis, the assassination of JFK, the OPEC I oil price shock, and the 9/11 terrorist attacks. In Bloom's model firm's region of inaction expands with uncertainty. This causes a drop in reallocation of capital and labor from low to high productivity firms slowing productivity growth. Tax revenue provides an interesting feedback loop to these shocks of uncertainty. First, tax revenue will be affected by the productivity growth shocks through their impact on wages, profits, and consumption, depending on the factors discussed in this paper. Second, tax revenue shocks increase the uncertainty of government expenditures and tax policy leading to increased uncertainty for firms. Therefore understanding how tax policy affects the resulting magnitude of tax revenue volatility due to wage, profit, and consumption shocks is important in dampening this feedback loop.

⁵"State Budget Update: July 2009," National Conference of State Legislatures.

Empirically, this paper demonstrates that changes in tax policy caused tax revenue volatility to increase more than the underlying volatility in the economy. This empirical work extends empirical work based on Groves and Kahn (1952) paper on optimal tax portfolios. Early work focused on the short-run elasticity of different tax revenue streams with respect to personal income as a measure of variability (Wilford, 1965; Legler and Shapero, 1968; Mikesell, 1977). Later work considered growth in revenues or the long-run elasticity of different tax revenue streams with respect to personal income (Williams and Lamb, 1973; White, 1983; Fox and Campbell, 1984). Recent work has focused on improving these estimates (Dye and McGuire, 1991; Bruce, Fox, and Tuttle, 2006). In addition to extending this empirical work my paper extends Groves and Kahn's theoretical model to produce a volatility-adjusted Ramsey rule.

I demonstrate volatility is an important consideration in optimal taxation. Previous optimal taxation studies extend the basic model that minimizes aggregate deadweight loss to account for distributional considerations (Mirrlees, 1971), externalities and complementarities (Corlett and Hague, 1953; Diamond and Mirrlees, 1971; Green and Sheshinski, 1979), administrative costs and tax avoidance (Allingham and Sandmo, 1972; Yitzhaki, 1974; Andreoni, Erard, and Feinstein, 1998), and dynamic considerations (Chamley, 1986; Judd, 1985; Summers, 1981). The normative model in this paper provides an additional consideration: costs due to volatility.

My paper considers the optimal tax policy with uncertain economic conditions in contrast to the literature, which considers the optimal uncertain tax policy. Stiglitz (1982) demonstrates using random tax rates can decrease excess burden, if the excess burden for an individual as a function of revenue raised is concave. Barro (1979), making the assumption that excess burden is convex in the amount of revenue raised, demonstrates the expected value of a tax rate tomorrow should be equal to the current tax rate. Skinner (1988) demonstrates the welfare gain from removing all uncertainty about future tax policy is 0.4 percent of national income. This literature focuses on the accumulated deadweight loss occurring with uncertainty. In contrast, my paper focuses on the costs of volatility in public and private consumption as a result of tax policy and trading these costs off with the costs from deadweight loss.

3. FACTS ABOUT TAX REVENUE VOLATILITY

3.1. Data. For this paper I collect data from numerous sources to be able to cross check inconsistencies due to the timing of policy changes. Income (both top and bottom rates), corporate, and sales tax rates for all states between the years 1951-2010 are collected from the Book of States, the World Tax Database, the Advisory Commission on Intergovernmental Relations biannual report “Significant Features in Fiscal Federalism,” and the Tax Foundation. Data on tax revenues for all state and local governments for the years 1951 through 2010 are collected from the Book of States and the U.S. Census of Governments.⁶ Data on property tax rates are unavailable because the rates are typically imposed at a local level.⁷ The analysis focuses on the income, sales, and corporate tax because these are the most important revenue sources for state governments.

The aggregated state and local tax revenues are used in this paper to account for different levels of decentralization across states. Data on state level economic conditions such as state level GDP and personal income are collected from the Bureau of Economic Analysis and exist for all states in all years between 1963 and 2010. Data on state populations are collected from the U.S. Census Bureau and are used as a control.⁸

Table 1 and figure 1 demonstrates the frequency and balance of the 1108 tax rate changes across 3000 state-year observations in the sample. These changes are roughly evenly divided between the tax bases; the sales tax rate changes the fewest times (252 times) and the top income tax rate changes the most (326 times). Of these changes, 603 are tax rate increases and 505 are tax rate decreases. The tax rate changes are spread across the years in the sample such that there is a tax rate change by at least one state for each tax base in over ninety-percent of the years observed. Furthermore, in about half of the years observed, at least one state increases a given tax rate and another state decreases the same tax rate.

⁶Approximately a dozen inconsistencies between the Book of States and the U.S. Census of Governments were found. When inconsistencies were found the data from the Book of States were used, though the analysis is robust to using the other sources.

⁷Data on property tax revenue is collected. In robustness specifications property tax revenue is used as an additional control variable to account for any possible horizontal externalities between the property tax and other tax bases. In these specifications these horizontal externalities do not appear to be important.

⁸All of the estimations are done in real aggregate terms controlling for population. Instead the estimations could have been run in real per capita terms without controlling for population. The first strategy is preferred because the second unnecessarily constrains the coefficient on population in the estimates.

TABLE 1. State Tax Rate Changes

Tax Rate	Observations			Years with	
	Changes	Increases	Decreases	At Least One Change	Increase and Decrease
Sales Tax	252	214	38	56	22
Corporate Tax	272	94	178	55	33
Top Income Tax	326	165	161	57	38
Bottom Income Tax	358	130	128	56	36

Data 1951-2010 tax rates by state collected by author from Book of States, the World Tax Database, the Advisory Commission on Intergovernmental Relations biannual report and the Tax Foundation. The 1108 tax rate changes across 3000 state-year observations demonstrate the variation used in the empirical analysis.

Despite the political climate, figure 1 provides little evidence that state governments changed tax rates fewer times in the 2000s relative to other decades. The number of increases and decreases in tax rates are misleading because tax rate increases tend to be larger than tax rate decreases. For example, the bottom income tax rate is increased 130 times and decreased 128 times but the average tax rate between 1950 and 2000 is 1.46 compared with the average rate of 2.14 between 2000 and 2010. Similarly the top income tax rate increased from 4.87 to 5.30, the sales tax from 3.15 to 4.82, and the corporate rate from 5.19 to 6.60. These changes in tax rates changed the relative importance of the income and sales tax in total tax revenues. Between 1952 and 2008 reliance on income tax revenues increased from 5 percent to 24 percent in contrast to the sales tax which increased only slightly from 33 percent to 34 percent.

The empirical design in this paper groups observations into years before the increase in volatility and those after. The groups are defined by a structural break found using a Quandt likelihood ratio (QLR) test (Quandt, 1958). Formally, the QLR, or sup-Wald, test statistic identifies structural breaks without presupposing in which year they occurred by performing repeated Chow tests, typically on all dates in the inner seventy-percent.^{9,10} The maximum QLR occurs in 2002 for the sales and corporate tax revenues and in 2000 for the income tax revenue. For all three tax revenues, the maximum QLR value (12.26 corporate, 17.78 sales, and 31.09 income) are larger than the critical

⁹Figure 10 in the appendix plots the QLR for the income, sales, and corporate tax revenues for all years between 1970 and 2003.

¹⁰The inner seventy-percent of years correspond to the years between 1970 and 2003 which is the suggested amount of observations for the QLR test.

FIGURE 1. Tax Rate Changes



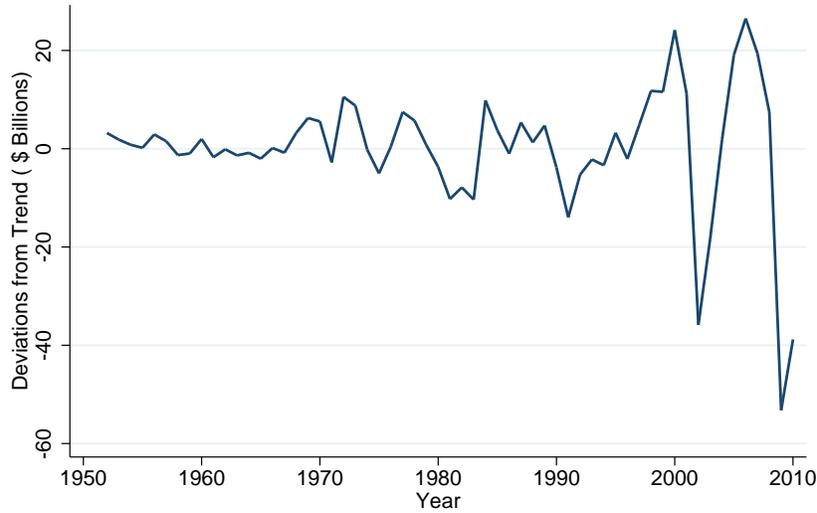
value at the one percent level, 3.57. Following these tests the before years are defined as 1963-2001 for the sales and corporate tax bases and 1963-1999 for the income tax base.

3.2. Basic Facts. Figure 2 demonstrates the increase in volatility in the 2000s by graphing tax revenue aggregated across states and its deviations from a time trend.¹¹ For the rest of the paper, I define volatility as the squared deviations from trend which is a short-run measure of variability and produces a data point for each state-year observation.¹² The absolute value of deviations from trend increased by \$19.1 billion in the 2000s and volatility increased by \$712 billion.

¹¹The time trend is estimated using a cubic time trend.

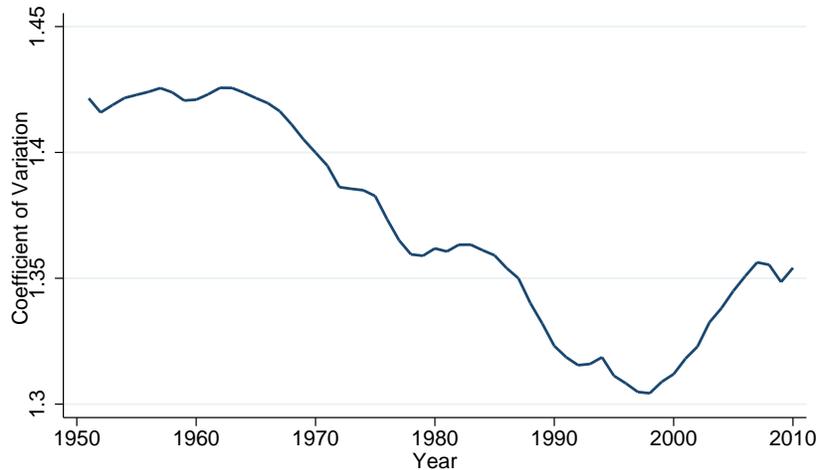
¹²Volatility in variable x is defined as $\tilde{x} = (x_t - \bar{x}_{time\ trend})^2$. This measures the short-run variability which is the focus of the paper. The variance of tax revenue, $\sigma_R = (R_t - \bar{R})^2$ conflates short-run variability and differences due to a time trend. For example, making a state's time trend steeper would increase the variance but would not change the short-run variability. The time trend, estimated for each state separately, in the baseline case is a cubic time trend. The results are robust to different time trends including a Hodrick-Prescott filter with a bandwidth of 6.25, as recommended by Ravn and Uhlig (2002) for yearly data, which is shown in table 3 and time trends with autoregressive processes, semi-parametric power series estimators, and moving averages.

FIGURE 2. Sum Income, Sales, Corporate Tax Revenue Deviations From Trend.



Cubic time trend with autoregressive process.
Data: US Census 1951 - 2010 total tax revenue.

FIGURE 3. State Tax Revenue Begins Diverging In 2000s.

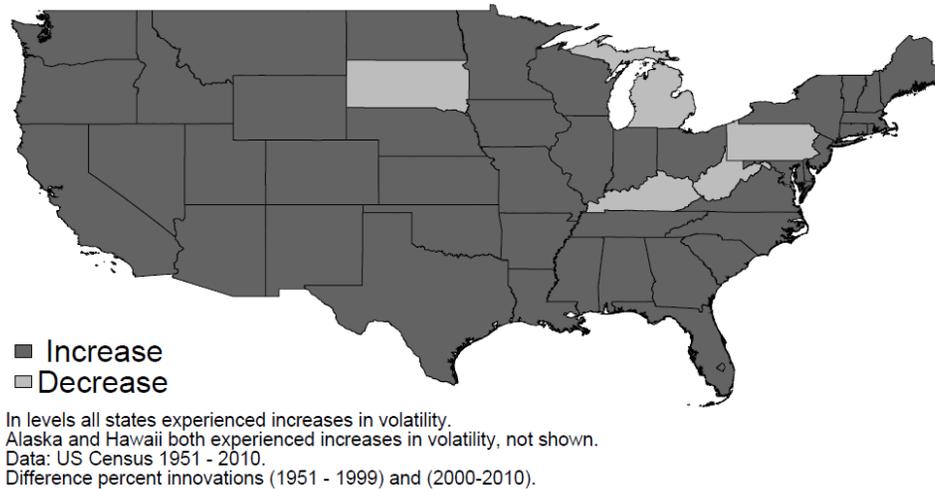


Coefficient of variation, ratio of standard deviation and mean, across state
Smoothed using a moving average with 7 year window on either side.
Data: US Census 1951 - 2010 income, sales, and corporate tax revenue
Similar trend with total tax revenue.

If state tax revenues became more correlated in the 2000s this could explain the increase in volatility. However, as figure 3 demonstrates tax revenues became less correlated in the 2000s. Figure 3 graphs the moving average of the coefficient of variation across states for each year between 1951 and 2010, demonstrating tax revenues began converging in the 1960s but in the late 1990s began diverging.¹³ The increase in volatility is robust to different specification and has been noted previously by Mattoon and McGranahan (2012) and Boyd and Dadayan (2009).

¹³The moving average uses a seven year window on either side and includes the specific year.

FIGURE 4. 45 States Experienced Increases in Volatility.



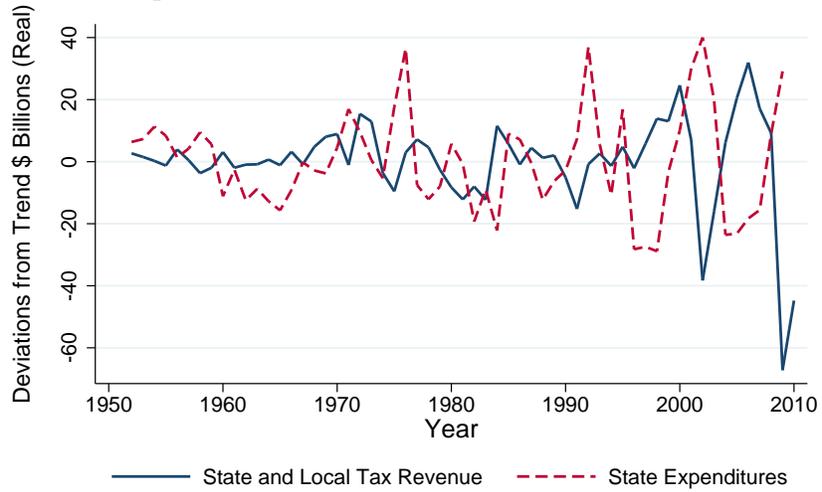
Volatility, as a percent of tax revenue, increased in the 2000s for forty five states mapped in figure 4. Tax revenue volatility increased per person in the 2000s for all fifty states, in levels. The trends depicted in the aggregate data hold for a majority of states and are not driven by a few outliers.

The increase in tax revenue volatility is especially important for state governments because of their self-imposed balanced budget rules. The rules differ in strictness and in some cases restrict the use of rainy day funds to smooth volatile revenue streams. The inability of state governments to smooth volatile tax revenues is demonstrated in figure 5 which plots the deviations from trend of aggregate state expenditures and tax revenues.¹⁴ Tax revenue volatility leads expenditure volatility, which is confirmed by a Granger causality test (Hiemstra and Jones, 1994; Baek and Brock, 1992).¹⁵ Expenditure volatility is especially costly at the state level because due to prior commitments, expenditure volatility is concentrated in a few items such as education, the timing causes state expenditures to be pro-cyclical which is costly to the extent state expenditures should be counter-cyclical, and swings in state government expenditures adds salient uncertainty to the economy.

¹⁴Figure 2 plots the deviations from trend of income, sales, and corporate tax revenue while figure 5 uses total tax revenue to compare with total expenditures.

¹⁵While tax revenue volatility Granger causes expenditure volatility the reverse is not true. The Granger causality test is not a test of causality but a descriptive statistic.

FIGURE 5. State Expenditure and Total Tax Revenue Deviations From Trend.



Cubic time trend with autoregressive process.
 US Census 1951 - 2010 state expenditures and state and local tax revenue.
 State tax revenue deviations from trend Granger cause state expenditure
 deviations from trend.

4. MODEL

In this section the government uses taxes to produce a public good in order to maximize a representative individual's utility in an economy with uncertainty. A technology shock generates uncertainty in the model and the representative individual has rational expectations over this shock. The technology shock is assumed to affect wages and profits differently causing wages and profits not to be perfectly correlated. The extent to which wages and profits are correlated determines the correlation between wage income and consumption in the model. The fact wage income and consumption are not perfectly correlated produces an incentive for the government to hedge wage income and consumption specific risk by taxing both sources. The purpose of this model is twofold; first to derive an equation for the variance of tax revenue which can be used in the empirical decomposition (section 5) and second to setup a normative model to determine the optimal tax policy in the environment where tax revenue volatility is costly (section 7).

A. Technology. The single-intermediate good, X , in the model is assumed to be produced by a single-input factor labor, L , and costlessly transformed into private and public consumption goods. The efficiency with which a representative firm converts the labor into the intermediary-output differs with the state of nature, θ .¹⁶

$$X(L, \theta) = \theta f(L) = \theta L^\gamma$$

$$\theta = \mu_t + v_t \quad v_t \sim \text{Log} - N(0, \sigma_v^2)$$

$$\mu_t = \phi \mu_{t-1} + (1 - \phi) \bar{\mu} + u_t \quad u_t \sim \text{Log} - N(0, \sigma_u^2)$$

The technology shock is given by a combination of two shocks, a persistent shock and a transitory shock. These shocks are assumed to affect wages and profits differently depending on the value of ω . Labor is paid its marginal product $w = \theta_w \gamma L^{\gamma-1}$ where $\theta_w = \mu_t + \omega v_t$. The representative individual owns and receives the profits of the representative firm $\pi = \theta_\pi (1 - \gamma) L^\gamma$ where $\theta_\pi = \mu_t + (1 - \gamma\omega)/(1 - \gamma)v_t$. The exogenous parameter ω determines the correlation between wage income and profit such that when $\omega = 1$ they are perfectly correlated. In the model ω is used to

¹⁶Writing intermediate production in this way implicitly assumes that an increase in the input increases the output by the same percentage in all states of nature.

model the empirical fact wages and profits are not perfectly correlated, hence there is a difference between taxing wage income and consumption.

B. *Individual Behavior.* The individual has utility over the supply of labor L , the public good g , and total private consumption c , which is split between taxed goods, βc , and untaxed goods, $(1 - \beta)c$. The individual chooses c , L , and β to maximize utility

$$\max_{c,L,\beta} \quad u = U(c, \beta, L, g)$$

subject to

$$c = (1 - \tau_c \beta)((1 - \tau_w)w(\theta)L + \pi) = (1 - \tau_c \beta)y$$

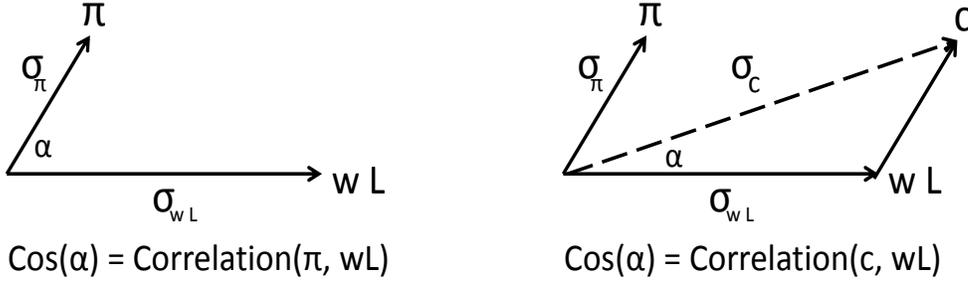
where τ_c and τ_w are the tax rates on consumption and wage income respectively. The correlation between wage income and profit determines the correlation between consumption and income. In figure 6 wage income, profit income, and consumption are characterized by vectors with lengths equal to their standard deviation. Using the law of cosines, the correlation between two vectors is depicted as the cosine of the angle between any two vectors. For example, if the vectors are parallel the variables are perfectly correlated and if the vectors are perpendicular the variables are independent.¹⁷ In the example depicted, if the standard deviation of profit income increased, holding wage income's standard deviation fixed, then the length and angle between consumption and wage income would both become larger. In this example increasing the standard deviation of profit income causes the standard deviation of consumption to increase and causes the correlation between consumption and wage income to decrease.¹⁸

Utility maximization requires: i) the marginal disutility from supplying labor equals the marginal utility of the income it produces and ii) the ratio of marginal utilities from total consumption c and β is equal to the consumption tax rate times income net of taxes. When the consumption tax

¹⁷First, let $\tau_c = \tau_w = 0$ for simplicity, allowing $c = wL + \pi$. Consumption can be represented as a vector equal to the sum of the vectors of wage and profit income where the lengths of all of the vectors equal the standard deviation of the variable. The cosine of the angle between wage income and consumption, using the law of cosines, can be written as $\cos(\theta) = (\sigma_c^2 + \sigma_{wL}^2 - \sigma_\pi^2)/(2\sigma_{wL}\sigma_c)$. The numerator can be reduced to $2\text{cov}(wL, c)$ using the variance formula $\text{var}(\pi) = \text{var}(c - wL) = \text{var}(c) + \text{var}(wL) - 2\text{cov}(wL, c)$. Therefore the cosine of the angle between wage and profit income is equal to the correlation between them; $\cos(\theta) = \text{cov}(wL, c)/(\sigma_{wL}\sigma_c) = \rho_{wL,c}$.

¹⁸If profit income and wage income were negatively correlated increasing the standard deviation of profit income could decrease the standard deviation of consumption. Therefore, even if profit is more volatile than wage income consumption may be less volatile than wage income.

FIGURE 6. Vector Representation of Shocks



(a) Wage Income and Profit Shocks

(b) Correlation Consumption and Wage Income

rate is zero there is no distortion between consumption goods, and the expected marginal utility with respect to β is zero. Composing utility in terms of total consumption c and a composition parameter β simplifies the exposition of deadweight loss because β encompasses all behavioral responses (substitution effects) between goods.¹⁹

$$(1) \quad U_1(c, \beta, L)(1 - \tau_c\beta)(1 - \tau_w)w = U_3$$

$$(2) \quad \frac{U_2}{U_1} = \tau_c((1 - \tau_w)wL + \pi)$$

C. Government The government produces the public good G and finances its production with taxes on consumption and wage income. Two assumptions are made for expository convenience: i) the supply of the public good is set equal to the tax revenue, $g = R$ and ii) the utility function is additive such that $U_{1,2} = 0$.²⁰ The expected utility of the individual can be completely characterized by the moments of private and public consumption. The analysis below focuses on the first two moments, which is sufficient if the production shocks are distributed with a joint distribution characterized fully by their first two moments (e.g. normal, log-normal, and uniform distributions) or if the utility function is quadratic, but the results are consistent with cases where expected utility is characterized

¹⁹For more details see the appendix.

²⁰Assuming the government must have a balanced budget abstracts away from debt issues which are not the focus of this paper. This assumption may be less of an abstraction for state governments, forty-nine of which have balanced budget requirements. In practice these balanced budget requirements do not preclude state debt but they do add additional costs. In this model the ability of the government to smooth revenue is modeled in its risk attitude.

by higher moments.^{21,22} The level of social welfare can be written as

$$(3) \quad E[u] = \int U(c, g) f(c, R, \sigma_R^2, \sigma_c^2) \equiv M(C, \sigma_c^2, \beta, L) + G(R, \sigma_R^2)$$

$$M_1 \geq 0, G_1 \geq 0, M_4 \leq 0, M_2 \leq 0, G_2 \leq 0$$

where R and c are the mean levels of the private and public consumption, σ_c^2 and σ_R^2 are the variances of private and public consumption respectively, and G represents the expected utility from public consumption.²³ The variance of tax revenue is a function of the tax rates, the tax bases, and the economic conditions.^{24,25}

$$(4) \quad \sigma_R^2 = \tau_w^2 L^2 \sigma_w^2 + \tau_c^2 \beta^2 \sigma_y^2 + 2\tau_w \tau_c \beta L \sigma_{y,w}$$

The variance of tax revenue given in equation (4) provides a structural equation for the empirical decomposition. First, aggregate tax revenue can be decomposed into its parts; income tax revenue volatility, sales tax revenue volatility, and the covariance of income and sales tax revenue. Second, each of these parts can be decomposed into its parts; the tax rate, the tax base, and the economic conditions as demonstrated in equation (5) for the sales tax. In equation (5) the sales tax revenue volatility, the sales tax rate, and the volatility of the economy are observed but the base β is unobserved because it is a complex combination of economic conditions, tax rates, and tax laws. The base is estimated in equation (5.1) as a function of tax rate variables $\boldsymbol{\tau}$ and economic condition variables \boldsymbol{x} .²⁶ The tax variables include tax rates from other bases (to account for tax shifting), information on the tax base (such as the number of brackets in the tax schedule), and τ_c . The economic variables include the volatility of state level GDP, personal income, population.

$$(5) \quad \log(\sigma_{R_c}^2) = 2\log(\tau_c) + 2\log(\beta) + \log(\sigma_y^2)$$

²¹In the case where two moments are sufficient, the indifference curves can be shown to be quasi-concave as long as $U'' < 0$.

²²Analysis in the appendix considers expected utility which is characterized by higher moments.

²³The shape of M can differ from the shape of G , allowing for different attitudes of risk in public and private consumption.

²⁴ $\sigma_c^2 = (1 - \tau_c \beta)^2 ((1 - \tau_w)^2 L^2 \sigma_w + \sigma_\pi + 2(1 - \tau_w) L \sigma_{w,\pi})$

²⁵Base factors L, β are choice variables allowed to vary with the state of nature. For expository ease they have been treated as constants but their variance can be included.

²⁶The equation for the income tax base assumes the unobservable characteristics ϵ is additively separable from the observable characteristics. This assumption is loosened in the empirical decomposition by using a weighting method.

$$(5.1) \quad \log(\beta) = \delta_0 + \log(\boldsymbol{\tau})\boldsymbol{\psi}_1 + \log(\boldsymbol{x})\boldsymbol{\psi}_2 + \nu$$

$$(5.2) \quad \log(\sigma_{R_c}^2) = \delta_0 + \log(\boldsymbol{\tau})\boldsymbol{\delta}_1 + \log(\boldsymbol{x})\boldsymbol{\delta}_2 + \varepsilon$$

For the empirical analysis the volatility is measured as the squared deviations from trend to focus on the short-run variability, discussed previously in the descriptive statistics section. Therefore, the volatility of state level GDP included in \mathbf{x} in equation (5.2) is given by $\sigma_{gdp,t}^2 = (gdp_t - gdp_{time\ trend})^2$.

5. EMPIRICAL DECOMPOSITION

The theoretical model demonstrates the increase in tax revenue volatility is due to changes in tax rates, amplified volatility in economic conditions, or tax base changes. Tax rates, economic conditions, and tax revenues are observable; however, tax base changes, such as the increase in e-commerce, are unobservable, which complicates the empirical decomposition. Adapting empirical decomposition methods pioneered by Oaxaca (1973), Blinder (1973), and DiNardo, Fortin, and Lemieux (1996) allows me to quantify tax base changes in a similar way as they quantify discrimination in pay or the effect of unions, which are also unobserved. The baseline model is estimated using a weighting method similar to DiNardo, Fortin, and Lemieux (1996) which can be thought of as a weighted extension of the decomposition method described by Oaxaca (1973) and Blinder (1973). For this reason I explain the method in terms similar to Oaxaca (1973).²⁷

Intuitively, the contributions of these factors are determined by comparing predicted tax revenue volatility in different counterfactual scenarios. For example, the contribution of tax factors is quantified by the difference between the actual tax revenue volatility in the 2000s with the predicted tax revenue volatility in the 2000s if the tax factors in the 2000s were equal to their values in the previous decades.²⁸ Similarly, the contribution of economic conditions can be quantified using

²⁷The weighting method, described in the appendix, is chosen as the baseline case because a test of nonlinearity in the Oaxaca (1973) estimate suggests nonlinearities exist. In this case the weighting method is preferred because it controls for nonlinearities and is asymptotically more efficient than matching or regression models, (Hirano, Imbens, and Ridder, 2003). In this context controlling for nonlinearities will decrease the upward bias in the structural factor estimates from the Oaxaca (1973) analysis.

²⁸Therefore the contribution of the base changes is the increase in volatility unexplained by the observed characteristics, similar to the treatment on the treated (TOT).

the observed difference in economic volatility. Changes in the tax base are captured by changes in the regression coefficients of the tax rates and economic conditions.²⁹ The difference between the coefficients estimated in the before and after periods estimates the change in the relationship between tax revenue volatility and the explanatory tax rates and economic conditions, which is the difference in the tax base.

5.1. Method. Equation (6) decomposes the three groups of factors where η_1 is an indicator function for the years after the structural break, η_{state} indicates the state fixed effects, and $\boldsymbol{\tau}$ and \boldsymbol{x} are matrices of all of the tax and economic factors respectively. This equation nests the following equations which estimate the volatility separately for the before and the after years denoted by $\boldsymbol{x}_{|0}$ and $\boldsymbol{x}_{|1}$ respectively.³⁰ In equation 6 $\delta_1 = \gamma_1$ and $\delta_2 = \gamma_2$. The coefficients on the economic and tax variables interacted with the time group dummy, δ_3 and δ_4 , are equal to the difference between the coefficients from the two separate equations, $\gamma_1 - \phi_1$ and $\gamma_2 - \phi_2$ respectively.

$$(6) \quad \begin{aligned} \log(\sigma_{R_i}^2) &= \delta_0 + \log(\boldsymbol{x})\boldsymbol{\delta}_1 + \log(\boldsymbol{\tau})\boldsymbol{\delta}_2 + (\eta_1 * \log(\boldsymbol{x}))\boldsymbol{\delta}_3 + (\eta_1 * \log(\boldsymbol{\tau}))\boldsymbol{\delta}_4 + \eta_1 + \eta_{state} + \varepsilon \\ \log(\sigma_{R_i|1}^2) &= \gamma_0 + \log(\boldsymbol{x}_{|1})\boldsymbol{\gamma}_1 + \log(\boldsymbol{\tau}_{|1})\boldsymbol{\gamma}_2 + \eta_{state} + \varepsilon_{|1} \\ \log(\sigma_{R_i|0}^2) &= \phi_0 + \log(\boldsymbol{x}_{|0})\boldsymbol{\phi}_1 + \log(\boldsymbol{\tau}_{|0})\boldsymbol{\phi}_2 + \eta_{state} + \varepsilon_{|0} \end{aligned}$$

The estimated difference in volatility is given in equation 7 and decomposed by rearranging terms and adding and subtracting $\bar{\boldsymbol{x}}_{|1}\hat{\boldsymbol{\phi}}_1 + \bar{\boldsymbol{\tau}}_{|1}\hat{\boldsymbol{\phi}}_2$, where $\bar{\boldsymbol{x}}_{|1}$ denotes the average value in the after period. The contribution of tax base changes is captured by the first three terms in equation 7 which encompass the change in intercept and the change in coefficients. The differences attributed to observable differences in economic conditions and tax rates are captured by the fourth and fifth

²⁹The empirical decomposition compares the observable characteristics and the relationship between observable characteristics between the before and after period. The identifying assumption states that on average differences in the residuals cancel, leaving only the observable characteristics and their relationships.

³⁰The before and after years represent the years before and after the structural break found by doing a Quandt likelihood ratio test. For more information on the Quandt likelihood ratio test see the appendix.

terms respectively.³¹

$$\begin{aligned}
(7) \quad \hat{\Delta} &= \widehat{\log(\sigma_{R_i|1}^2)} - \widehat{\log(\sigma_{R_i|0}^2)} \\
&= \hat{\gamma}_0 + \log(\bar{\mathbf{x}}_{|1})\hat{\gamma}_1 + \log(\bar{\tau}_{|1})\hat{\gamma}_2 - \hat{\phi}_0 - \log(\bar{\mathbf{x}}_{|0})\hat{\phi}_1 - \log(\bar{\tau}_{|0})\hat{\phi}_2 \\
&= \hat{\gamma}_0 - \hat{\phi}_0 + \log(\bar{\mathbf{x}}_{|1})(\hat{\gamma}_1 - \hat{\phi}_1) + \log(\bar{\tau}_{|1})(\hat{\gamma}_2 - \hat{\phi}_2) \\
&+ (\log(\bar{\mathbf{x}}_{|1}) - \log(\bar{\mathbf{x}}_{|0}))\hat{\phi}_1 \\
&+ (\log(\bar{\tau}_{|1}) - \log(\bar{\tau}_{|0}))\hat{\phi}_2 \\
&= \underbrace{\hat{\eta}_1 + \log(\bar{\mathbf{x}}_{|1})\hat{\delta}_3 + \log(\bar{\tau}_{|1})\hat{\delta}_4}_{\text{Tax Base}} + \underbrace{(\log(\bar{\mathbf{x}}_{|1}) - \log(\bar{\mathbf{x}}_{|0}))\hat{\delta}_1}_{\text{Economic Conditions}} + \underbrace{(\log(\bar{\tau}_{|1}) - \log(\bar{\tau}_{|0}))\hat{\delta}_2}_{\text{Tax Rates}}
\end{aligned}$$

5.2. Identification and Specification Checks. This decomposition relies on the conditional mean of the error being zero. This assumption allows the counterfactual volatility to be written as $\phi_0 + E[x|\eta_1 = 1]\phi_1 + E[\tau|\eta_1 = 1]\phi_2$ because the error term conveniently drops out. Intuitively, this assumption assigns the difference in tax revenue volatility between the before and after periods to either differences in the observable characteristics (tax policy or economic conditions) or differences in the estimated coefficients (tax base) but not unobservable characteristics.

Identifying Assumption:

The conditional mean of the error is equal to zero, $E[\varepsilon|x, \tau, \eta_1, \eta_{state}] = 0$

The identification in this decomposition is threatened if there are endogenous or omitted variables which cause the identifying assumption not to hold. The panel data is useful both for providing additional controls and for allowing a series of specification checks. First, to check for omitted variables the regressions are run with and without state-neighbor interacted with time fixed effects. These additional controls check for common unobservable variables across groups of states for example, unobserved shocks that would affect both state GDP and tax revenues in the Northeast. Different state-neighbor groups are used and the estimated coefficients are robust to these controls.

³¹The formulas in equation 7 are more complicated in the two robustness specifications run and reported in the appendix. First, when state fixed effects are allowed to differ between the two groups the term does not drop out. However, an F-test fails to reject the null all coefficient estimates are the same when the state fixed effects are allowed to differ. Second, when state-neighbor fixed effects are included these terms would not drop out. These additional variables would be included in the unobserved group.

This specification test alleviates some concern of omitted variables but is unable to account for state-year specific shocks.

Second, the importance of state spill-over effects is checked by including the tax rates and economic variables from neighboring states. Third, concerns of simultaneity of tax revenue volatility and tax policy are checked by replacing the tax rates with their two year lags. The contemporaneous tax rate is highly correlated with the tax rate from two years prior but the contemporaneous volatility of tax revenue could not be used to influence the tax rate from two years earlier. Intuitively, the volatility of tax revenue is defined as the squared deviations from trend, or transitory shocks, which makes conditioning policies on them difficult.³² The estimated coefficients are robust to both of these specification checks, further alleviating concerns of the validity of the identifying assumption.

Finally, the importance of tax base measures are checked by including controls for the number of tax brackets a state's income and corporate taxes have. Intuitively and statistically, the number of brackets in a given tax code is an important factor in the tax base. In addition, the number of brackets is an independent variable highly correlated with other variables, specifically tax rates. Therefore measuring the change in coefficients from omitting the controls for tax brackets provides an estimate of the effect of omitting other time varying tax policy changes which can be complex and nuanced.³³ Time invariant state specific tax laws are controlled for with the state fixed effects. The robustness of the estimates to this specification check alleviates concerns over the impact of other time varying tax law changes.

The ability to causally interpret the decomposition depends on three key factors. First, the identifying assumption must hold. Second, empirical decompositions suffer from general equilibrium effects when the counterfactual of interest is out of sample, for example changes in the counterfactual world without unions. However, the counterfactuals of interest in this paper are policies in previous decades, meaning the counterfactuals are within sample. Finally, decompositions of differences

³²As an additional robustness check an autoregressive process is estimated to filter out any time correlation leaving only transitory shocks. The estimates are reasonably robust to this specification.

³³For example, in Milwaukee County, Wisconsin marshmallows are subject to the local food and beverage tax unless they contain flour and in 2009 Wisconsin changed the law such that ice cream sandwiches sold in grocer's frozen food section are no longer subject to this tax. (<http://www.revenue.wi.gov/faqs/pcs/expo.html>. Tax 11.51 Guidelines "Marshmallows unless they contain flour.")

between non-manipulatable groups, such as the before and after years in this analysis, are subject to Holland's (1986) choice critique. However, the grouping in this analysis fundamentally differs from race and gender, which Holland refers to, because this analysis observes the same states in both before and after groups, thus alleviating some of these concerns.

6. RESULTS

Aggregate tax revenue volatility increased, on average, by \$712 billion in the 2000s. The first stage of the decomposition given in equation (8) reports 52 percent of the aggregate increase in tax revenue volatility is due to increases in the volatility of the income tax, 20 percent due to the sales tax, 14 due to the corporate income tax, and the remaining 14 percent due to the covariances. The first stage decomposition is consistent with the explanation tax revenue volatility increased due to an increase in the reliance on the income tax.

$$(8) \quad \hat{\Delta}_A = \underset{(52\%)}{\hat{\Delta}_I} + \underset{(20\%)}{\hat{\Delta}_S} + \underset{(14\%)}{\hat{\Delta}_C} + \underset{(7\%)}{\hat{\Delta}_{I,S}} + \underset{(4\%)}{\hat{\Delta}_{I,C}} + \underset{(3\%)}{\hat{\Delta}_{S,C}}$$

Column (1) of Table 2 reports the second stage decomposition of aggregate tax revenue volatility into tax rates, economic conditions, and tax base changes. Changes in tax rates are the most important factors explaining aggregate tax revenue volatility, explaining 70.26 percent of the increase. Changes in the economic conditions explain 28.95 percent and changes in the tax base explain only 0.78 percent. The ninety-five percent confidence intervals are calculated by bootstrapping the sample, clustering by state, and reporting the 2.5 and 97.5 percentiles. These estimates are robust to extreme outliers and produce asymmetric confidence intervals. An F-test rejects the null that changes in economic conditions are more important than changes in tax rates in explaining the increase in tax revenue volatility, confirming the intuition from the ninety-five percent confidence intervals. Columns (2) through (4) report the decomposition separately for the income, sales, and corporate tax respectively. Similar to the changes in aggregate tax revenue volatility, changes in income and corporate tax revenue volatility is explained principally by changes in tax rates followed by changes in economic conditions. In contrast, the evidence that tax rate changes explain the increase in sales tax revenue volatility is weaker because the ninety-five percent confidence interval includes zero.

TABLE 2. Results

	Percent Explain	Income	Sales	Corporate
Δ Tax Rates	70.26 % [58.42 , 88.49]	66.18 % [50.62 , 72.56]	52.08 % [-40.99 , 67.43]	84.14 % [73.18 , 88.78]
Δ Economic Conditions	28.95 % [10.69 , 40.69]	33.04 % [18.93 , 39.59]	47.35 % [9.99 , 66.77]	15.04 % [4.66 , 19.74]
Δ Tax Base	0.78 % [0.70 , 0.87]	0.80 % [0.70 , 0.83]	0.69 % [0.14 , 0.81]	0.82 % [0.76 , 0.84]
State FE	Yes	Yes	Yes	Yes
Observations	2350	2350	2350	2350

Bootstrapped 95 percentile confidence interval (3000 replications) clustered by state.

Base Case: cubic time trend and kernel matching to produce weights.

Weighted estimates of equation 6.

Volatility of revenue and economic variables calculated as $(x - x_{time\ trend})^2$.

Intuitively, the confidence intervals are determined by the variation across states in how much each factor explains the increase in tax revenue volatility. For the tax base, the confidence intervals are very precise implying there is very little variation across states in the amount of the increase in tax revenue volatility explained by changes in tax base. In contrast, the confidence intervals for tax rate changes for the sales tax are large. Intuitively, there is a lot of variation across states in the changes in sales tax rates because of the variation in how states implement the sales tax. Therefore, the estimate of the importance of changes in tax rates for the sales tax varies across bootstrap samples depending on the states in the sample, causing a large confidence interval.

Changes in the tax base are not economically important in explaining the tax revenue volatility. Intuitively, for changes in the tax base to be an important factor, the changes in the tax base would have to change the volatility of the base. For example, if online shopping caused the sales tax base to be left with only large durable goods, such as cars, then this change in the tax base would have caused a large increase in tax revenue volatility because large durable goods are more volatile than the sales tax base as a whole. In contrast, if the consumption goods being bought online are a representative bundle of the sales tax base, at least with respect to volatility, then even if the sales tax base decreased significantly because of online shopping the volatility of the base may not

change. These results suggest changes in the tax base have not changed the volatility of the tax base.

Columns (2) and (3) of Table 3 report two alternative methods for estimating tax revenue volatility. The baseline case given in the first column estimates a cubic time trend and uses a kernel estimation to produce weights. The second column reports the results with inverse probability weights estimated by a probit. The third column reports the results with a time trend estimated by a Hodrick-Prescott filter (Hodrick and Prescott, 1997). The results are qualitatively and quantitatively similar in both of the alternative methods.

TABLE 3. Alternative Model Specifications

	Baseline	IPW	HP Filter
Income			
Δ Tax Rates	66.18 % [50.62 , 72.56]	64.19 % [35.38 , 71.68]	80.88 % [61.5 , 89.83]
Δ Economic Conditions	33.04 % [18.93 , 39.59]	35.06 % [19.08 , 44.28]	18.26 % [-7.19 , 26.91]
Δ Tax Base	0.8 % [0.7 , 0.83]	0.76 % [0.64 , 0.82]	0.87 % [0.82 , 0.89]
Sales			
Δ Tax Rates	52.08 % [-40.99 , 67.43]	50.44 % [-62.15 , 66.75]	49.38 % [26.81 , 57.58]
Δ Economic Conditions	47.35 % [9.99 , 66.77]	48.98 % [0.23 , 72.25]	49.82 % [30.58 , 58.07]
Δ Tax Base	0.69 % [0.14 , 0.81]	0.63 % [-0.04 , 0.78]	0.79 % [0.66 , 0.84]
Corporate			
Δ Tax Rates	84.14 % [73.18 , 88.78]	83.79 % [71.35 , 88.6]	73.23 % [58.17 , 80.71]
Δ Economic Conditions	15.04 % [4.66 , 19.74]	15.45 % [4.14 , 20.66]	25.97 % [6.25 , 32.84]
Δ Tax Base	0.82 % [0.76 , 0.84]	0.78 % [0.71 , 0.82]	0.79 % [0.72 , 0.82]

Bootstrapped 95 percentile confidence interval (3000 replications) clustered by state.

Bootstrap clustered by state.

Inverse probability weights constructed from probit estimates.

Weighted estimates of equation 6 with different model specifications.

Volatility of revenue and economic variables calculated as $(x - x_{time\ trend})^2$.

7. RAMSEY PROBLEM DECOMPOSITION

The government's objective function differs from those in traditional optimal taxation because the mean and variance of both private and public consumption enters explicitly. The government maximizes the expected utility of the representative individual who has utility over both private and public consumption. Previously in this paper it was shown that the expected utility function can be written as a function of the mean and variance of public and private consumption with minimal

additional assumptions.³⁴ Aggregate production uncertainty, which is assumed to be uninsurable, enters the individual's income through uncertainty in wages and profits such that wages and profits are not perfectly correlated. The aggregate production uncertainty is split between public and private consumption depending on the tax rates on wage income and consumption.

This section begins with the full government's problem, which consists of costs from volatile public and private consumption as well as the typical costs from behavioral changes by the representative individual due to the use of distortionary taxes. This analysis produces a volatility-adjusted Ramsey rule which characterizes the government's optimal tax rates when uncertainty and behavioral distortions exist. The analysis then turns to three cases which decompose this condition into its separate parts. These cases are depicted in the box below. The first case considers the planner's problem of distributing *certain* aggregate production between public and private consumption. The second case considers the planner's problem of distributing *uncertain* aggregate production between public and private consumption. Finally, the third case considers the government's problem of taxing the representative individual's *certain* wage income and consumption to provide public consumption.

	Certain	Uncertain
Planner	Case 1 Pareto Optimum	Case 2 Volatility Modified
Competitive	Case 3 Behavioral Changes (PF literature)	Full Model

The timing of the model differs between the certain and uncertain cases as given below. In the certain cases (cases 1 and 3) nature decides the aggregate production state of the world before the government or individual makes their decisions. In the uncertain case (case 2) the government

³⁴The expected utility function can be written as a function of the higher moments of public and private consumption. When the distribution functions of public and private consumption are characterized by the first two moments (e.g. normal, log-normal, and uniform distributions) the expected utility function reduces to a function of the mean and variance of public and private consumption. This is discussed previously in the model section.

must make its state-independent decision before the aggregate production state is determined, causing uncertainty for the government. In both the certain and uncertain cases the individual's decisions are made after the aggregate production state of the world is determined; hence, there is no uncertainty for the individual in any case.

<u>Certain Case</u>		<u>Uncertain Case</u>	
<i>Order of Decisions</i>	<i>Choices</i>	<i>Order of Decisions</i>	<i>Choices</i>
1st - Nature	θ	1st - Government	τ or ρ
2nd - Government	τ	2nd - Nature	θ
3rd - Individual	c, L, β	3rd - Individual	c, L, β
4th - Production occurs		4th - Production occurs	
5th - Utility realized		5th - Utility realized	

Full Government's Problem. The government chooses the tax rates on wage income and consumption before the state is realized and the individual chooses the amount of labor to supply and the consumption composition after the state is realized. Each of the government's tax bases are state-dependent, meaning conventional approaches to evaluating alternative tax structures (e.g., deadweight loss for equal revenue streams) encounter complications because differing tax structures will change the pattern of returns across states of nature. If the government is risk neutral comparing the expected loss of utility for an expected level of revenue will be sufficient. However, if the government is sensitive to both the level and volatility associated with a revenue stream, then comparing expected utility losses will be inadequate. The government's attitude toward risk depends upon the individual's preferences and the ability of the government to smooth revenue.³⁵

$$\max_{\tau_c, \tau_w} M(c, \sigma_c^2, \beta, L) + G(R, \sigma_R^2)$$

subject to

$$\begin{aligned} c &= (1 - \tau_c \beta)(wL(1 - \tau_w) + \pi) & \sigma_c^2 &= (1 - \tau_c \beta)^2 \sigma_y^2 \\ R &= \tau_c \beta (wL(1 - \tau_w) + \pi) + \tau_w wL & \sigma_R^2 &= \tau_c^2 \beta^2 \sigma_y^2 + \tau_w^2 L^2 \sigma_w^2 + 2\tau_c \beta \tau_w L \sigma_{y,w} \end{aligned}$$

³⁵ $y = wL(1 - \tau_w) + \pi$ and $\sigma_y = (1 - \tau_w)^2 L^2 \sigma_w^2 + \sigma_\pi^2 + 2(1 - \tau_w)L\sigma_{w,\pi}$

The government's first-order conditions given in equations (SC_{τ_c}) and (SC_{τ_w}) encompass the full tradeoff between the costs from volatile public and private consumption and the deadweight loss due to behavioral changes by the individual in response to distortionary taxes. The first-order conditions can be broken into three parts; the marginal benefit of public and private consumption, the loss due to behavioral changes, and the loss due to volatility.

The loss due to behavioral changes consists of the weighted sum of the elasticities of labor and β with respect to the given tax rate where the weights scale the elasticities by their impact on utility.³⁶ Similarly, the loss due to volatility consists of the weighted sum of the elasticities of the variance of private and public consumption with respect to the given tax rate.³⁷ The losses due to behavioral changes and volatility create wedges that cause the marginal benefit of public consumption to differ from the marginal benefit of private consumption.

$$(SC_{\tau_c}) \quad FOC_{\tau_c} : \quad G_1 = M_1 \underbrace{-\omega_{\beta, \tau_c} \varepsilon_{\beta, \tau_c} - \omega_{L, \tau_c} \varepsilon_{L, \tau_c}}_{\text{Behavioral}} + \underbrace{\omega_{\sigma_c^2, \tau_c} \varepsilon_{\sigma_c^2, \tau_c} + \omega_{\sigma_R^2, \tau_c} \varepsilon_{\sigma_R^2, \tau_c}}_{\text{Volatility}}$$

$$(SC_{\tau_w}) \quad FOC_{\tau_w} : \quad G_1 = M_1 - \omega_{\beta, \tau_w} \varepsilon_{\beta, \tau_w} - \omega_{L, \tau_w} \varepsilon_{L, \tau_w} + \omega_{\sigma_c^2, \tau_w} \varepsilon_{\sigma_c^2, \tau_w} + \omega_{\sigma_R^2, \tau_w} \varepsilon_{\sigma_R^2, \tau_w}$$

The wedge due to behavioral changes is always nonnegative but the wedge due to volatility can be positive or negative because the variance of tax revenue is U-shaped with respect to an individual tax rate. Therefore, the marginal cost from volatility with respect to a given tax rate is positive if the tax rate is relatively larger than the other tax rates, causing it to be on the upward sloping part of the tax revenue curve.

The efficient provision of public consumption is determined by the volatility-adjusted Samuelson conditions given in equations (SC_{τ_c}) and (SC_{τ_w}) which differs from the traditional Samuelson condition by the presence of the two wedge terms. The provision of public consumption with these two wedge terms can be greater or less than the provision without the wedge terms because the wedge due to volatility can be positive or negative. If the volatility wedge is sufficiently negative then the

³⁶Weights on the base elasticities: $\omega_{\beta, \tau_c} = G_1$, $\omega_{L, \tau_c} = \frac{G_1 w L (\tau_c \beta (1 - \tau_w) + \tau_w)}{\tau_c \beta y}$, $\omega_{\beta, \tau_w} = \frac{G_1 \tau_c \beta y}{(1 - \tau_c \beta) w L \tau_w}$, $\omega_{L, \tau_w} = \frac{G_1 w L (\tau_c \beta (1 - \tau_w) + \tau_w)}{(1 - \tau_c \beta) w L \tau_w}$.

³⁷Weights on the variance elasticities: $\omega_{\sigma_c^2, \tau_c} = \frac{-M_2 \sigma_c^2}{\tau_c \beta y}$, $\omega_{\sigma_R^2, \tau_c} = \frac{-G_2 \sigma_R^2}{\tau_c \beta y}$, $\omega_{\sigma_c^2, \tau_w} = \frac{-M_2 \sigma_c^2}{(1 - \tau_c \beta) w L \tau_w}$, $\omega_{\sigma_R^2, \tau_w} = \frac{-G_2 \sigma_R^2}{(1 - \tau_c \beta) w L \tau_w}$.

efficient public good provision is larger than in the case without these wedges (e.g. case 1).³⁸ The volatility wedge becomes more negative as the individual becomes more risk averse with respect to private consumption. When this occurs, the government has an incentive to raise its tax rates to shift risk into the public good, but raising the tax rates also increases the provision of public consumption.

The optimal tax rates are characterized by the volatility-adjusted Ramsey rule given in equation (9) and produced by combining the first-order conditions in equations (SC_{τ_c}) and (SC_{τ_w}) . The volatility-adjusted Ramsey rule states the sum of the elasticity of the tax base and the elasticity of the cost from volatility, both with respect to a given tax rate and weighted by their contribution to utility, should be equal across tax rates.^{39,40} The utility weights determine the relative importance of behavioral changes and volatility. The welfare weight on volatility encompasses the risk preferences of the representative individual. These risk preferences can be thought of as encompassing the relative ability of the individual and government to smooth volatile income streams which, for simplicity, has been left out of this model.

$$(9) \quad \omega_{B,\tau_c} \varepsilon_{B,\tau_c} + \omega_{\sigma,\tau_c} \varepsilon_{\sigma,\tau_c} = \omega_{B,\tau_w} \varepsilon_{B,\tau_w} + \omega_{\sigma,\tau_w} \varepsilon_{\sigma,\tau_w}$$

This condition nests the traditional Ramsey rule, which in a special case reduces to setting tax rates that are inversely proportional to their elasticities of demand. Similar intuition holds in the volatility-adjusted Ramsey rule. A tax base will be taxed relatively higher as the individual becomes less responsive to the tax rate, captured by the base elasticities. In addition, the volatility-adjusted Ramsey rule demonstrates that the tax base with smaller costs due to volatility will be taxed relatively higher, captured by the volatility elasticities. There are two considerations with the costs of volatility. First, changing the tax rates on wage income and consumption changes the distribution of risk between public and private consumption. Therefore, by taxing state-dependent tax bases the government is able to share some of the aggregate production risk within public good consumption. Second, the government is able to hedge some of the idiosyncratic risk involved with

³⁸The volatility wedge is sufficiently negative when the sum of the volatility wedge and the behavioral wedge is negative.

³⁹The Ramsey rule expanded:

$$-\omega_{L,\tau_c} \varepsilon_{L,\tau_c} - \omega_{\beta,\tau_c} \varepsilon_{\beta,\tau_c} + \omega_{\sigma_c^2,\tau_c} \varepsilon_{\sigma_c^2,\tau_c} + \omega_{\sigma_R^2,\tau_c} \varepsilon_{\sigma_R^2,\tau_c} = -\omega_{L,\tau_w} \varepsilon_{L,\tau_w} - \omega_{\beta,\tau_w} \varepsilon_{\beta,\tau_w} + \omega_{\sigma_c^2,\tau_w} \varepsilon_{\sigma_c^2,\tau_w} + \omega_{\sigma_R^2,\tau_w} \varepsilon_{\sigma_R^2,\tau_w}$$

⁴⁰ $\omega_{B,\tau_i} = -\omega_{\beta,\tau_i}$, $\omega_{\sigma,\tau_i} = \omega_{\sigma_c^2,\tau_i}$, $\varepsilon_{B,\tau_i} = \varepsilon_{\beta,\tau_i} + (\omega_{L,\tau_i}/\omega_{\beta,\tau_i})\varepsilon_{L,\tau_i}$, and $\varepsilon_{\sigma,\tau_i} = \varepsilon_{\sigma_c^2,\tau_i} + (\omega_{\sigma_R^2,\tau_i}/\omega_{\sigma_c^2,\tau_i})\varepsilon_{\sigma_R^2,\tau_i}$

a given tax base by taxing multiple tax bases. Therefore, it is possible to decrease the volatility of public consumption by raising a tax rate.

To decompose the volatility-adjusted Ramsey rule the analysis turns to three special cases. Each of these cases highlights a different part of the full tradeoff faced by the government.

Case 1: Planner's Problem with Certainty. In this case the planner chooses L, c, R , and β (labor, the level of public and private consumption, and the composition of private consumption) after the state of nature is realized. The planner produces the efficient allocation without losses due to behavioral changes and there is no uncertainty to result in costs from volatility.

$$\max_{c, \beta, R, L} \quad M(c, \sigma_c^2, \beta, L) + G(R, \sigma_R^2)$$

subject to

$$\theta f(L) = c + R$$

The first-order condition with respect to labor states the marginal cost of supplying labor should equal the marginal benefit. The first-order condition with respect to β states the marginal benefit should be zero, implying that a shift of consumption either towards or away from taxable consumption would decrease utility. The first-order conditions with respect to public consumption dictate the marginal benefits from public and private consumption should be equal.⁴¹

$$FOC_L : \quad M_4(c, \sigma_c^2, \beta, L) = M_1(c, \sigma_c^2, \beta, L)\theta f'(L)$$

$$FOC_\beta : \quad M_3 = 0$$

$$(SC.1) \quad FOC_R : \quad G_1(R, \sigma_R^2) = M_1(c, \sigma_c^2, \beta, L)$$

The last first-order condition, given by equation (SC.1), is the Samuelson condition characterizing the efficient provision of public consumption. In contrast to the Samuelson condition given in the

⁴¹Equal marginal benefits between public and private goods results from the assumption of a representative individual and the assumption that the intermediate good is costless to transform into public and private consumption.

full government's problem this condition does not have the wedges due to behavioral or volatility costs. The behavioral wedge can be thought of as an additional cost to the government from transforming private consumption into public consumption. The volatility wedge can be either an additional cost or an additional benefit of transforming private consumption into public consumption. Therefore, the provision of public consumption in this case could be greater or smaller than in the full government's problem depending on the magnitudes of the wedges in the full government problem.

Case 2: Planner's Problem with Uncertainty. In this case the planner chooses ρ , L , and β (the fraction of uncertain production to allocate to the public sector, labor, and the composition of private consumption). The planner allocates resources without losses due to behavioral changes but incurs a cost from volatility due to the uncertainty in aggregate production.

$$\max_{\rho, L, \beta} \quad M(c, \sigma_c^2, \beta, L) + G(R, \sigma_R^2)$$

subject to

$$c = (1 - \rho)\theta f(L) \qquad \sigma_c^2 = (1 - \rho)^2 f(L)^2 \sigma_\theta^2$$

$$R = \rho\theta f(L) \qquad \sigma_R^2 = \rho^2 f(L)^2 \sigma_\theta^2$$

The first-order condition with respect to labor states that the marginal cost of supplying labor should equal the marginal benefit, where the change in volatility is included. The first-order condition with respect to β states the marginal benefit should be zero, which is the same as in the first case and therefore omitted below. Finally, the first-order condition with respect to ρ states the marginal benefit of public consumption should equal the marginal benefit of private consumption plus the marginal cost due to volatility from shifting aggregate production from private consumption to

public consumption.

$$FOC_L : \quad M_4 = ((1 - \rho)M_1 + \rho G_1) \theta f'(L) + ((1 - \rho)^2 M_2 + \rho^2 G_2) 2f(L)f'(L)\sigma_\theta$$

$$(SC.2) \quad FOC_\rho : \quad G_1 = M_1 + \frac{2\rho\sigma_\theta^2}{R\varepsilon_{R,\rho}} ((1 - \rho)M_2 - \rho G_2)$$

The last first-order condition given in equation (SC.2) is the Samuelson condition, in this case with uncertainty. In this case, the provision of public consumption can be greater or less than the case with certainty (case 1) depending on the benefits of risk sharing. Specifically, if the marginal cost from private consumption volatility is larger than the marginal cost from public consumption volatility then the government has an additional incentive to increase the provision of public consumption.⁴²

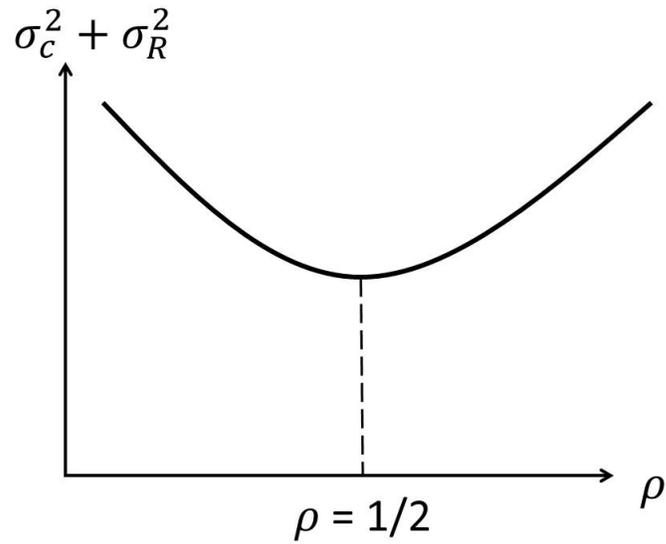
Table 4 demonstrates the marginal effects on the mean and variance of private and public consumption as production is shifted to the public sector. Because the planner can shift production without loss due to behavioral changes, the marginal effects cancel for the mean, shown in the first column of table 4. The variance of public and private consumption is convex in production meaning a shift in production can increase or decrease the sum of the variances.⁴³ For example, if the risk preferences for public and private consumption are represented by the same linear function, the best allocation of risk occurs when $\rho = 1/2$ and the cost of risk increases convexly away from this point as demonstrated in figure 7.

⁴²This can be seen by noting that the provision of public consumption is larger in this case than in the first case when the marginal benefit of public consumption is less than the marginal benefit of private consumption $G_1 < M_1$. This occurs when the volatility wedge is negative. The volatility wedge is negative when the marginal cost from private consumption volatility is larger than the marginal cost from public consumption volatility $[-(1 - \rho)M_2] > [-\rho G_2]$, given that $M_2 < 0$ and $G_2 < 0$.

⁴³Notice however volatility in the economy does not depend on ρ since volatility in the economy is simply σ_θ^2 . This is also apparent if public and private consumption are considered perfect substitutes, in which case, the planner would care about the variance of $c + R$. The variance of $c + R$ is the variance of c plus the variance of R plus 2 times the covariance. In this case: $\sigma_c^2 + \sigma_R^2 + 2\sigma_{c,R} = (1 - \rho)^2\sigma_\theta^2 + \rho^2\sigma_\theta^2 + 2(1 - \rho)\rho\sigma_\theta^2 = \sigma_\theta^2$.

TABLE 4. Shifting Production Income and Risk Effects

	$\frac{\partial}{\partial \rho}$	$\frac{\partial \sigma_i^2}{\partial \rho}$
<i>C</i>	$-E[\theta f(L)]$	$-2(1 - \rho)\sigma_\theta^2$
<i>R</i>	$E[\theta f(L)]$	$2\rho\sigma_\theta^2$
Total	0	$-2\sigma_\theta^2 + 4\rho\sigma_\theta^2$

FIGURE 7. Risk is U-shaped With Respect To ρ 

Case 3: Government's Problem with Certainty. In this case the government and representative individual make their choices after the state of nature is realized. The government chooses the tax rates and the individual chooses both β and L . The government's objective function is given below.

$$\max_{\tau_c, \tau_w} M(c, \sigma_c^2, \beta, L) + G(R, \sigma_R^2)$$

subject to

$$c = (1 - \tau_c \beta)(wL(1 - \tau_w) + \pi)$$

$$R = \tau_c \beta (wL(1 - \tau_w) + \pi) + \tau_w wL$$

The first-order conditions with respect to the consumption and wage income tax rates are the Samuelson conditions for this case and state that the marginal benefit of public consumption should equal the marginal benefit of private consumption plus the marginal cost from behavioral changes. The wedge due to the marginal cost from behavioral changes is nonnegative and hence, in this case the provision of public consumption is less than the provision in the first case.⁴⁴ The wedge consists of the elasticities of the tax base factors β and L which characterize the responsiveness of the individual to a given tax. If the individual is very responsive to a tax rate change the elasticities will be large in magnitude.⁴⁵ The utility weights ω_β and ω_L scale the changes in β and labor by their impact on utility.

$$(SC.3_{\tau_c}) \quad FOC_{\tau_c} : \quad G_1 = M_1 - \omega_{\beta, \tau_c} \varepsilon_{\beta, \tau_c} - \omega_{L, \tau_c} \varepsilon_{L, \tau_c}$$

$$(SC.3_{\tau_w}) \quad FOC_{\tau_w} : \quad G_1 = M_1 - \omega_{\beta, \tau_w} \varepsilon_{\beta, \tau_w} - \omega_{L, \tau_w} \varepsilon_{L, \tau_w}$$

The result that tax rates should be set proportional to the inverse of their price elasticities is a special case of equations $(SC.3_{\tau_c})$ and $(SC.3_{\tau_w})$ where the cross-price elasticities are equal to zero.

⁴⁴The wedge is nonnegative because $\omega_{\beta, \tau_i} > 0$, $\omega_{L, \tau_i} > 0$, $\varepsilon_{\beta, \tau_i} < 0$ and $\varepsilon_{L, \tau_i} < 0$.

⁴⁵The elasticities are negative; hence, the more responsive the individual is to a tax rate change the more negative the elasticity is.

Below, this result is demonstrated for the consumption tax rate and its distortion on β from equation ($SC.3_{\tau_c}$). However, this could also be done with equation ($SC.3_{\tau_w}$) for the wage income tax rate. The price of β captures the difference in prices between the taxed and untaxed set of consumption goods, assumed to be one in this model.⁴⁶ Although this result holds only in the special case where the cross-price elasticity is equal to zero and there is certainty in aggregate production, the intuition that tax bases with larger elasticities should be taxed at a lower level holds generally.

$$\begin{aligned}
G_1 &= M_1 - \omega_{\beta, \tau_c} \varepsilon_{\beta, \tau_c} - \omega_{L, \tau_c} \varepsilon_{L, \tau_c} & \omega_{\beta, \tau_c} &= G_1, \text{ assume } \varepsilon_{L, \tau_c} = 0 \\
G_1 &= M_1 - G_1 \frac{\partial \beta}{\partial P_\beta} \frac{\tau_c P_\beta}{\beta P_\beta} & & \text{Multiply by } \frac{P_\beta}{P_\beta} \\
\frac{M_1 - G_1}{G_1} &= \frac{\partial \beta}{\partial P_\beta} \frac{P_\beta}{\beta} \frac{\tau_c}{P_\beta} \\
v &= \varepsilon_{\beta, P_\beta} \frac{\tau_c}{P_\beta} \\
\frac{v}{\varepsilon_{\beta, P_\beta}} &= \frac{\tau_c}{P_\beta}
\end{aligned}$$

Combining the two first-order conditions produces the Ramsey rule in the case of certain aggregate production, given in equation (10) below. The Ramsey rule states the marginal distortion caused by a given tax should be equal across tax bases. The Ramsey rule is a generalization of the inverse elasticity rule demonstrated above. The Ramsay rule for this case (certainty in aggregate production) is generalized to the case with uncertainty in aggregate production in the full government's problem by volatility-adjusted Ramsey rule.

$$(10) \quad \underbrace{\omega_{\beta, \tau_c} \varepsilon_{\beta, \tau_c} + \omega_{L, \tau_c} \varepsilon_{L, \tau_c}}_{\omega_{B, \tau_w} \varepsilon_{B, \tau_w}} = \underbrace{\omega_{\beta, \tau_w} \varepsilon_{\beta, \tau_w} + \omega_{L, \tau_w} \varepsilon_{L, \tau_w}}_{\omega_{B, \tau_w} \varepsilon_{B, \tau_w}}$$

8. IMBALANCED STATE GOVERNMENT PORTFOLIOS

This section produces a sufficient condition for determining whether a government inefficiently relies on a given tax base by rewriting the volatility-adjusted Ramsey rule. The sufficient condition is then estimated using data from U.S. states to determine which states inefficiently rely on the

⁴⁶The elasticity of β with respect to its price is negative, as is v , therefore the left hand side is positive.

income and sales tax bases. The previous section writes the volatility-adjusted Ramsey rule in terms of the elasticities of labor and β to highlight the behavioral costs from taxation. This section writes the volatility-adjusted Ramsey rule in terms of the elasticity of tax revenue with respect to a tax rate to produce a sufficient condition that does not depend on the functional form of utility.

The volatility-adjusted Ramsey rule can be written as the weighted sum of the weighted elasticities of tax revenue and the variance of tax revenue as in equation (11).⁴⁷ The weighted elasticities are the elasticities weighted by the relative amount of tax revenue collected by that base.⁴⁸ The welfare weights in this equation are the same for the consumption and wage income tax rates but depend on the functional form of utility.⁴⁹ However, if the weighted elasticities of both tax revenue and the variance of tax revenue are larger in magnitude for a given tax base relative to another tax base then the volatility-adjusted Ramsey rule is violated irrespective of the welfare weights. Therefore, it is sufficient to demonstrate that the weighted elasticities are both larger for a given tax base to demonstrate a government inefficiently relies on that tax base.

$$(11) \quad \omega_R \hat{\varepsilon}_{R, \tau_c} + \omega_{\sigma_R^2} \hat{\varepsilon}_{\sigma_R^2, \tau_c} = \omega_R \hat{\varepsilon}_{R, \tau_w} + \omega_{\sigma_R^2} \hat{\varepsilon}_{\sigma_R^2, \tau_w}$$

The four elasticities in equation (11) are estimated to determine whether a government relies inefficiently on a given tax base. This section estimates the elasticity of tax revenue and the elasticity of the variance of tax revenue with respect to the income and sales tax rate for each U.S. state.⁵⁰

$$(12) \quad \log(R_i) = \pi_0 + \log(\tau_i)\pi_1 + \log(\boldsymbol{\tau})\boldsymbol{\pi}_2 + \log(\mathbf{x})\boldsymbol{\pi}_3$$

The elasticity of tax revenue with respect to a tax rate is π_1 . From equation 13 the elasticity of the variance of tax revenue with respect to the tax rate can be estimated in a similar manner where $\varepsilon_{\sigma_R, \tau_i} = \xi_1$. Finally, both of these elasticities are appropriately weighted to produce the weighted

⁴⁷The additional assumption that $\hat{\varepsilon}_{\sigma_c, \tau_c} = \hat{\varepsilon}_{\sigma_c, \tau_w}$ is made for simplicity. This assumes the variance of private consumption depends on the amount of revenue collected but not how it is collected. Without this assumption, another elasticity for each tax base would need to be estimated to determine the sufficient condition.

⁴⁸The weighted elasticities are given by the following expressions: $\hat{\varepsilon}_{R, \tau_c} = \frac{R}{\tau_c \beta y} \varepsilon_{R, \tau_c}$, $\hat{\varepsilon}_{\sigma_R^2, \tau_c} = \frac{\sigma_R^2}{\tau_c \beta y} \varepsilon_{\sigma_R^2, \tau_c}$, $\hat{\varepsilon}_{R, \tau_w} = \frac{R}{\tau_w w L(1-\tau_c \beta)} \varepsilon_{R, \tau_w}$, $\hat{\varepsilon}_{\sigma_R^2, \tau_w} = \frac{\sigma_R^2}{\tau_w w L(1-\tau_c \beta)} \varepsilon_{\sigma_R^2, \tau_w}$.

⁴⁹The welfare weights in the volatility-adjusted Ramsey rule are: $\omega_R = G_1$ and $\omega_{\sigma_R^2} = G_2$.

⁵⁰This section focuses on the income and sales tax because they are the two major sources of tax revenue for most states.

elasticities in the volatility-adjusted Ramsey rule.

$$(13) \quad \log(\sigma_{R_i}^2) = \xi_0 + \log(\tau_i)\xi_1 + \log(\tau)\xi_2 + \log(\mathbf{x})\xi_3$$

I estimate equations (12) and (13) using a three step process. The first step estimates inverse probability weights which estimate the similarity between states in their observable characteristics.⁵¹ The second step estimates a weighted, seemingly unrelated regression of equations (12) and (13) for each state. These equations could be estimated for each state using only data from the state for the years 1963-2010 but other state's experiences are informative and are used to supplement the state's data by weighting other states based on how informative its experience is. For example, Wisconsin's data has a high weight in Minnesota's estimation but a low weight in California's because Wisconsin and Minnesota are more similar than Wisconsin and California.

The third and final step uses the estimated elasticities and calculates time-varying elasticities. The time-varying elasticities are calculated by multiplying the estimated mean elasticities by the mean of the ratio of the dependent and independent variables and the ratio of the independent and dependent variable for a given year, shown in equation (14).⁵² These calculations produce four elasticities for each state for all years between 1963 through 2010.⁵³ Comparing these elasticities determines whether the sufficient condition for imbalance is met for a given state in a given year.

$$(14) \quad \varepsilon_{\sigma_{R_i}^2, \tau_i, t} = \hat{\varepsilon}_{\sigma_{R_i}^2, \tau_i} \frac{\bar{\tau}_i}{\sigma_{R_i}^2} \frac{\sigma_{R_i, t}^2}{\tau_{i, t}} \quad \varepsilon_{R, \tau_i, t} = \hat{\varepsilon}_{R, \tau_i} \frac{\bar{\tau}_i}{\bar{R}_i} \frac{R_{i, t}}{\tau_{i, t}}$$

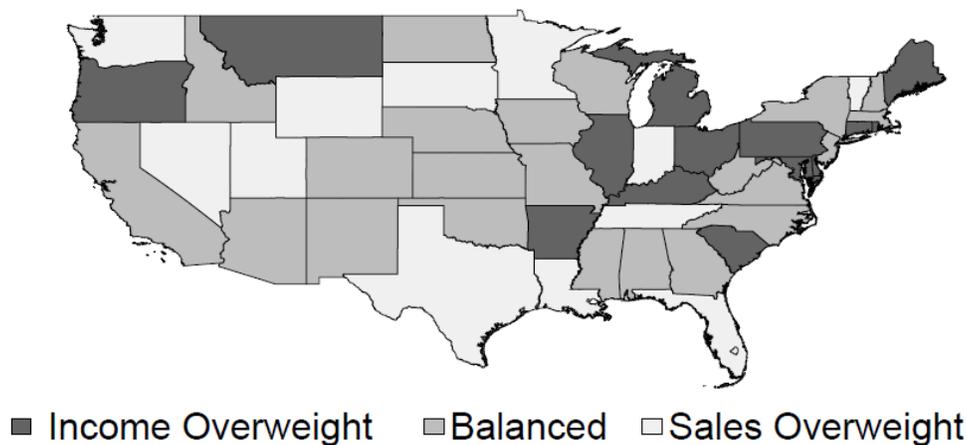
In 1965 fourteen states relied too heavily on the income tax and twelve relied too heavily on the sales tax, with the remainder not satisfying the sufficient condition for imbalance, mapped in figure 8. The number of states that inefficiently relied on the income tax increased by twelve between 1965 and 2005. In contrast, the number of states that relied inefficiently on the sales tax decreased by two in the same time period. Figure 9 maps the twenty-six states that relied too heavily on the

⁵¹The weights can be calculated parametrically using a probit or semi-parametrically using a kernel estimation. The baseline results reported use a probit and the results are robust to using a kernel estimation.

⁵²This final step assumes the derivative in the elasticity is constant over time.

⁵³These calculations produce 9400 elasticities. The weighted tax rates and revenues are used to impute tax rates and revenues that are zero.

FIGURE 8. Imbalanced State Tax Portfolios 1965.



Alaska and Hawaii balanced, not shown.

Data: US Census 1951 - 2010.

Estimates of elasticity of base and volatility with respect to tax rate for each state for the sufficient condition.

States without sales tax; DE, MT (3% local), OR (5 % local). States without income tax; FL, NV, SD, TN, TX, WA, WY.

Variance elasticities differ at the five percent level across bases for all states except: KY, MS, MO, ND, AS, NM.

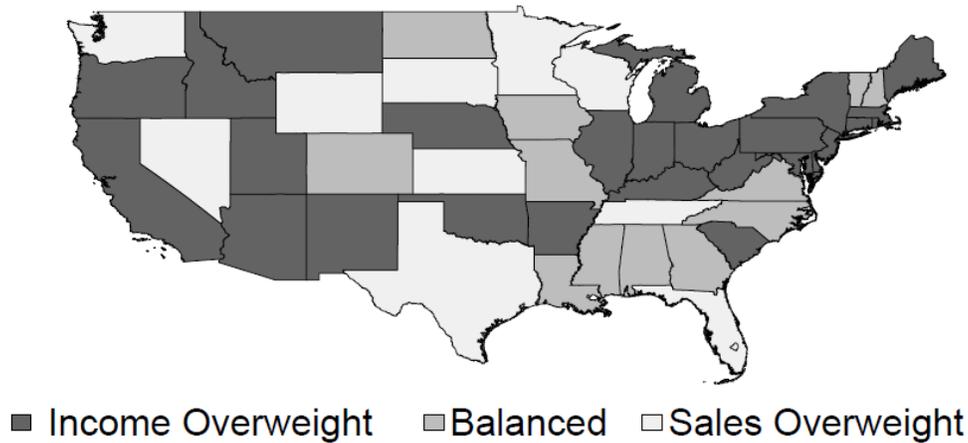
Base elasticities differ at the five percent level across bases for all states except: CA, KS, MT, ND, NY, OK, WI.

income tax and the ten states that relied too heavily on the sales tax in 2005. Comparing these two maps reinforces the result that tax policy is important in explaining the increase in volatility by demonstrating the increased reliance on the income tax between 1965 and 2005.⁵⁴

State governments expose their tax revenues to unnecessary levels of risk when they rely inefficiently on one tax base. In decades with little economic volatility, tax revenue from states that rely inefficiently on a tax base look similar to those that do not. However, in decades with increased economic volatility, such as the 2000s, states that rely inefficiently on a tax base experience elevated levels of tax revenue volatility. I find a positive correlation between states that hold imbalanced tax portfolios and states with the largest increases in volatility in the 2000s. The correlation is positive

⁵⁴To determine whether a state's tax portfolio is imbalanced two sets of elasticities are compared. The differences are reported in figures 8 and 9. These differences are statistically significant at the five percent level for all states except for Kentucky, Mississippi, Missouri, North Dakota, Arkansas, and New Mexico for the variance elasticities and California, Kansas, Montana, North Dakota, New York, Oklahoma, and Wisconsin for the base elasticities.

FIGURE 9. Imbalanced State Tax Portfolios 2005.



Alaska and Hawaii balanced, not shown.

Data: US Census 1951 - 2010.

Estimates of elasticity of base and volatility with respect to tax rate for each state for the sufficient condition.

States without sales tax; DE, MT (3% local), OR (5 % local). States without income tax; FL, NV, SD, TN, TX, WA, WY.

Variance elasticities differ at the five percent level across bases for all states except: KY, MS, MO, ND, AS, NM.

Base elasticities differ at the five percent level across bases for all states except: CA, KS, MT, ND, NY, OK, WI.

for both states that inefficiently rely on the income tax and those that inefficiently rely on the sales tax, demonstrating the importance of balance and not just stable tax bases.

9. CONCLUSION

The main contribution of this paper is to provide theoretic and empirical evidence of the importance of tax revenue volatility. Empirically, tax revenue volatility at the state level has increased dramatically in the 2000s. This paper provides strong evidence that changes in tax policy, specifically the increased reliance on the income tax base, explains most of the increase in tax revenue volatility.

An optimal tax system must consider the costs of revenue volatility. I update the Ramsey rule to include the costs of volatility, demonstrating the tradeoff governments face between costs from volatility and deadweight loss due to behavioral changes. The volatility-adjusted Ramsey rule is

applied to the data to test whether state governments set their tax rates efficiently. Between 1965 and 2005 the number of states that set their tax rates inefficiently increased by almost forty percent such that by 2005 almost three-fourths of all states could change their tax portfolios to lower the costs from volatility and deadweight loss due to behavioral changes without decreasing their level of tax revenue.

The methods in this paper can be applied to other governments and can help diagnose the causes of their tax revenue volatility allowing them to create policies to dampen it. Dampening tax revenue volatility may be even more important for developing countries because of the capital market frictions they face which make smoothing tax revenue shocks costly. The empirical test of the volatility-adjusted Ramsey rule provides governments with a benchmark to test whether they are inefficiently relying on a given tax base. Efficiently relying on different tax bases is important because it may dampen the feedback loop between government uncertainty and production uncertainty which can cause slow productivity growth.

This paper focused on tax policy which is only one of three important ways governments can handle uncertainty. The interplay between tax policy and government expenditures and savings (through the use of rainy day funds) remains an important area of research. For example, the extent to which tax revenues should be procyclical depends crucially on whether government expenditures are complements or substitutes to private consumption. It would be interesting to know whether governments that spend more on goods and services complementary to private consumption have tax revenues that are more procyclical. My paper serves as a starting point for these investigations into how governments manage uncertainty to minimize its negative impacts to the economy.

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10. APPENDIX

10.1. Consumption Decomposition. In the text private consumption of the representative agent is decomposed into consumption that is taxed and consumption that is untaxed such that the fraction β of total consumption is taxed and $(1 - \beta)$ is untaxed. This decomposition changes the variables from two consumption goods into total consumption and the fraction spent on taxable items. This section demonstrates the change of variables and its benefits.

First, start with two goods B, N such that the consumption of B is taxed and the consumption of N is not taxed and the representative agent has utility $V(B, N)$ over the two goods. By definition $B = \beta c$ and $N = (1 - \beta)c$. The utility function can be written as a function of β and c by substituting these equations in for B and N .⁵⁵ The budget constraint is given below written both as a function of B and N and β and c .

$$\begin{aligned} W &= (1 + \tau_c)B + N \\ &= (1 + \tau_c)\beta c + (1 - \beta)c \\ &= c(1 + \beta\tau_c) \end{aligned}$$

10.2. Higher Order Terms. In the text the expected utility is assumed to be fully characterized by the first two moments of the public and private good, which is sufficient when the goods are jointly normally distributed or when the utility function is quadratic. Generally, the expected utility can be written as in equation 15 below where Ω consists of the second moment and higher that is necessary to fully characterize the joint distribution between public and private goods. This composition of the expected utility is much more general than the case where the joint distribution is normal but is not fully general because not every distribution can be uniquely characterized by

⁵⁵If the utility function is homothetic then the utility function can be written as $V(B, N) = v(\beta)U(c)$ otherwise $V(B, N) = U(c, \beta)$.

its moments. However, in the case that the joint distribution is normal the distribution can be fully characterized by the first two moments and Ω consists solely of the second moments of the private and public good. If the utility function is additive, such that $U_{1,2} = 0$, then the level of social welfare can be written as the second line in the equation below.

$$\int U(c(\theta), G)f(\bar{c}, \bar{R}, \Omega) \equiv M((\bar{c}, \bar{R}, \Omega)$$

(15)

$$= M((\bar{c}, \Omega_c) + G((\bar{R}, \Omega_R) \quad \text{When } U_{1,2} = 0$$

(16)

To generalize the formulas in the text to the case where higher moments are needed to characterize the expected utility replace all of the partial derivatives of the second moment with the partial derivative of Ω .

To demonstrate this transformation consider a Cobb Douglas utility where total consumption is assumed to be distributed uniformly with mean μ and standard deviation σ . Writing the utility function in terms of total consumption c and the shift parameter β gives the following form where the density function is $\frac{1}{2\sigma\sqrt{3}}$ for $c \in [-\sqrt{3}\sigma, \sqrt{3}\sigma]$ and zero everywhere else.

$$E[U(c, \beta)] = E[\log c + \alpha \log \beta + (1 - \alpha) \log(1 - \beta)]$$

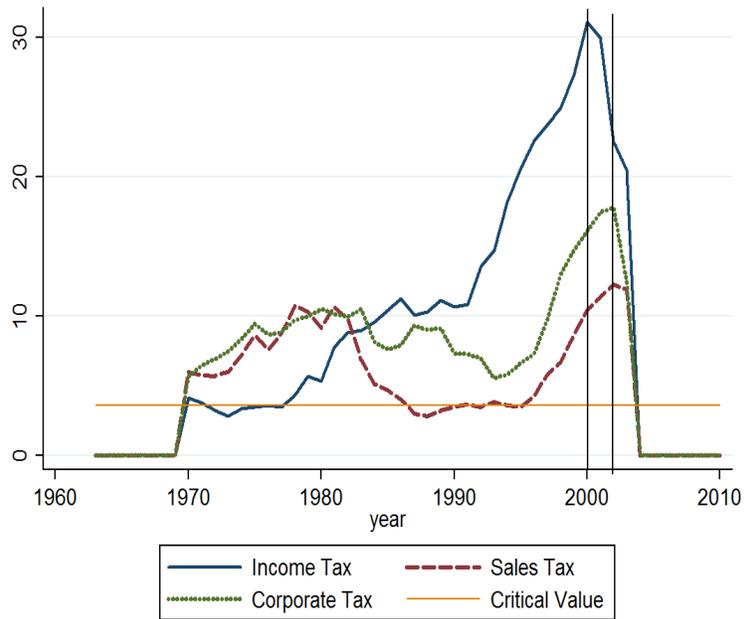
$$= E[\log[c]] + \alpha \log \beta + (1 - \alpha) \log(1 - \beta)$$

$$= \int_{-\sqrt{3}\sigma}^{\sqrt{3}\sigma} \log c \frac{1}{2\sigma\sqrt{3}} dc + \alpha \log \beta + (1 - \alpha) \log(1 - \beta)$$

$$M(\mu, \sigma^2, \beta) = \frac{(\sigma\sqrt{3} + \mu)(\log(\mu + \sigma\sqrt{3}) - 1) + (\sigma\sqrt{3} - \mu)(\log(\mu - \sigma\sqrt{3}) - 1)}{2\sigma\sqrt{3}}$$

$$+ \alpha \log \beta + (1 - \alpha) \log(1 - \beta)$$

FIGURE 10. Quandt Likelihood Ratio: Finding Structural Breaks



The preceding line is a function of the mean, standard deviation, and β alone.

10.3. Statistical Tests. Figure 10 depicts the Quandt Likelihood Ratio which determines the structural break for the model. This figure demonstrates the break occurred in the early 2000s and is statistically significant. Figure 11 demonstrates that the regressions contain variables that are stationary. The volatility measures are stationary because they are measures that have filtered out the time trend and the Adjusted Dickey-Fuller test formally shows this. Finally, figure 12 is a scatter plot of the corporate tax rate by year for all states. This figure demonstrates the data before and after 2000 look similar and formally have enough overlap to run the weighted regressions.

10.4. Weighting Decomposition Method. The decomposition method introduced by DiNardo, Fortin, and Lemieux (1996) provides a method for estimating counterfactual distributions without assuming linearity (assumption 1). Similarly to the regression decomposition the estimated counterfactual distributions of the volatility are used to decompose the contribution of each of the factors. The actual and counterfactual distributions, given in equation 17, differ by the densities they are

FIGURE 11. Adjusted Dickey-Fuller Test Statistics: Stationarity

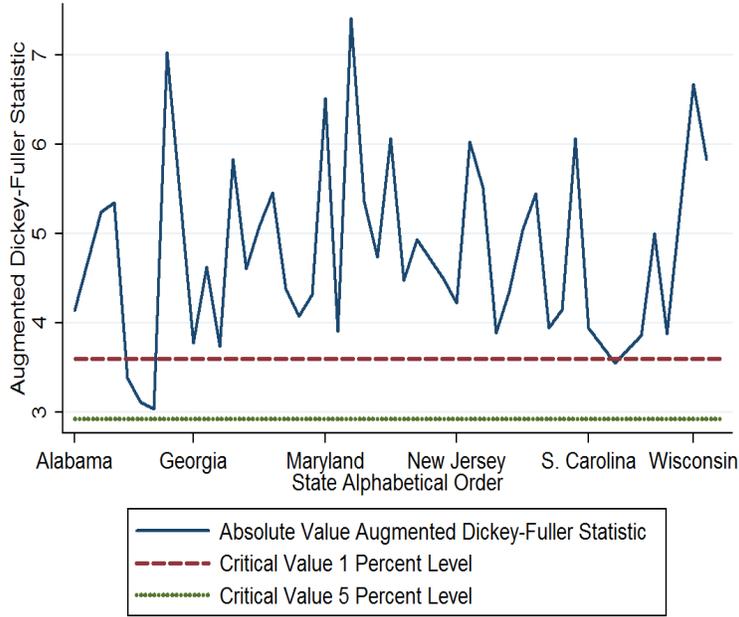
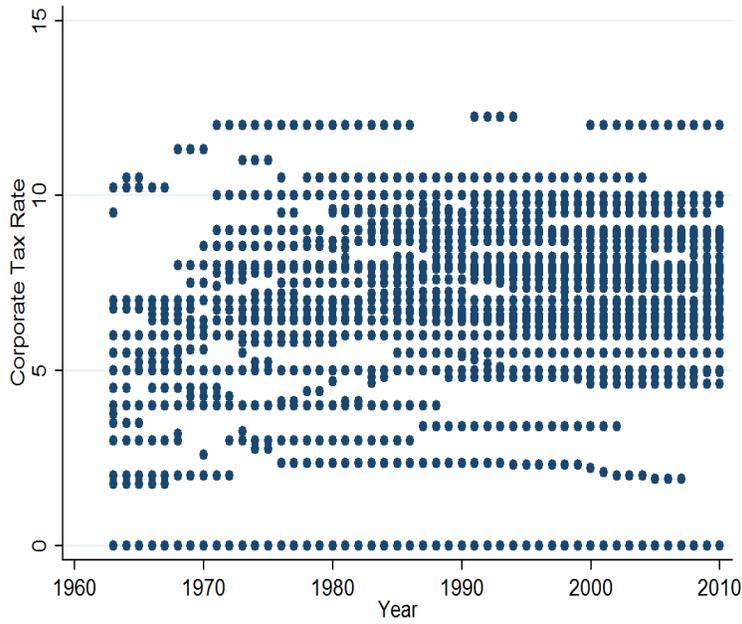


FIGURE 12. Scatter Corporate Tax Rate by Year



integrated over.⁵⁶

$$\text{Actual Distribution } f_1^1(\text{Log}(\text{Revenue}_{i,t})) \equiv \int f(\text{Log}(\text{Revenue}_{i,t})|z)h(z|D = 1)dz \quad (17)$$

$$\text{Counterfactual Distribution } f_0^1(\text{Log}(\text{Revenue}_{i,t})) \equiv \int f(\text{Log}(\text{Revenue}_{i,t})|z)h(z|D = 0)dz$$

⁵⁶In equation 17 z represents all observable characteristics, tax and economic.

The important insight of DiNardo, Fortin, and Lemieux (1996) is that the counterfactual distribution can be written as a weighted function of the actual distribution. The weight is the ratio of the conditional density functions which by Bayes' rule can be rewritten as the ratio of propensity scores normalized by the number of observations in each group, $\omega = P(D = 1|z)/P(D = 0|z)(P(D = 1)/P(D = 0))$.⁵⁷ This realization by DiNardo, Fortin, and Lemieux (1996) transforms a possibly impossible problem of integration over many variables into a simple reweighting problem where the weights can be estimated by a logit or probit model.

$$\text{Counterfactual Distribution } f_0^1(\text{Log}(\text{Revenue}_{i,t})) \equiv \int \omega f(\text{Log}(\text{Revenue}_{i,t})|z)h(z|D = 1)dz$$

The increase in volatility of tax revenue can be decomposed using different counterfactual distributions. The increase that cannot be explained by differences in observable characteristics is again attributed to the structural change, which captures the second hypothesis. Formally, this is given by the difference between the mean of the actual distribution of the years after the structural break and the mean of the counterfactual distribution that would have occurred if all of the observable characteristics had been similar to those after the structural break. This is similar to the effect of the treatment on the treated (TOT).

The rest of the increase in volatility is what can be explained by observable characteristics. The marginal effect that can be explained by economic factors is given by the difference in the means of the counterfactual distribution that would have occurred if all observable variables would have been similar to the characteristics in the years after the structural break and the counterfactual that would have occurred if only the tax variables would have been similar to the characteristics of the states after the structural break. Similarly, the marginal tax effect can be found by the difference of the means of the two counterfactual distributions formally given in equation 18. The conditional weights $\omega_x = P(D = 1|\tau)/P(D = 0|\tau)(P(D = 1)/P(D = 0))$ and $\omega_\tau = P(D = 1|x)/P(D =$

⁵⁷The weight is $h(z|D = 0)/h(z|D = 1)$ where $h(z|D = 1) = h(z_j = z_0)P(D = 0|z_j = z_0)/P(D = 0)$ by Bayes' rule.

$0|x)))(P(D = 1)/P(D = 0))$ are used to calculate the other two counterfactual distributions.

$$\begin{aligned}
 & \text{Tax Base Factors} && \int \text{Log}(\text{Revenue}_{i,t})f(\text{Log}(\text{Revenue}_{i,t})|z)h(z|D = 1)dz \\
 & && - \int \omega \text{Log}(\text{Revenue}_{i,t})f(\text{Log}(\text{Revenue}_{i,t})|z)h(z|D = 1)dz \\
 (18) \quad & \text{Business Cycle Factors} && \int \omega \text{Log}(\text{Revenue}_{i,t})f(\text{Log}(\text{Revenue}_{i,t})|z)h(z|D = 1)dz \\
 & && - \int \omega_x \text{Log}(\text{Revenue}_{i,t})f(\text{Log}(\text{Revenue}_{i,t})|z)h(z|D = 1)dz \\
 & \text{Tax Policy Factors} && \int \omega \text{Log}(\text{Revenue}_{i,t})f(\text{Log}(\text{Revenue}_{i,t})|z)h(z|D = 1)dz \\
 & && - \int \omega_\tau \text{Log}(\text{Revenue}_{i,t})f(\text{Log}(\text{Revenue}_{i,t})|z)h(z|D = 1)dz
 \end{aligned}$$

This method controls for nonlinearities and is asymptotically more efficient than matching or regression models (Hirano, Imbens, and Ridder, 2003). In this context controlling for nonlinearities will decrease the upward bias in the structural factor estimates from the regression analysis. The typical concern with this method is a selection bias, for example, when individuals choose their group based on unobservable characteristics. This selection bias is a violation of the second assumption above, $E[\epsilon|x, \tau, D, I.state] = 0$. While the selection bias is not an issue in this context because states cannot choose their groups, the second assumption may still be violated if endogenous variables are included. Finally, this method depends on the occurrence of observations that “look similar” in both groups of years, formally that there is sufficient overlap of independent variables. Overlap would be a problem if the set of state tax rates in the early years were disjoint from the set of tax rates in the later years. Figure 4 is a scatter plot of the corporate tax rates for all states for the year 1963 to 2010 and demonstrates graphically sufficient overlap.

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