Question 1: (courtesy of Justin) Given a vector space $V$ over a field $k$ and $v_1, \ldots, v_n \in V$, we obtain a function $T : k^n \to V$ defined by $T(a_1, \ldots, a_n) = a_1v_1 + \cdots + a_nv_n$.

(a) prove that $T$ is a linear function

(b) prove that $T$ is injective if and only if $v_1, \ldots, v_n$ are linearly independent.

(c) prove that $T$ is surjective if and only if $\text{Span}(v_1, \ldots, v_n) = V$.

Question 2: Given a field $k$ and a non-negative integer $n$, show that any two vector spaces over $k$ of dimension $n$ are isomorphic.

Question 3: Given a vector space $V$ and a subvector space $W$, we defined $V/W$ in class. Check that addition and scalar multiplication are well-defined. Prove that $V/W$ is a vector space under these operations.