Name: Date:

Problem Set # 2

Instructions: *Numerical answers require units and appropriate numbers of significant digits.* ***Show your work!***

Review Problem:

R-1. (4 points) Consider the right triangle shown:

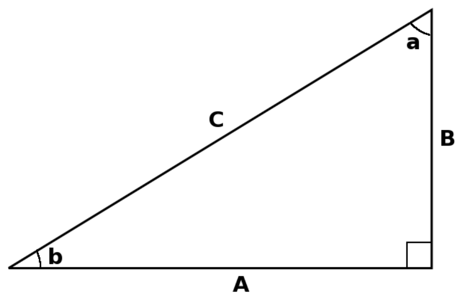

Figure 1: A right triangle

Show that:

(a) $b = 90 - a$
(b) $\sin^2 a + \cos^2 a = 1$
(c) $\tan b = \dfrac{\sin b}{\cos b}$
(d) $\sin b = \cos(90 - b) = \cos a$

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1. (3 points) Backpackers, sailors, and others caught outdoors in a storm often estimate how far away it is from the number of seconds between the time they see lightening and the time they hear thunder. Since light travels much faster than sound, assume that the light arrives the instant it leaves the storm, while the sound travels at about 1000 feet per second. Give a simple rule relating the distance of the storm in miles to the time difference in seconds.

2. (5 points) Using raypaths to demonstrate the propagation of the 2011 Japan tsunami will result in Fig. 2(a). Comparing with Fig. 2(b) which is a map of ocean bathymetry, explain why we observe the focusing and defocusing in the ray paths (Hint: use the equation for tsunami speed from problem set 1).

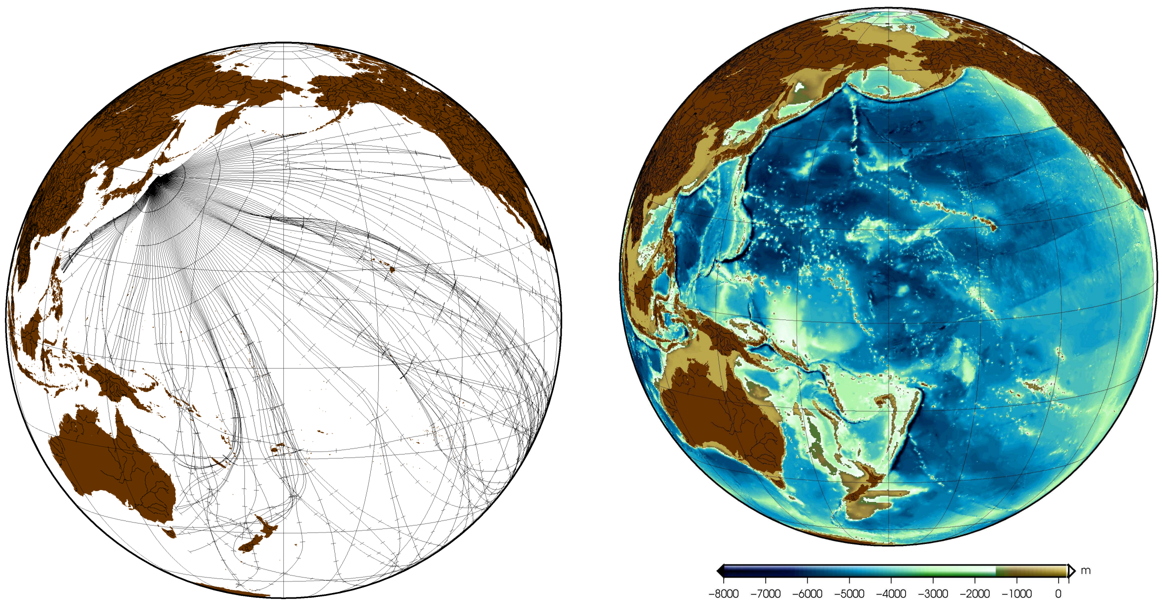


Figure 2: (a) Raypath representation of 2011 Japan Tsunami and (b) Bathymetry of the Pacific.

2. (4 points) On a seismometer located at an earthquake epicenter PcP and ScS arrive 8 minutes, 34 seconds and 15 minutes, 36 seconds respectively after the earthquake. If the earth's radius at this point is 6371 km, and the core's radius is 3471 km, find the average P and S wave velocities in the earth's mantle. (Remember it takes time to go both up and down!). When would the phase PcS arrive? (Note: assume that both paths are vertically incident).

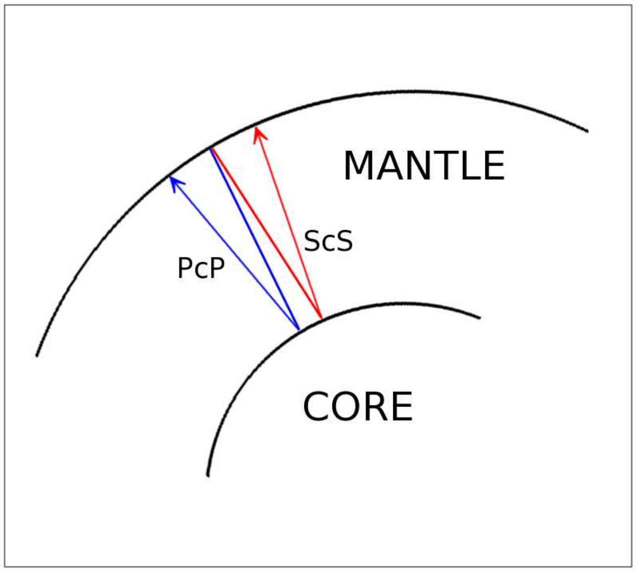


Figure 3: *PcP* and *ScS*

3. (5 points) Consider two media, one with density ρ_1 and velocity v_1, and the other with density ρ_2 and velocity v_2. How are the reflection coefficients related between a wave incident from layer 1 (R_{12}) and a wave incident in layer 2 (R_{21})? Relate the same for the transmission coefficients of a wave going from layer 1 to layer 2 (T_{12}), and a wave going from layer 2 to layer 1 (T_{21}).

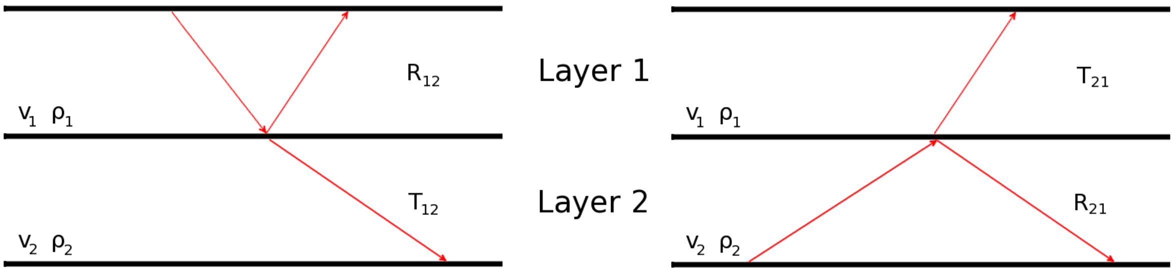
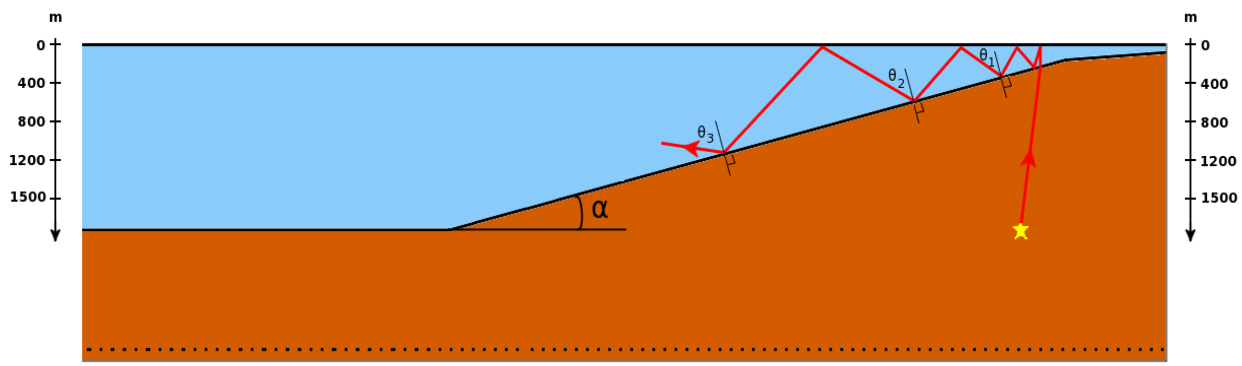


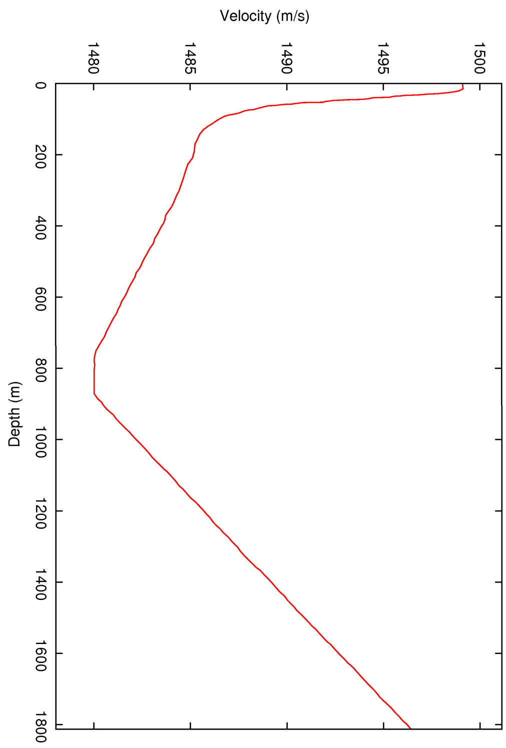
Figure 4: The 2-layer setup.

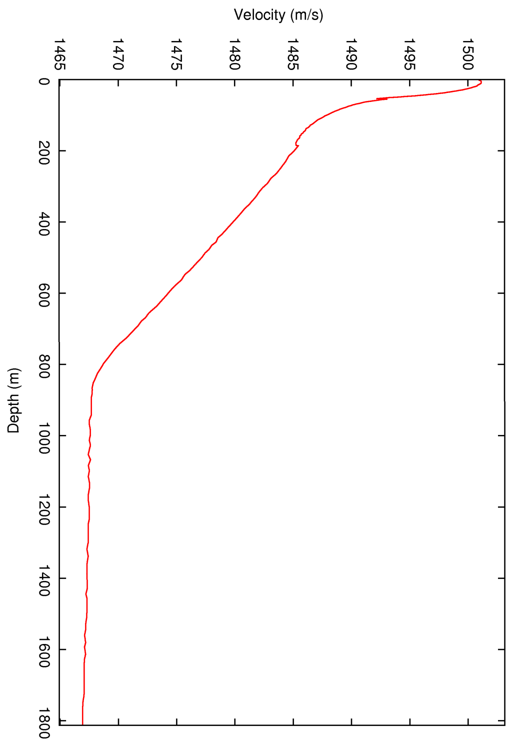
4. (25 points) In the figure below, the rays (=waves) from an earthquake (yellow star) arrive at the surface and reverbate between the sea floor and the ocean surface. An example of such rays is shown in red. Answer the following questions.



a. In the presence of a beach sloping at an angle α, show how much the rays are deflected at each successive reflection from the sea floor (Hint: What are the values of angles θ_2, θ_3, compared to θ_1?)

b. From the two profiles below, choose the one that shows how the wave velocity changes with depth in the ocean. Explain your answer.





c. Using your answer to (b), sketch the raypath in the ocean by continuing the red ray (you can either draw it on the figure, or sketch it on a separate piece of paper).

5. (10 points) Figure 5 shows waves travelling on a string whose right and left sides have a different velocity and density.

(a) Use the positions of the waves at successive times to find the velocity on either side.

(b) Measure the amplitude of the wave incident on the junction (dashed line) at time 1, and the amplitudes of the reflected and transmitted waves at time 2. Find the reflection and transmission coefficients from the ratio of these amplitudes.

(c) Derive a relationship, using the expression for the transmission coefficient, for the ratio between the densities of the two media. Which medium is the denser?

(d) If the density of the right hand medium is $\rho = 5$ g/cm^3 determine the density of the left hand medium.

(e) Do these waves more nearly represent P-waves or S-waves? Why?

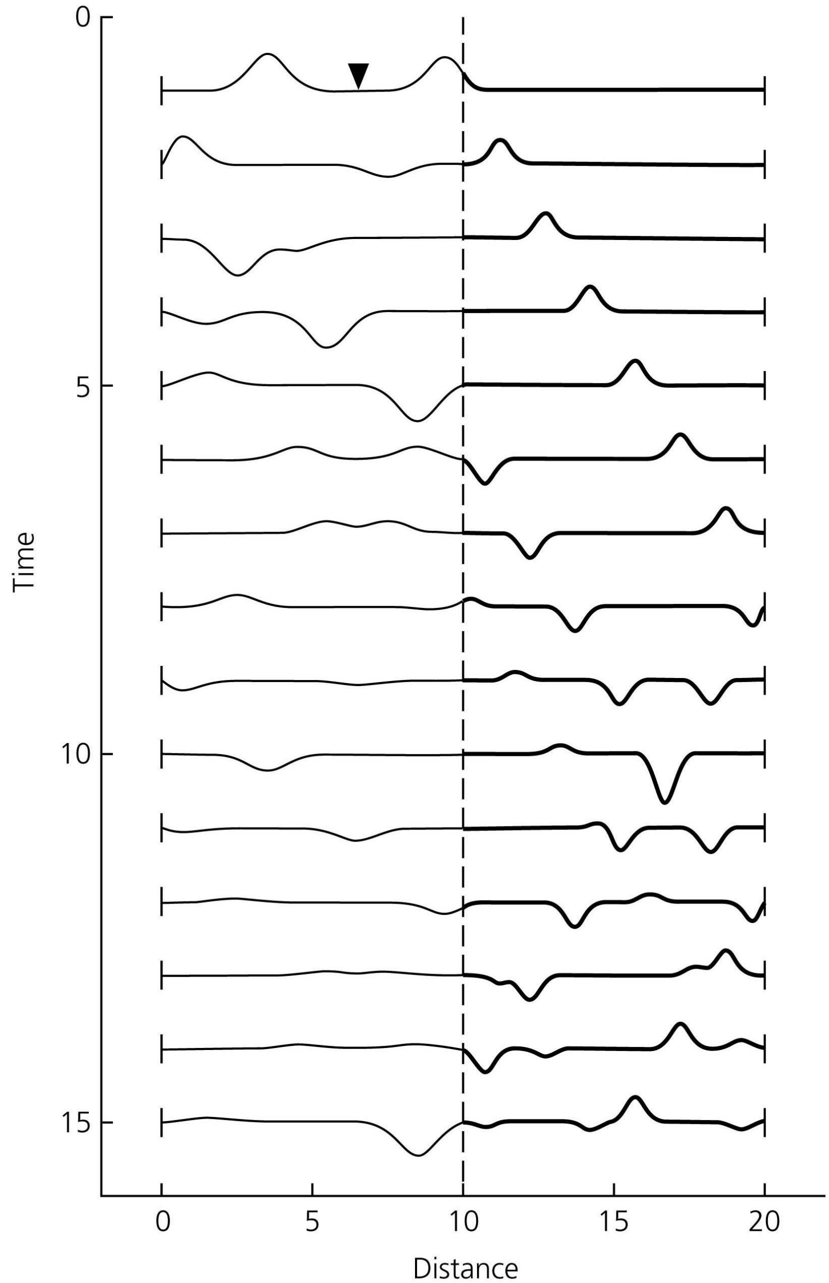
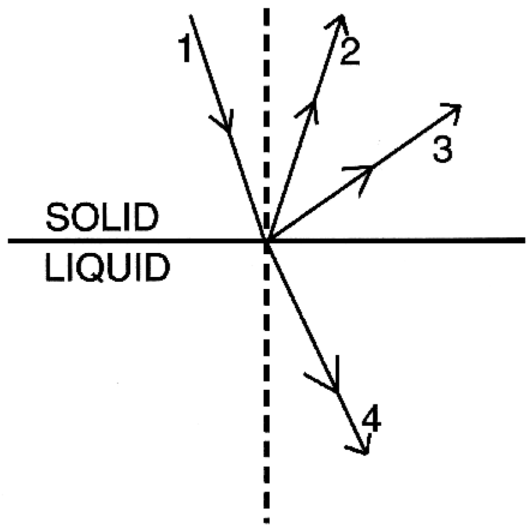


Figure 5: Waves in the string (Fig. 2.2.6 in Stein & Wysession).

6. (7 points) Identify the four wave types shown (P or S). Assuming the P velocity in the solid is 12 km/s, find the P and S velocities in both media. (Hint: Use a protractor to measure the angles).



7. (3 points) Estimate the depth of the earthquake using the seismogram below from the time separation between P and pP. Assume that the velocity near the surface is 6.5 km/s and that the time difference is due to pP going up to the surface and back down, travelling vertically.

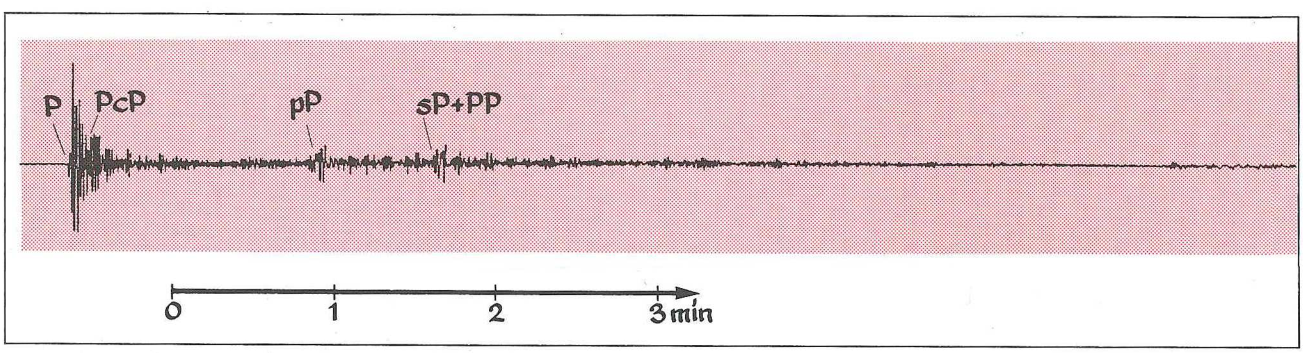
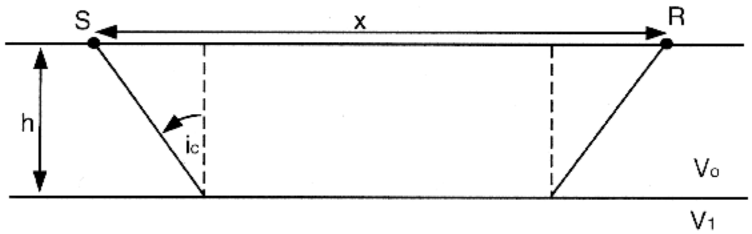


Figure 6: From Kulhanek (1990).

8. (15 points) For a layer of thickness h, composed of material with velocity v_0 above a half-space which consists of material with a velocity v_1 as shown below.



(a) Express the travel time $T_{direct}(x)$ of the direct wave to the receiver as a function of the source-receiver distance x.

(b) Show that the travel time of the reflected wave $T_{reflect}(x)$ can be represented as,

$$T_{reflect} = \frac{(x^2 + 4h^2)^{\frac{1}{2}}}{v_0}$$

To test this, determine the travel time for the reflection at zero distance (zero offset).

(c) Express the critical angle i_c as a function of the velocities v_0 and v_1.
(d) Show that the travel time for a critically refracted (or head) wave at a distance x is:

$$T_{refract} = \frac{x}{v_1} + 2h\frac{(v_1^2 - v_0^2)^{\frac{1}{2}}}{v_0 v_1}$$

(e) Derive an expression for the minimum distance at which the refracted wave may be observed in terms of the layer thickness h and the velocities.

(f) Show that the cross-over distance (the distance at which the direct and refracted waves arrive at the same time) is given by

$$X_c = 2h(\frac{v_1 + v_0}{v_1 - v_0})^{\frac{1}{2}}$$

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C-1. (9 points) Write a spreadsheet or program to calculate the travel times for the direct wave, the reflected wave and the refracted wave as a function of distance.

(a) Plot T_{direct}, $T_{reflect}$, and $T_{refract}$ using $h = 40$ km, $v_0 = 6$ km/s and $v_1 = 8$ km/s.

(b) Calculate the critical angle and the crossover distance using $h = 40$ km, $v_0 = 6$ km/s and $v_1 = 8$ km/s.

(c) Calculate the minimum distance at which the refracted wave may be observed using $h = 40$ km, $v_0 = 6$ km/s and $v_1 = 8$ km/s.

In addition to your answers for parts (a) to (c), please hand in a printout or screenshot of your code or excel spreadsheet.