Name: Date:

Problem Set # 1

Instructions: *Numerical answers require units and appropriate numbers of significant digits.* ***Remember to show your work!***

Review Problems:

R-1. (3 points) For the function

$$f(x) = 4x^3 - 6x + 2 \tag{1}$$

(a) Find the indefinite integral, $F(x)$, of this function.
(b) Evaluate the constant of integration if $F(x = 2) = 23$.
(c) Check your answer by differentiating $F(x)$ to show that

$$\frac{dF(x)}{dx} = f(x) \tag{2}$$

R-2. (4 points) Unit conversion:

Unit conversion is an important part of any scientific research. Please view the following link:

`http://youtu.be/Xr9L7rZqT34`

Here, we are going to work on a simple example.

Speed of tsunamis, c, depends *only* on the ocean depth and is given by the following formula in the high seas.

$$c = (gh)^{\alpha} \tag{3}$$

where g is the acceleration due to gravity and h is the ocean depth at any point.

(a) Using the units (dimensions), find the exponent α.

(b) Assuming that the typical ocean depth is 4000 m, calculate the typical speed of tsunami waves in m/s.

(c) Calculate the tsunami speed in km/h.

(d) Calculate the tsunami speed in mi/h.

(e) How do these numbers compare with the speed of a jet airliner?

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1. (4 points) Consider the equations for the gravitational force and the electrostatic force:

$$F_g = \frac{Gm_1m_2}{r^2} \qquad F_E = \frac{kq_1q_2}{r^2}$$

(a) Compare the two equations. In what ways do they differ?

(b) A large ship (mass $= 6 \times 10^7$ kg) is out at sea when a small fish (mass $= 2$ kg) swims 1 meters below it. How much force does the fish exert on the ship? How much force does the ship exert on the fish? Should the fish be worried about getting pulled to the ship? Why or why not?

2. (5 points) To determine the size of the moon, use the fact that it appears to be the same size as a dime held 154 cm from the observer (this is called having the same *angular diameters*). If the moon is 3×10^5 km away, what is the *radius* of the moon? How does the moon compare in volume to the earth?

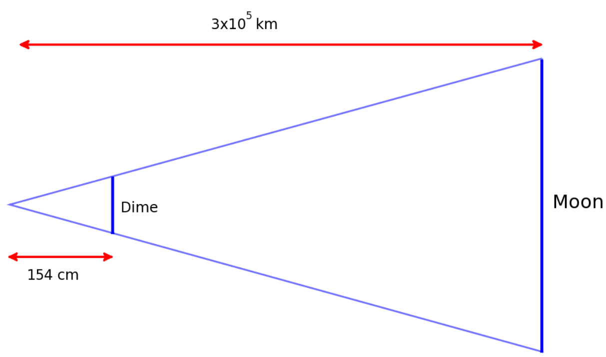

Figure 1: Dime and the Moon.

3. (5 points) The acceleration of gravity on the lunar surface is about $\frac{1}{6}$ that on the earth's surface. Given the moon's radius from problem 2, find its mass and average mean density.

4. (3 points) Your lazy roommate thinks that instead of actually working on problem 3, you can instead say that if the acceleration of gravity on the moon was $\frac{1}{6}$ that on earth, its mean density is $\frac{1}{6}$ that of earth. Why isn't this correct?

5. (3 points) A model often used for the moon is that it is made of (green) cheese. Test this model by comparing its density to a block of Monterey Jack with dimensions 4 cm $\times$ 6 cm $\times$ 6 cm and a mass of 143 grams.

6. (4 points) Show how gravity at the Moon's surface would change if its:

(a) Radius is kept constant, but density is doubled;
(b) Radius is doubled, but density is kept constant.

7. (6 points) We saw in class that a body in orbit has an orbital period which depends on its distance from the center of the planet being orbited. Some satellites are put in *synchronous orbit* – that is, with a period equal to one planetary day so that they stay above the same point on the surface as the planet rotates. For the planet Mars, how large is the radius of this orbit? How high above Mars's *surface* is the satellite?

8. (4 points) Moment of inertia factors I/MR^2 (and other information) for the various planets can be found on the website `http://nssdc.gsfc.nasa.gov/planetary/planetfact.html` or on the class website `http://www.earth.northwestern.edu/people/seth/202` by clicking on the planetary fact sheet, which is found underneath "TOPIC 1: Size, Mass, & Density of the Earth". Find the values for the Sun, Venus, Mars, the moon, Earth, and Jupiter. Put these values in order from largest to smallest and explain what they tell about the density distribution (the way that the density changes from the surface to the center of an object).

9. (8 points) The formulas for calculating the mass and moment of inertia of a planet are:

$$M = 4\pi \int_0^a \rho(r) r^2 \mathrm{d}r$$
$$I = \frac{8}{3}\pi \int_0^a \rho(r) r^4 \mathrm{d}r$$

Given these formulas, show that a two-shell planet with mantle density ρ_m, core density ρ_c, radius a, and core radius r_c, has a mass and moment of inertia:

$$M = \frac{4}{3}\pi[\rho_m a^3 + (\rho_c - \rho_m) r_c^3]$$
$$I = \frac{8}{15}\pi[\rho_m a^5 + (\rho_c - \rho_m) r_c^5]$$

C-1. (8 points) Using the results of problem 9:

(a) Write a spreadsheet or program that takes as inputs a, ρ_m, ρ_c, and r_c and computes r_c/a, I, M, and I/Ma^2 (Hint: In Excel the symbol ˆ is used to raise a quantity to a power).
(b) Test this by setting $\rho_m = \rho_c$. What is I/Ma^2 and why?
(c) Test this by setting $\rho_c = 0$ and $r_c = 0.99a$. What is I/Ma^2 and why?
(d) Derive a plausible two-layer model for Mercury, assuming $I/Ma^2 = 0.346$.

In addition to your answers for parts (a) to (d), please hand in a printout or screenshot of your code or excel spreadsheet.