In-Lab Project Sample Solution: Testing for Population Proportion
Stat 350 Winter Semester 08, Lab 5

Sample Solution:

1. Question to investigate: Do a majority of UM students like Valentine’s Day?
   Appropriate null and alternative hypotheses:
   \( H_0: p = 0.5, \ H_a: p > 0.5 \)
   where \( p \) represents the population proportion of UM students who like Valentine's Day.

2. Results by Qwizdom from lab 016:

<table>
<thead>
<tr>
<th>Number of students responding</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students who responded yes</td>
<td>8</td>
</tr>
<tr>
<td>Sample proportion of students who responded yes ( \hat{p} = 8/22 = 0.36 )</td>
<td></td>
</tr>
</tbody>
</table>

3. In order to use normal approximation of binomial distribution, we need to check

\[
np_0 = 22 \times 0.5 = 11 \geq 10 \\
n(1-p_0) = 22 \times (1-0.5) = 11 \geq 10
\]

Here \( p_0 = 0.5 \) is used as the calculation is under the null hypothesis.

4. Test statistic (z-statistic in this case):

\[
z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.36 - 0.5}{\sqrt{\frac{0.5(1-0.5)}{22}}} = -0.14 \quad 0.1066 = -1.3133
\]

The distribution of the test statistic under the null hypothesis is the standardized normal distribution, or \( Z \sim \mathcal{N}(0,1) \).

5. Finding the corresponding p-value:

\[
p-value = P(Z \geq -1.3133) = 0.9049
\]

6. Decision at 5% significant level: do not reject \( H_0 \), since p-value > \( \alpha = 5\% \).
   Conclusion: based on the data, the results are not statistically significant at level 5% to conclude that the majority of UM students like Valentine’s Day.

7. When testing whether at least 90% of the students (instead of a majority) like Valentine’s Day, the new hypotheses are

\( H_0: p = 0.9, \ H_a: p > 0.9 \)

In this case, the condition of normal approximation of binomial distribution is not satisfied since

\[
n(1-p_0) = 22 \times 0.1 = 2.2 < 10
\]

Review of Some Key Points:

- Calculation of p-value: the direction in (3) (\( \geq \) in this case) is decided by the direction in \( H_a: p > 0.5 \), as the p-value is the probability of getting an observed test statistic as extreme. \( H_a \) is decided according to the problem (a majority).
- Difference between \( p_0 \) and \( \hat{p} \), in particular when checking assumptions or calculating z-statistic as in (1) (2).
- Z-statistic has always the standard normal distribution \( \mathcal{N}(0,1) \).