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POST-LIQUEFACTION STATE OF SAND, STRESS CORROSION CRACKING, AND RELAXATION OF DEVIATORIC STRESS IN PREVIOUSLY LIQUEFIED SAND BED

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ABSTRACT

Liquefaction in sand is associated with the break in inter-granular force chains leading to zero effective stress. Upon dissipation of excess pore water pressure and formation of new contacts between grains, a macroscopic stress state in the sand bed is restored, but, this stress state differs from the pre-liquefaction state. It is argued that the horizontal stress in post-liquefaction sand bed is substantially lower than the pre-liquefaction stress, causing the deviatoric stress to increase after liquefaction. Hence, the post-liquefaction stress state brings the soil bed closer to the yielding state. This is the reason why the cone penetration resistance in dynamically compacted sand often exhibits a drop immediately after compaction despite increased density (strength). A gradual increase in the penetration resistance occurs, however, in a matter of weeks and months. An explanation of this phenomenon is sought in the process of *stress corrosion cracking* of the micro-morphological features on the surface of the sand grains at the contacts. This process, also referred to as *static fatigue*, leads to *grain convergence*, and a change in the macroscopic elastic moduli of the sand. The latter is the cause of the increase in the macroscopic horizontal stress in sand under one-dimensional strain conditions. The consequence of this process is the time-dependent (delayed) increase in the penetration resistance of the sand bed after dynamic compaction.

Keywords: Dynamic compaction of sand, Sand liquefaction, Stress corrosion cracking, Static fatigue

INTRODUCTION

Dynamic compaction of sand beds (blasting, vibro-compaction) is an effective means of increasing the strength of foundation soils. Saturated sands subjected to dynamic compaction liquefy in the process; the evidence of compaction is in subsidence of the ground surface. Surprisingly, cone penetration tests immediately after compaction often indicate a reduction in the penetration resistance when compared to the pre-compacted state. The penetration resistance does increase, however, in weeks and months after compaction. Attention to this phenomenon was brought by Mitchell and Solymar (1984), Schmertmann (1987), and others, but the phenomenon is still referred to as a continuing *Enigma* (Mitchell 2008).

Early attempts to explain this phenomenon focused on possible mineral dissolution and bonding between grains. This bonding would be destroyed in the process of liquefaction caused by dynamic compaction, but be re-built in time after compaction. Dissolution of minerals and pressure solution are phenomena that play an important role in formation of sedimentary rocks, but the evidence of these processes being

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important in silica sand at the engineering temporal scale is not available, beyond experiments with halite (rock salt) that is particularly susceptible to dissolution (Tada and Siever 1985).

Current efforts toward explanation of the peculiar behavior of sands after dynamic compaction shifted the emphasis on the processes of rearrangement of particles (or a *secondary compression*-like process, Mitchell 2008). This rearrangement, however, cannot occur without a cause, and this article focuses on the phenomenon that is likely to be a source of the time-dependent process observed in sand beds after dynamic compaction.

DYNAMIC DISTURBANCE AND POST-LIQUEFACTION STRESS STATE

Compaction of sand requires a change in its packing, so that the volume of pores can be reduced. When the sand is liquefied, the contact forces between grains are lost, and the contacts are reconfigured so that a tighter packing can be achieved. Compaction leads to both the change in the scalar measures of packing (void ratio) as well as the fabric of packing. In this section we will discuss how the post-liquefaction stress state relates to the pre-liquefaction state.

While the explanation of the delayed effect of the increase in cone penetration resistance will be sought at the microscopic level, the engineering analyses are carried out at the macroscopic scale in terms of stresses averaged over the representative volume V, according to formula (see, e.g., Drescher and De Jong 1972)

$$\bar{\sigma}_{ij} = \frac{1}{V}\int_S x_i t_j \, dS , \qquad i, j = 1, 2, 3 \tag{1}$$

where S is the surface bounding volume V, and t_j is the force at co-ordinate x_i on boundary S. The stress so defined is a macroscopic measure of the state.

Provided the soil surface is horizontal, the macroscopic stresses $\bar{\sigma}_{ij}$ in a sand bed will be characterized with vertical and horizontal principal directions, with the ratio of horizontal-to-vertical (K_0) somewhere below unity (normally consolidated bed). Macroscopic stresses are reduced to zero during liquefaction, but the stress state is rebuilt upon dissipation of the excess pore water pressure. The vertical stress is governed by gravity, but the horizontal stress does not have a comparable governing rule, and it only needs to be large enough so that the stress state is admissible, *i.e.*, it does not violate the yield condition. When the effective vertical macroscopic stress is increasing from zero during liquefaction to a value that balances the buoyant weight of the soil at any depth, the horizontal stress will trail with its value such, so that the stress state is just below the yield condition. This horizontal stress is likely to be significantly lower than the stress in the pre-liquefaction sand bed. To indicate that this conjecture is reasonable, we perform discrete element simulations.

Discrete Element Simulation

To support the last statement we illustrate the after-liquefaction development of the effective stress state in a granular assembly using the discrete element simulation. An assembly of nearly 12,000 grains was generated using the *PFC*^{3D} (2008) computer code. These grains were predominantly spherical with radii between 2.3 and 3.7 cm. Of the total number of particles in the specimen 25% had elongated shape constructed of two overlapping and bonded spherical grains ("clumps"). The "clumps" are marked darker in Fig. 1(a). The assembly was formed in a cube of 1.2 m in size, and the porosity of the assembly was 0.35. The contact normal and tangential stiffness was set to K_n = 4.0 and K_t = 1.6 MN/m, respectively. The coefficient of grain interface friction μ = 0.65 was used (equivalent to a friction angle of 33°); the mass density of the grains was set to 2,650 kg/m^3, and gravity acceleration was taken as 9.81 m/s^2. The ratio of horizontal-to-vertical macroscopic stress in the prepared specimen was $k = 0.468$. A triaxial compression test of the specimen was then simulated with a confining pressure of 50 kPa. Interpreting the strength of the granular assembly as frictional with a linear envelope, the peak internal friction angle was found to be 33.5°.

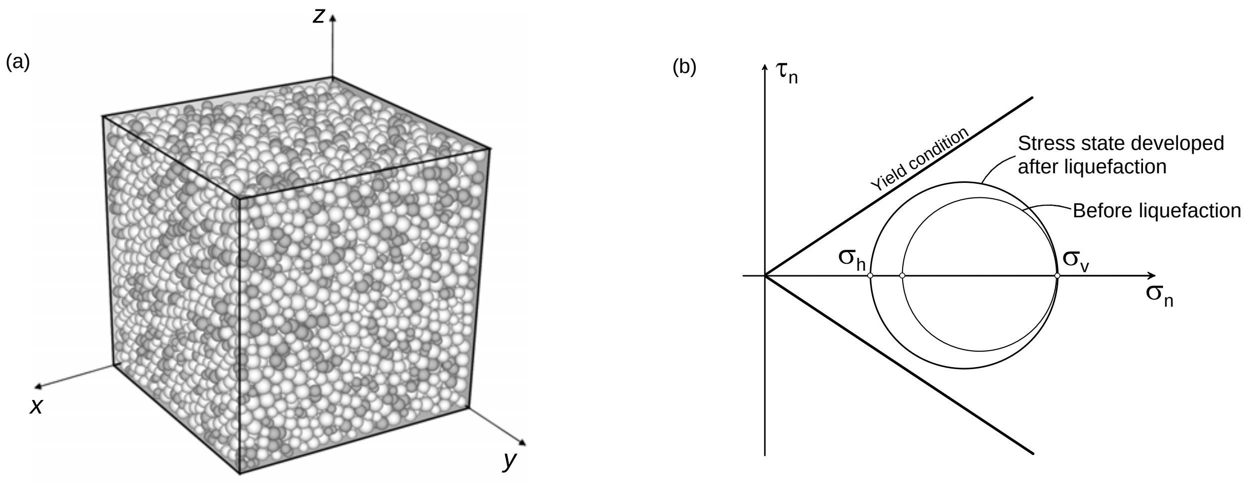

Figure 1. (a) Discrete element model of a granular assembly, and (b) stress state before and after liquefaction (DEM simulation using *PFC^{3D}* code).

The fluid in pore space is not modeled; rather, liquefaction was simulated by a small reduction in the size of all grains (0.7% reduction in grain diameters), causing a sudden loss of all contacts (without producing a meaningful reduction in the void ratio). Upon re-gaining the firm contacts between the particles, macroscopic vertical stress has reached its linear distribution governed by gravity as before “liquefaction,” but the horizontal stress rose to a magnitude lower than that before loss of contacts. The average ratio of the macroscopic horizontal-to-vertical stress was 0.468 before “liquefaction” and it dropped to 0.355 after re-gaining the contacts. This ratio was calculated as the average of $\bar{\sigma}_x$ and $\bar{\sigma}_y$ stresses to the average of $\bar{\sigma}_z$, and it is consistent with the definition of K_0. This simulation supports the hypothesis of the reduced horizontal stress (or increased deviatoric stress) in the post-liquefaction sand bed, even though it is only numerical evidence. The average stress state in the specimen before and after liquefaction is illustrated in Fig. 1(b).

Prior to compaction, the stress state in a sand bed is affected by geologic history, and in normally consolidated soils the ratio of the horizontal-to-vertical macroscopic stress is less than unity but far away from the ratio describing “active” state at failure. Dynamic compaction leads to an increase in strength of the soil, but also it causes the change in the stress state (increase in deviatoric stress) that has an adverse effect on the cone penetration resistance.

DELAYED INCREASE IN PENETRATION RESISTANCE AFTER DYNAMIC COMPACTION

Once the contacts between particles have been formed after liquefaction, the post-liquefaction stress state developed is characterized by an increased deviatoric stress. This stress state, however will not remain constant because of the process of the time-dependent increase in elastic moduli of sand. A hypothesis is presented in this section indicating *stress corrosion cracking* as the origin of the time-dependent behaviour of sand.

Stress Corrosion Cracking, Grain Convergence, and Increase in Elastic Moduli

Silica sand grain surfaces typically have a rich micro-morphology that is a source of irreversible and time-dependent behaviour. Examples of surface features of Ottawa 20-30 sand are shown in Fig. 2. When two particles with such surfaces are brought to a contact, the asperities and the crystalline debris become loaded giving rise to *stress corrosion cracking*. Stress corrosion cracking is a delayed microfracturing, accelerated by environmental factors, such as moisture, and it does not occur simultaneously at all contacts. This process, also known as *static fatigue*, was believed to be the cause of creep in brittle rocks (Scholz 1968), and, more recently, it was brought up by Lade and Karimpour (2010) as a possible cause of rate effects in sand, though they considered delayed fracture of grains, rather than static fatigue of the surface features.

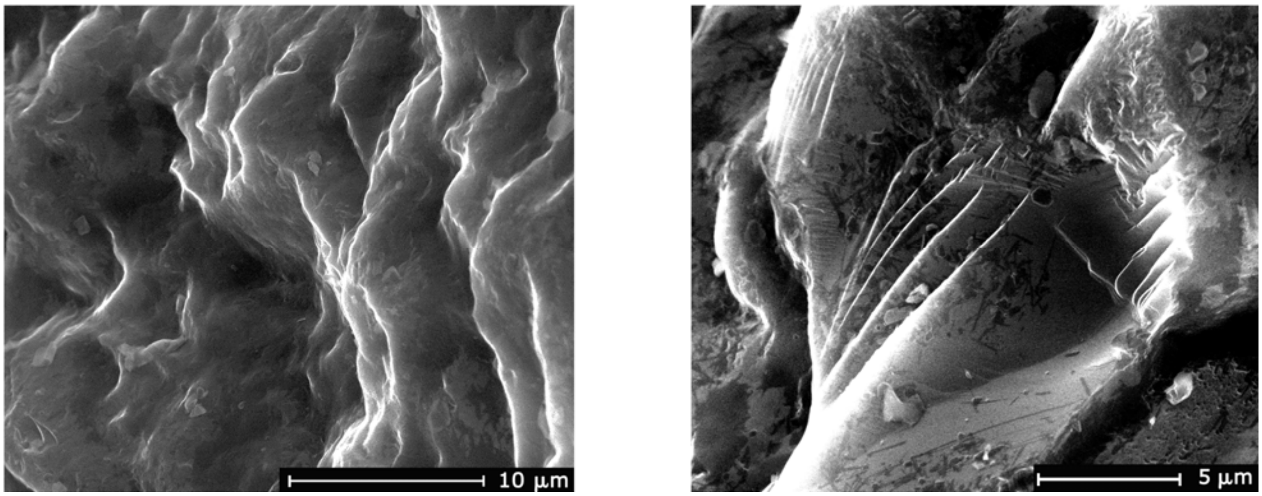

Figure 2. Morphology of sand grain surface: Ottawa 20-30 sand.

Stress corrosion cracking of the features on the surface brings the grains closer together. This is a minute change in configuration of the order of a size of the surface asperity. We refer to this process as *grain convergence*. This grain convergence causes an increase in the contact stiffness that propagates through the spatial scales, and manifests itself at the macroscopic level as an increase in elastic stiffness expressed in terms of stresses as defined in eq. (1). This increase in stiffness is instrumental for the explanation of the delayed increase in the cone penetration resistance after dynamic compaction.

Evidence supporting this hypothesis is in measurements of the shear wave velocity in specimens of sand subjected to sustained load. Afifi and Woods (1971) reported that the speed of the elastic waves increases in time. In air-dry Ottawa sand, the shear modulus was found to increase about 2% to 5% per log cycle of time. They also noted that the relative increase in shear modulus was larger with the decrease in the particle size. In specimens of the same size, the number of contacts increases with the decrease in the grain size. Therefore, one would expect the macroscopic effect of the process that occurs at inter-granular contacts to be more distinct in finer materials.

The fundamental conclusion from the measurements of the speed of the shear wave propagation indicates that the elastic moduli of sand increase in time if the sand is subjected to persistent load. We refer to elastic moduli rather than one elastic modulus, as the sand deposited under gravity conditions is typically cross-anisotropic, and it is characterized by at least two moduli and three Poisson's ratios. The authors propose the hypothesis that it is the stress corrosion cracking of the microscopic features at the contacts between grains that is a direct cause of *grain convergence*, leading to the increase in elastic moduli of sand. This hypothesis was very recently presented by Michalowski and Nadukuru (2010), though the detailed description of the behavior of sands after dynamic disturbance and the DEM simulations were not included in that reference.

Another evidence for time-delayed increase in stiffness of sand subjected to sustained loads comes from triaxial tests of Daramola (1980), who indicated that the sustained load on sand did not produce an increase in strength, but it did result in the increase in stiffness. This is because the strength is governed predominantly by the meso-scale characteristics, *i.e.*, the soil specific volume (or void ratio) and fabric, and these characteristics are altered during compaction, but they are not changed during the following period of rest under sustained load. It is the stiffness, however, that does change in time. We suggest the hypothesis that it is stress corrosion cracking (or static fatigue) and grain convergence that cause the increase in the elasto-plastic stiffness in the results of Daramola (1980), as well as an increase in elastic moduli detected by Afifi and Woods (1971).

Relaxation of Deviatoric Stress

Static fatigue and grain convergence are the cause of an increase in elastic moduli. Under one-dimensional strain conditions in a sand bed, this change in elastic properties will cause an increase in the horizontal stress. To demonstrate this increase consider a spring between two unyielding supports, Fig. 3(a). The force in the spring is analogous to the horizontal stress in the soil bed.

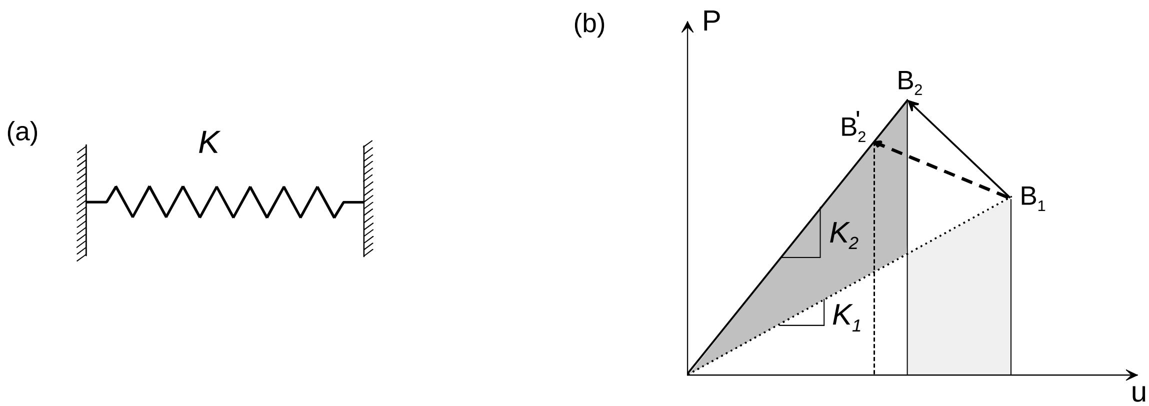

Figure 3. (a) Spring between unyielding walls, and (b) balance of energy associated with an increase in the spring constant.

Once the spring constant increases from K_1 to K_2 (Fig. 3(b)), the force in the spring changes from B_1 to B_2, preserving the elastic energy (the shaded areas are equal to one another). Should the change in material property K require expenditure of energy, the force in the spring may still increase, though to some lower level at B_2'.

The increase in the horizontal stress in the sand bed after dynamic compaction is analogous to the increase in the spring force in Fig. 3. This process is delayed, because the stress corrosion cracking (the origin of the process) is a function of time. The vertical stress in the sand bed, however, remains unchanged, since it is governed by gravity. Consequently, the increase in the horizontal stress in the sand bed produces a reduction in the deviatoric stress, and moves the stress state further away from the yield condition. This is illustrated in Fig. 4 with a reduction in the diameter of the stress circle, as the minor principal stress increases.

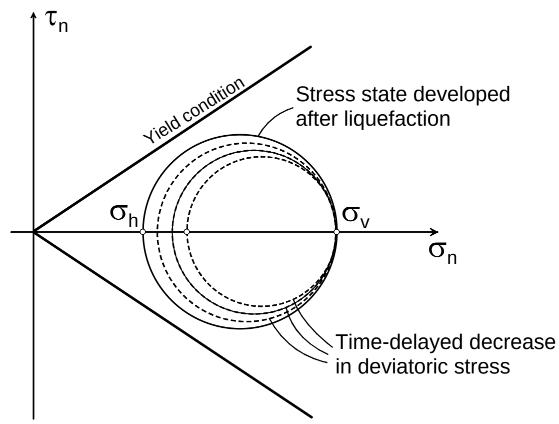

Figure 4. Time-delayed reduction in deviatoric stress.

It is suggested that the drop in the cone penetration resistance immediately after dynamic compaction is the result of superposing the load (from the penetrating cone) over the stress state in the soil that is already close to yielding. Subsequently, in the weeks and months after compaction, the horizontal stress in the soil bed increases, as a result of the change in elastic stiffness under one-dimensional strain conditions. The latter is caused by the process of stress corrosion cracking (static fatigue) at inter-granular contacts. Consequently, the cone penetration resistance gradually increases. An increase in the horizontal stress was considered earlier by Schmertmann (1987) and Mesri *et al*. (1990) as a possible reason for time-dependent increase in cone penetration resistance. We indicate stress corrosion cracking as playing the primary role in this process.

Preliminary finite element simulations of the cone penetration process into a sand bed confirm the conjecture that a gradual increase in the stiffness of the sand leads to the change in the macroscopic stress state, causing the delayed increase in the cone penetration resistance of the soil bed.

CONCLUSIONS

The development of the post-liquefaction stress state in sands has far reaching consequences on the time-dependent behavior of sand. A hypothesis was presented indicating that the decrease in the cone penetration resistance of sands immediately after dynamic compaction is caused by an increase in deviatoric stress in the sand bed. Some evidence was presented based on the discrete element simulation. While the numerical evidence may not be convincing, the simulated process of force chains formation after liquefaction is quite plausible. The subsequent increase in the cone penetration resistance in the weeks and months following compaction is due to an increase in horizontal stress in the soil bed caused by an increase in elastic moduli under one-dimensional strain conditions. The latter is caused by static fatigue at inter-granular contacts, and grain convergence. Static fatigue is a result of stress corrosion cracking of morphological features at inter-granular contacts.

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REFERENCES

Afifi, S.S. and Woods, R.D. (1971). "Long-term pressure effects on shear modulus of soils." ASCE J. Soil Mech. Found. Div., Vol. 97, No. 10, pp. 1445-1460.

Daramola, O. (1980). "Effect of consolidation age on stiffness of sand." Géotechnique, Vol. 30, No. 2, pp. 213-216.

Drescher, A. and De Josselin De Jong, G. (1972). "Photoelastic verification of a mechanical model for the flow of a granular material." J. Mech. Phys. Solids, Vol. 20, pp. 337-351.

Lade, P.V. and Karimpour, H. (2010). Static fatigue produces time effects in granular materials. ASCE Proc.: *Advances in Analysis, Modeling & Design*, GeoFlorida, Feb. 20-24, 2010.

Mesri, G., Feng, T.W. and Benak, J.M. (1990). "Postdensification penetration resistance of clean sands." J. Geot. Engineering, Vol. 116, No. 7, pp. 1095-1115.

Michalowski, R.L. and Nadukuru, S.S. (2010). Stress corrosion cracking and relaxation of deviatoric stress after dynamic compaction of sand. Keynote, *37*th *Solid Mechanics Conference,* (*SolMech 37*), Sept. 6-10, Warsaw, Poland (extended abstract).

Mitchell, J.K. and Solymar, Z.V. (1984). "Time-dependent strength gain in freshly deposited or densified sand." J. Geot. Engineering, Vol. 110, No. 11, pp. 1559-1576.

Mitchell, J.K. (2008). Aging of sand – a continuing Enigma? *6*th *Int. Conf. on Case Histories in Geotechnical Engineering*. Arlington, VA, Aug. 11-16, 2008, pp. 1-21.

PFC^{3D} (2008). Particle Flow Code in 3 Dimensions. Itasca Consulting Group, Inc. Minnepolis, MN.

Schmertmann, J.H. (1987). "Time-dependent strength gain in freshly deposited or densified sand." Discussion, J. Geot. Engineering, Vol. 113, No. 2, pp. 173-175.

Scholz, C.H. (1968). "Mechanism of creep in brittle rock." J. Geophys. Res., Vol. 73, No. 1, pp. 3295-3302.

Tada, R. and Siever, R. (1985). "Experimental knife-edge pressure solution of halite." Geochimica et Cosmochimica Acta, Vol. 50, pp. 29-36.