Arching in Distribution of Active Load on Retaining Walls

Srinivasa S. Nadukuru, S.M.ASCE¹; and Radoslaw L. Michalowski, F.ASCE²

Abstract: Traditional methods for calculations of active loads on retaining structures provide dependable forces, but these methods do not indicate reliably the location of the resultant load on the walls. The Coulomb method does not address the load distribution because it utilizes equilibrium of forces, whereas the Rankine stress distribution provides linear increase of the load with depth. Past experimental studies indicate intricate distributions dependent on the mode of displacement of the wall before reaching the limit state. The discrete element method was used to simulate soil-retaining structure interaction, and force chains characteristic of arching were identified. Arching appears to be the primary cause affecting the load distribution. A differential slice technique was used to mimic the load distributions seen in physical experiments. The outcome indicates that rotation modes of wall movement are associated with uneven mobilization of strength on the surface separating the moving backfill from the soil at rest. Calculations show that the location of the centroid of the active load distribution behind a translating wall is approximately 0.40 of the wall height above the base, but for a wall rotating about its top point, the location of the resultant is at approximately 0.55H. In the third case, rotation about the base, the location of the calculated centroid of the stress distribution on the wall is slightly below one-third of the wall height. DOI: 10.1061/(ASCE)GT.1943-5606.0000617. © 2012 American Society of Civil Engineers.

CE Database subject headings: Arches; Retaining structures; Load distribution; Numerical analysis; Discrete elements.

Author keywords: Arching; Retaining walls; Active load; Numerical analysis; Discrete element method.

Introduction

The methods for calculation of loads on retaining walls have a two-century history, but with no significant modifications these methods are widely used in design today. They include the methods originally conceived by Coulomb (1773) and Rankine (1857). The two methods are different in their scope. Coulomb used what today is referred to as the limit equilibrium method, with an intuitive use of minimum and maximum principles. In his original work, an equilibrium of three forces (global equilibrium) acting on a wedge behind a wall was considered, with the reaction of the wall being sought. Such an approach was shown to be equivalent to the rigorous kinematic approach of limit analysis (Michalowski 1989). Although the case considered is referred to in geotechnical literature as an active load, the reaction of the wall was sought, and the kinematic limit analysis leads to a lower bound on that reaction. The adjective active does not pertain to the unknown calculated, but to the load of the soil on the wall, which does not appear explicitly in the equation of equilibrium (of course, it is equal in magnitude to the reaction). Note that Coulomb did not use trigonometry to calculate the wall reaction, which led to a rather elaborate result. Only later was the solution transformed into a simpler form using trigonometric functions (Maynial 1808, and others).

The important consequence of the method devised by Coulomb is that only the total (resultant) reaction of the wall can be calculated, and not its distribution. The linear distribution of the wall load often attributed to Coulomb comes from an analysis of a series of wedges of different height. Coulomb only briefly noted the use of such an analysis when calculating a moment about the base. However, although the analysis is valid for walls of any height, analyses with different heights are not valid all at the same time for one wall (unless the plastic state is assumed within the wedge).

The outcome of the Coulomb solution is typically represented by the coefficient of active pressure $K_a$. This coefficient relates the resultant force on the wall $P_a$ to the wall height $H$ and the (unit) weight $γ$ of the soil

$$P_a = \frac{γH^2}{2}K_a$$ (1)

For a smooth wall and horizontal backfill, $K_a$ assumes a well-known expression

$$K_a = \frac{1 - \sin φ}{1 + \sin φ} = \tan^2\left(\frac{π - φ}{4}\right)$$ (2)

where $φ$ = internal friction angle. Coulomb coefficient $K_a$ is a global coefficient that has interpretation only in terms of the total load on the wall. Because it is associated with the limit state in the soil at the interface at which the wedge slides over the stationary soil, it is the lowest load that the soil can exert on the wall.

The second early contribution to earth loading on walls was published in Philosophical Magazine approximately 80 years later by Rankine (1857). Rankine’s paper was primarily about the limit stress state in a sloping soil or “loose earth” (infinite slope). The approach using the stress equilibrium at every point in the soil is very different from Coulomb’s consideration of the force (global) equilibrium of a wedge. Rankine (1857) criticized the global force approach, referring to the “wedge of least resistance” as a “mathematical artifice.” Rankine developed two limit stress fields in a sloping ground.
(infinite slope), referred to today as active and passive; both of them are statically admissible. For the active state, Rankine wrote the relation of the stress parallel to the slope surface \( R_v \) acting on a vertical plane to the vertical stress \( R_u \) acting on a plane parallel to the slope (\( R_v \) and \( R_u \) is “a pair of conjugate pressures”)

\[
\frac{R_v}{R_u} = \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}
\]  

(3)

where \( \beta \) = inclination angle of the slope and \( \phi \) = internal friction angle. Once a vertical cut of finite depth is made and is a retaining (revêtement) wall is placed to prevent the soil from failing, stress \( R_v \) has to be supported by the wall. From that premise, the Rankine coefficient of active earth pressure is derived; replacing \( R_v \) with \( \sigma_n \) (stress on the wall) and \( R_u \) with \( \sigma_v \cos \beta \) (\( \sigma_v \) = vertical stress on a horizontal plane) in Eq. (3) results in

\[
\sigma_n = \sigma_v \cos \beta \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}} = \sigma_v K_a
\]  

(4)

or

\[
K_a = \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}
\]  

(5)

Arching in Soils

As Terzaghi (1943) alluded, arching may play a role in distribution of loads on retaining structures. Whereas the term arching has been accepted in the geotechnical literature, the concept does not relate to the formation of a physical arch, but rather a distribution of stress (or variation in the stress field) for which stiffer components of the system attract more load. This is illustrated in a discrete element method (DEM) simulation of a prismatic sand mound in Fig. 1. This simulation was carried out using the PFC3D code (Itasca 2008). Approximately 12,000 particles were used in this simulation, of which 75% were spherical and the remaining ones were clumps with an elongated shape built of two spheres overlapping by one radius. A friction coefficient of 0.65 between particles was used. The grains simulated poorly graded gravel with particles between 2.3 and 3.7 cm (the remaining details are omitted as they are not central to this illustration).

The particles were first rained into a box with a base of 4.8 × 0.6 m; subsequently, instability was induced by removing side walls and a prismatic heap was formed during a process of particles’ rolling and sliding off under gravity. After the wedge-shaped heap was formed, as in Fig. 1(a), the two halves of the base were rotated about their end edges to produce the maximum subsidence \( \Delta \) at the center equal to 0.67% of the heap’s height \( \Delta/H = 0.0067 \). Fig. 1(b) indicates the force chains characteristic of soil arching (thickness of the force chains is indicative of the intensity of forces between grains). The distribution of the normal macroscopic stress at the base immediately after formation of the heap and after deflection of the base is shown in Fig. 1(c). Not surprisingly, the load on the central portion of the base was reduced in the process of deflection, at the expense of the less compliant portions farther away from the center. Consequently, the stress distribution exhibits a distinct local minimum at the center (a “dip”), the subject of a previous study (Michalowski and Park 2004).

Analogous to the redistribution of the base reaction caused by deflection, the rotation of a retaining structure should be expected to affect the stress distribution and, possibly, induce arching.

DEM Simulations

DEM Model

The PFC3D code was used to model granular soil behind a wall, as seen in Fig. 2. Approximately 30,000 particles form the backfill, with 70% of particles being spherical with diameter ranging from 4.60 to 7.36 cm, and 30% being clumps in the shape of two overlapping spheres. The number of particles significantly affects the time of simulations, and the size of the particles was selected to enable a 3 m wall to be simulated with 30,000 particles. The particles’ normal and shear stiffness were assumed as 8.3 MN/m and
3.32 MN/m, respectively; the coefficient of interparticle friction was set to \( \mu = 0.65 \) and the particles’ density \( \rho = 2,650 \text{ kg/m}^3 \).

The backfill assembly was generated by raining particles into a box with smooth side walls and the dimensions shown in Fig. 2. The backfill had the porosity of approximately 0.35 and the ratio of horizontal to vertical average stress \( \sigma_h/\sigma_v = 0.385 \). A numerical triaxial test performed on a cylindrical specimen (1.2 m in height and diameter, with 8,200 particles) of the same mixture of grains at the confining pressure of 200 kPa indicated internal friction angle \( \phi = 39^\circ \).

**Arching at the Wall**

Three cases of retaining wall movement were simulated: horizontal translation, rotation about the base, and rotation about the top edge. The results from all three are reported in the next section; in this section, only the rotation about the top edge is illustrated, where arching is most distinct.

A rough wall with a coefficient of friction of \( \mu = 0.35 \) was simulated. After the model was numerically generated, rigid movement of the wall was induced, and the displacements associated with the wall rotation about the top edge are illustrated in Fig. 3(a). The displacements shown were recorded at the wall rotation of 0.008 rad (magnification factor is 15). The force chains are illustrated in Fig. 3(b), and they indicate an arching effect similar to that seen in Fig. 1(b). This arching phenomenon is interpreted schematically in Fig. 3(c). In the case of translation and rotation about
the base, all displacement vectors tend to be parallel (not shown), and inclined at some angle to a straight line that could be interpreted as a failure plane behind a wedge. However, the velocity field in the case of rotation about the top point is more complicated, with a curved surface separating the material at rest from the moving region [Fig. 3(c)]. The presence of the inflection point indicates the possibility of two regions that rotate about two different centers.

Dem Simulations versus Physical Tests

Loads on Walls

The load on the wall immediately after the deposition process is referred to as the pressure-at-rest. The total at-rest load on a wall 3.1 m high and 0.5 m wide (Fig. 2) was approximately 15.4 kN. The reduction of the load associated with three modes of wall movement is illustrated in Fig. 4 for both a smooth and a rough wall (DEM simulations). Each of the three processes was simulated starting with the same at-rest condition. For the rotation cases, \( \Delta \) in the relative displacement measure \( \Delta/H \) is defined as the wall displacement at the edge opposite to the point of rotation. The active force for a horizontally translating smooth wall and the wall rotating about the top edge reach the steady-state value before the wall rotating about its base does. A rough wall [Fig. 4(b)] attracts a lower load (as expected). At small displacements, the reduction in the load exerted on the wall is qualitatively similar to that for the smooth wall, but for larger displacements, the load on the wall rotating about the base approaches that for translation, a striking difference from the smooth wall case. For the cases of a horizontally translating rough wall or a wall rotating about the top edge, the active force reaches its steady-state at approximately 0.003 relative
displacement [Fig. 4(b)]; rotation about the base has not reached the asymptotic value at $\Delta/H = 0.008$. The gradient of the force-displacement curves is affected by the combination of the size of the spheres relative to the wall deflection and the contact stiffness. However, the final state was found to be a function of the internal friction angle of the model soil and not the grain size.

In presentation of the numerical results, the wall height is normalized (equal to unity), and the stress on the wall is normalized by the value $\rho g(1-n)H$ [see Eq. (6) for description of symbols]. The wall was divided into six segments (Fig. 2), and the average stress on each segment was calculated by dividing the sum of all wall-particle contact forces on the segment by the area of the segment.

The stress distribution at-rest and distributions associated with each of the simulated wall movements are illustrated in Fig. 5(a) (RT is rotation about the top edge, and RB is rotation about the base). The numerical results are for a rough wall with coefficient of friction $\mu = 0.35$. Various wall movement modes produced distinctly different stress distributions. The rigid translation induced an active load distribution far from linear, with the lowest total load on the wall, leading to an active wall load coefficient equal to 0.214. This coefficient was calculated as

$$K = \frac{F_{\text{total}}}{\frac{1}{2}\rho(1-n)gH^2w}$$  \hspace{1cm} (6)

where $F_{\text{total}}$ = total force on the wall, $\rho = $ density of grains (2,650 kg/m$^3$), $n = $ porosity (0.36), $g = $ gravity acceleration, $H = $ wall height (3.1 m), and $w = $ wall width (0.5 m). The location of the stress distribution centroid was found at the level 0.39 of the wall height above the base. The results are reported in Table 1. Similar results for smooth wall simulations are reported in Table 2. Although all the results calculated are for active loads, the coefficient reported is denoted as $K$ to distinguish it from traditional coefficient $K_a$ of active pressure associated with specific methods, Eqs. (2) and (5).

### Comparison to Physical Tests

Although the force chains in DEM simulations with their visual interpretation are very appealing to engineering intuition, the assembly of particles considered in DEM is only a model, and it still needs to be validated through comparison with physical test results. We chose the tests by Fang and Ishibashi (1986) for comparison.

The physical test results after Fang and Ishibashi (1986) are shown in Fig. 5(b). These tests were performed for a 40-inch wall, and the results in Fig. 5(b) are normalized as the DEM results were (wall height is set to unity and stresses are normalized by $\gamma H$). All numerical tests, as shown in Fig. 5(a), were performed for a model material with $\phi = 39^\circ$, whereas the physical tests in Fig. 5(b) are all for slightly different angles $\phi$ (adapted from Figs. 6, 11, and 12 of the reference by Fang and Ishibashi 1986). Nevertheless, the qualitative similarity of the distributions from DEM simulations and physical tests is evident. The stress distribution at-rest and distributions associated with each simulated wall movement are illustrated in Fig. 5(a) (RT is rotation about the top edge, and RB is rotation about the base). The numerical results are for a rough wall with coefficient of friction $\mu = 0.35$. Various wall movement modes produced distinctly different stress distributions. The rigid translation induced an active load distribution far from linear, with the lowest total load on the wall, leading to an active wall load coefficient equal to 0.214. This coefficient was calculated as

$$K = \frac{F_{\text{total}}}{\frac{1}{2}\rho(1-n)gH^2w}$$  \hspace{1cm} (6)

where $F_{\text{total}}$ = total force on the wall, $\rho = $ density of grains (2,650 kg/m$^3$), $n = $ porosity (0.36), $g = $ gravity acceleration, $H = $ wall height (3.1 m), and $w = $ wall width (0.5 m). The location of the stress distribution centroid was found at the level 0.39 of the wall height above the base. The results are reported in Table 1. Similar results for smooth wall simulations are reported in Table 2. Although all the results calculated are for active loads, the coefficient reported is denoted as $K$ to distinguish it from traditional coefficient $K_a$ of active pressure associated with specific methods, Eqs. (2) and (5).

### Table 1. Load Coefficient $K$ and Location of Stress Distribution Centroid from DEM Simulations and Physical Tests (rough wall)

<table>
<thead>
<tr>
<th>Mode</th>
<th>DEM simulation ($\phi = 39^\circ$)</th>
<th>Fang and Ishibashi (1986)$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>At-rest</td>
<td>0.385</td>
<td>—</td>
</tr>
<tr>
<td>Translation</td>
<td>0.214</td>
<td>0.20</td>
</tr>
<tr>
<td>Rotation about base</td>
<td>0.217</td>
<td>0.25</td>
</tr>
<tr>
<td>Rotation about top edge</td>
<td>0.251</td>
<td>0.26</td>
</tr>
</tbody>
</table>

$^a$Interpolated for $39^\circ$ from Fang and Ishibashi (1986).

$^b$Location of stress distribution centroid above base as fraction of wall height.

### Table 2. Load Coefficient $K$ and Stress Distribution Centroid Location from DEM Simulations (Smooth Wall)

<table>
<thead>
<tr>
<th>Mode</th>
<th>$K$</th>
<th>Centroid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translation</td>
<td>0.230</td>
<td>0.401</td>
</tr>
<tr>
<td>Rotation about base</td>
<td>0.315</td>
<td>0.253</td>
</tr>
<tr>
<td>Rotation about top edge</td>
<td>0.275</td>
<td>0.536</td>
</tr>
<tr>
<td>Classical (Coulomb)</td>
<td>0.228</td>
<td>—</td>
</tr>
</tbody>
</table>

---

**Fig. 5.** (a) load distribution on a rough wall (DEM) and (b) physical test results adapted from Fang and Ishibashi (1986)
from the physical tests is striking. Only the pressure-at-rest differs significantly because in DEM, the sand bed was deposited by raining, whereas in physical experiments it was vibrated to achieve the targeted density, leading to increased horizontal stresses. The $K_0$ achieved was 0.63, far more than the value expected on the basis of the Jaky formula of 0.441 (Michalowski 2005). In contrast, the DEM value of 0.385 is quite close to the expected value of 0.371. Quantitative differences in the active load can be attributed to the differences in the internal friction angle of the backfills tested by Fang and Ishibashi (1986) and a slightly different wall friction coefficient (0.45 in physical tests versus 0.35 in DEM). Numerical comparison in terms of the load coefficient in Eq. (6) and the location of the distribution centroid are given in Table 1. Fang and Ishibashi (1986) reported physical test results for a variety of internal friction angles of the backfill, and the numbers reported in Table 1 were interpolated for $\phi = 39^\circ$ on the basis of the data they reported. The DEM-calculated numbers compare reasonably well with those from the physical tests.

The DEM-calculated location of the stress distribution centroid for a wall that rotates about the base was found at 0.31$H$ and 0.25$H$ above the base for rough and smooth walls, respectively. These values are consistent with the physical test value for a rough wall of 0.26$H$ (Fang and Ishibashi 1986) and were consistently larger for the translation mode: 0.39 and 0.40 of the wall height for DEM and 0.43 for the physical test. For the rotation about the top edge, the centroid was found very consistently above the midpoint (0.55, 0.54 for DEM simulations and 0.55 for the physical test).

### Mixed Translation-Rotation Mode

Retaining structures may be subjected to a combination of distinct movement modes considered previously. These modes can be affected, for instance, by the differential settlement of the wall footing. Of interest to this research was whether a mixed mode of wall movement will result in a stress distribution common to both modes, or whether it will retain a distribution characteristic to only one of the modes. The distributions of the active load on a rough wall ($\mu = 0.35$) illustrated in Fig. 6 include cases of wall translation, rotation about the top edge [as in Fig. 5(a)], and a combination of the two (or rotation about the point $\frac{1}{2}H$ above the wall top point). Clearly, the resulting distribution for the mixed mode borrows its characteristics from the two primary modes. The coefficient of active wall load for this mixed mode was found to be $K = 0.238$ and the location of the distribution centroid was found to be 0.491.

### Differential Slice Approach

With the shape of the load distribution determined from DEM simulations, an attempt is made to mimic this distribution using the differential slice method (DSM). DSM is one of the older engineering tools, first applied to granular materials by Janssen (1895) to calculate stresses on silo walls. This technique was modified for
applications to nonsymmetric hoppers (Michalowski 1983), and subsequently applied to retaining walls (Michalowski 1984). Slice methods of different flavors can be found in Drescher (1991).

The differential slice technique solution offers the stress distribution on the wall. The question addressed is: under what conditions can the stress distribution on a retaining wall seen in experiments be replicated by the differential slice technique? An answer to this question is expected to shed some light on yielding (or mobilization of strength) in the backfill behind the wall.

The differential slice approach described in this paper follows the earlier development in Michalowski (1984), and a schematic for the distribution on the wall. The question addressed is: under what conditions is considered. The length of the slice in direction $x$ is

$$ l = z (\tan \theta + \tan \delta) $$

and the only infinitesimal slice dimension is that in the $z$-direction. The distribution of the normal stress along the slice is assumed to be linear, and the relations of the normal and tangential stress along the wall ($\sigma_{n}, \sigma_{t}$) and along the shear surface ($\sigma'_{n}, \sigma'_{t}$) are considered known. In a typical analysis, the former is determined by the wall-soil interface friction angle $\delta$ and the latter is dependent on the internal friction angle of the soil, $\phi$

$$ \sigma_{n}' = \sigma_{n} \tan \delta, \quad \sigma_{t}' = \sigma_{t} \tan \phi $$

In addition, the relation of the normal stress $\sigma_{n}$ and the normal stress on the differential sides are assumed to be given as

$$ \sigma_{n}' = k' \sigma_{n}, \quad \sigma_{n}'' = k' \sigma_{n}'' $$

The relation in Eq. (9) at the failure surface (right-hand side) can be determined from the Coulomb method, is a global coefficient that relates the total force (resultant) acting on the wall to the geometry of the wall and the soil weight, as in Eq. (1). As indicated previously, global equilibrium of the wedge behind the wall suggests that the total load on the wall must be independent of its distribution for as long as the strength ($\phi$) is fully mobilized on the plane failure surface $AC$ behind the backfill wedge [in Fig. 7(a)] and the direction of the total load on the wall is well-defined. The following question is now posed: what would the distribution of the stress ratio $k'$ at the wall need to be to mimic the wall load distribution found in the physical experiments?

The translation case of a vertical rough wall with interface friction angle of 24.7° is first considered (as in physical tests of Fang and Ishibashi 1986). The inclination of the failure surface is determined by angle $\theta$ (Fig. 7), and this angle will be found from the requirement that the resultant load on the wall is the maximum. The differentials in the set of ordinary differential equations in Eq. (10) were substituted with finite differences, and they were solved using the Runge-Kutta (RK4) method.

The limit state is assumed to be reached in the soil along the plane $AC$. Together with the assumption of the wall friction limit being reached (well-defined inclination of the wall reaction), the resultant wall reaction becomes determinate, with a unique solution. We now look for the variation of coefficient $k'$, as defined in Eq. (9), to replicate the distribution of the wall stress as illustrated in Fig. 5(b). Of many distributions of $k'$ tested, the following one

$$ k' = \frac{\sigma_{n}'}{\sigma_{n}''} = \frac{\cos^{2} \phi}{1 + \sin \phi \sin (\phi + 2\theta)} $$

resulted in the wall stress shown as the solid line in Fig. 8(c) ($d =$ depth). A fully mobilized internal friction of 34° was assumed at the failure surface behind the wedge, and $k'$ was calculated from Eq. (12). The stress distribution has a characteristic waviness, as the one from Fang and Ishibashi (1986) in Fig. 5(b).

The two remaining distributions in Fig. 8(c) illustrate cases of rotation about the base and the top edge. An effort was made to replicate the respective distributions from physical tests in Fig. 5(b); therefore, the internal friction angle was different for each case as it was in the physical tests.

To replicate the two distributions from the physical tests for the wall rotating about the base and about the top edge, internal friction angles of 33.4° and 40.4°, respectively, are taken. However, the

\begin{align*}
A' & = 4k' (\tan \theta + \tan \delta) - 3 \tan \theta \\
A'' & = 4k' (\tan \theta + \tan \phi) - 3 \tan \theta' \\
B' & = 2 \tan \theta - \tan \theta' - 2k' (\tan \theta + \tan \delta) \\
B'' & = 2 \tan \theta - \tan \theta' - 2k' (\tan \theta + \tan \phi) \\
D' & = k' (\tan \theta \tan \delta - 1) \\
D'' & = k' (\tan \theta \tan \phi - 1)
\end{align*}
internal friction angle was assumed to be fully mobilized only at the top of the failure surface for rotation about the base, and vice versa, as illustrated in Fig. 8(b) (see Appendix for analytical expressions). These distributions of the mobilized internal friction angle were driven by matching the calculated load on the wall to that from the physical tests, and did not utilize an analysis of strains along the failure surface. Coefficient $k'$ was then calculated from Eq. (12) with $\phi$ replaced by the mobilized value of the friction angle. Coefficient $k'$ along the wall was assumed such so that the distribution of the wall stress from physical experiments could be mimicked (the distribution of $k'$ is shown in Fig. 8(a), and the analytical expressions are given in the Appendix). For rotation about the base, the inclination of the failure surface was found from the requirement of maximum wall reaction and, for rotation about the top edge, a varied coordinate system had to be used because of changing $\theta'$. For the two cases in which mobilization of internal friction angle was varied, the resultant force on the wall is no longer independent of its distribution.

The stress distributions for the three different wall movements are presented in Fig. 8(c); these distributions closely resemble those from the physical tests in Fig. 5(b). They differ significantly from the lab tests only in the very bottom portion of the backfill, which was caused by approaching a singularity in differential Eq. (10) at $z = 0$ [as discussed earlier, Michalowski (1984); however, oscillations close to $z = 0$ have little influence on the integral load]. A comparison of load coefficient $K$ and the location of the centroid from the differential slice method and from the physical results is presented in Table 3. The reasonably good agreement of these results and distribution patterns is an indication that the yield criterion may, indeed, not be fully mobilized in the rotation regimes. This conjecture is confirmed by comparison of physical tests in Fang and Ishibashi (1986) for translation and base rotation cases, for different internal friction angles. The translational mode accommodates the dilatancy of the soil during deformation (increase in volume), whereas the rotational modes restrain this ability, not allowing for full mobilization of strength. Physical tests indicate that the load for translation and base rotation is very similar for small internal friction angles, but the two diverge with an increase in the internal friction angle (thus, an increase in dilatancy).

### Conclusions

The discrete element method (DEM) is a good tool to demonstrate arching, although the phenomenon itself is elusive and difficult to quantify. DEM calculations with the number of particles approaching the true number involved in engineering problems are not feasible because of the computational demand. However, even with a relatively small number of grains (or relatively large grains), the characteristic features of arching and stress distribution on retaining structures were demonstrated to be replicable. This statement was validated by comparing the computational outcome with the physical experiments. The distribution of stress on a retaining wall from DEM calculations was found to fall surprisingly close to that from physical experiments, and this was confirmed for three different modes of wall movement.

DEM calculations indicate that of the three modes of wall movement considered, the translation mode leads to the lowest active load for both smooth and rough walls. Rotation about the base produced the highest active load for a smooth wall but it approached the load on the translating wall for the rough wall case and large displacements.

The distribution of stress on the wall in the computational model (DEM) was approximately linear after the backfill was generated (rained). For the wall translation and rotation about the top, the distributions at the active state became distinctly nonlinear. For the rotation about the top edge, the wall stress gradient in the upper portion of the wall is very large and is more characteristic of

### Table 3. Load Coefficient $K$ and Location of Stress Distribution Centroid: Differential Slice Method (DSM) versus Physical Experiment (Rough Wall, $\delta = 24.7^\circ$)

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\phi$</th>
<th>Differential slice technique</th>
<th>Physical experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$K$</td>
<td>Centroid $^a$</td>
</tr>
<tr>
<td>Translation</td>
<td>34°</td>
<td>0.254</td>
<td>0.363</td>
</tr>
<tr>
<td>Rotation about base</td>
<td>33.4°</td>
<td>0.312</td>
<td>0.292</td>
</tr>
<tr>
<td>Rotation about top edge</td>
<td>40.4°</td>
<td>0.292</td>
<td>0.492</td>
</tr>
</tbody>
</table>

$^a$Location of stress distribution centroid above base as fraction of wall height.
the passive case, but this gradient becomes negative at about 3/4 of the wall height. The physical experiments by Fang and Ishibashi (1986) confirm this characteristic. The coefficient of active wall load $K_a$, as in Eq. (1), is a global concept that relates to the wall, and it does not indicate what the stress distribution might be. The Rankine method is different in that respect, as it explicitly yields a linear distribution.

In the translation mode of wall displacement, a moving wedge in the DEM displacement field can be identified behind the wall (very much like in the classical Coulomb solution). If the limit state in the granular soil is reached along the plane behind the sliding wedge, and the direction of wall reaction is well-defined, then the total load on the wall is uniquely determined by the global equilibrium of the wedge, and the direction of wall reaction is well-defined, then the total load is independent of the specific distribution of the stress on the wall. Therefore, no methods utilizing global equilibrium will produce solutions to the problem. Calculations revealed that the mixed mode borrows distribution characteristics from the primary modes, having the appearance of interpolation between the modes. In any case, the location of the resultant force on the wall should not be used in design lower than 0.3 of the wall height, and for walls with displacement restrained at the top, taking this location not lower than the midpoint is prudent. These values fall close to those alluded to in the literature (e.g., Terzaghi et al. 1996), although they came from different considerations.

A differential slice technique was used in an effort to replicate the physical test results numerically. These simulations indicated that, for the translational mode of wall displacement, the distribution of stress that mimics the experimental one can be found when the strength of the soil is fully mobilized along the failure plane behind the sliding backfill wedge. However, for rotational modes, the strength is likely not fully mobilized along the contour defining the wedge. For wall rotation about the base, a good match of the experimental distribution was obtained when the strength was fully mobilized in the upper portion of the backfill, and vice versa. Intuitively, such variation is reasonable because the strain along the slip surface varies for rotational modes.

### Appendix

Equilibrium of all forces in the $x$-direction acting on the slice in Fig. 7(b) yields

$$\frac{d\bar{r}}{dz} - \sigma_z^* \frac{D_y}{l} + \sigma_z^* \frac{D_y}{l} + \bar{r} + \gamma \sin \beta = 0$$

where $\gamma$ = unit weight of the soil, angle $\beta$ is illustrated in the figure, and coefficients $D_y$ and $D_y'$ are given in Eq. (11). Equilibrium in the $z$-direction produces the following equation

$$\frac{d\sigma_z^*}{dz} + \frac{d\sigma_z^*}{dz} - \sigma_z^* \frac{W_l}{l} + \sigma_z^* \frac{W_r}{l} + 2\gamma \cos \beta = 0$$

and the moment equilibrium equation leads to

$$\frac{d\sigma_z^*}{dz} - \frac{d\sigma_z^*}{dz} = \sigma_z^* \frac{R_l}{l} + \sigma_z^* \frac{R_r}{l} - \frac{12}{l} \bar{r} = 0$$

where

$$W_l = -(A_l + B_l)$$

$$W_r = -(A_r + B_r)$$

and coefficients $A$ and $B$ are given in Eq. (11). After some algebraic transformation, Eqs. (14)–(16) take form as in Eq. (10).

Point $O$ on the $z$-axis is defined in the differential slice technique as a cross-section of the lines tangent to the slice, as shown in Fig. 7(b). For a given slice and a curved slip surface (varied $\theta'$) the location of this point below the slice is found from $z = l/(\tan \theta + \tan \theta')$, where $l$ = slice width.

The following functions were used in the differential slice technique for calculations in the case of the wall rotating about the base ($d$ is depth)

$$\phi^* = \phi \left(1 - \frac{d}{H}\right)^{0.13}, \quad \phi = 33.4^\circ$$

$$k_l = 0.2602 \left[1 + \left(\frac{d}{H}\right)^{15}\right]$$

where $\phi^*$ = mobilized internal friction angle; and for rotation about the top edge

$$\phi^* = \phi \left(\frac{d}{H}\right)^{0.25}, \quad \phi = 40.4^\circ$$

$$k_l = 0.1961 \left[1 + 20\left(1 - \frac{d}{H}\right)^4\right]$$

### Acknowledgments

This material used work supported by the National Science Foundation under Grant No. CMMI-1129009. This support is greatly appreciated. The authors also acknowledge Itasca Consulting Group, Inc., for the academic loan of their PFC3D code. This collaboration is also very much appreciated.

### References


