Static Fatigue, Time Effects, and Delayed Increase in Penetration Resistance after Dynamic Compaction of Sands

Radoslaw L. Michalowski, F.ASCE¹; and Srinivasa S. Nadukuru, S.M.ASCE²

Abstract: Dynamically compacted sands often exhibit a drop in cone penetration resistance immediately after compaction, but a gradual increase in the resistance occurs in a matter of weeks and months. An explanation of the former is sought in analysis of the stress state immediately after a dynamic disturbance, and a justification for the latter is found in the micromechanics process of static fatigue (or stress corrosion cracking) of the micromorphologic features at the contacts between sand grains. The delayed fracturing of contact asperities leads to grain convergence, followed by an increase in contact stiffness and an increase in elastic modulus of sand at the macroscopic scale. Time-dependent increase in small-strain stiffness of sand under a sustained load is a phenomenon confirmed by earlier experiments. It is argued that the initial drop in the cone penetration resistance after dynamic compaction is caused by a drop in the horizontal stress after the disturbance. The subsequent gradual increase in the penetration resistance is not a result of increasing strength, but it is owed to the time-delayed increase in stiffness of sand, causing increase in horizontal stress under one-dimensional strain conditions. This process is a consequence of static fatigue at contacts between grains. The strength of sand after dynamic compaction increases as soon as the fabric of the compacted sand is formed and is little affected by the process of grain convergence in the time after compaction. Contact stiffness, with its dependence on static fatigue, holds information about the previous loading process, and it is a memory parameter of a kind; this information is lost after a disturbance, such as dynamic compaction, in which new contacts are formed. The scanning electron microscope (SEM) observations, discrete element simulations, and energy considerations are carried out to make the argument for the proposed hypothesis stronger. DOI: 10.1061/(ASCE)GT.1943-5606.0000611. © 2012 American Society of Civil Engineers.

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Introduction

It is well-documented in the literature that sand deposits subjected to densification through dynamic means, such as blasting or vibrocompaction, may exhibit a drop in cone penetration resistance immediately after compaction. However, the resistance to cone penetration increases over time, far beyond the time needed for the excess pore water pressure to dissipate. Attention to this peculiar behavior was brought by Mitchell and Solymar (1984), who presented the data from the Jebba Hydroelectric Development project in Nigeria. The dam in this project was founded on alluvial sand with maximum depth of 70 m. Blasting was carried out at a depth of 25 to 40 m, and vibrocompaction was used at depths less than 25 m. Blasting was found to reduce the penetration resistance immediately afterward, even though the densification was evident by the surface subsidence. Nine days after blasting, the static cone penetration resistance was still slightly lower than that before blasting, but a considerable increase in penetration resistance was found after 11 weeks. A similar effect was found in soils after vibrocompaction, Fig. 1. Other examples of this phenomenon can be found in Baxter (1999). Among several possible causes for this delayed effect, preference was given, at the early stage of research, to mineral dissolution and cementation at interparticle contacts developed in time after dynamic compaction. Since then, attention has shifted to the secondary compressionlike process, but the behavior is still referred to as continuing Enigma (Mitchell 2008).

The delayed increase in penetration resistance after blast densification of saturated sand was shown by Dowding and Hryciw (1986) in well-controlled laboratory experiments. Blasting was carried out in a 66-cm deep sand tank with a 107-cm diameter. Fine, poorly graded silica sand was used, the charges were set at the center line of the tank, and a small cone was used to test the effect of blasting. Dowding and Hryciw confirmed the peculiar behavior of dynamically compacted sand; they indicated that a sand bed prepared in the test drum but not subjected to blasting also exhibited an increase in penetration resistance but to a lesser extent.

Thomann and Hryciw (1992) indicated that the initial loss of stiffness of sand depends on the extent of disturbance (measured by shear strain). In laboratory tests, a significant increase in stiffness occurred in the first few hours after disturbance, and this increase slowed down considerably afterward.

The time-dependent changes in sand affect the small-strain shear modulus. Afifi and Woods (1971) subjected sand to a constant confining stress and measured the shear modulus using a resonant column. In air-dry Ottawa sand, the shear modulus was found...
to increase approximately 2 to 5% per log cycle of time. They also noted that the percentage increase in shear modulus amplifies with the decrease in the particle size, which is an important finding supporting the explanation of the time-delayed effect advocated in this paper.

A series of experiments, very relevant to the study of time effects in sand, was carried out by Daramola (1980). Specimens of Ham River sand were loaded with a confining pressure of 400 kPa and then subjected to a drained compression test in a triaxial apparatus. One specimen was tested immediately after applying the confining pressure, and the remaining specimens were tested after 10, 30, and 152 days. The sand appeared to gain in stiffness but not in strength, as shown in Fig. 2. This will prove to be important evidence supporting the hypothesis suggested in this paper.

The process behind the time-delayed effect observed after dynamic compaction of sand was termed in the literature as “aging” (e.g., Schmertmann 1991). The terms “aging” and “creep” have been used with some liberty in many studies. Neither term reveals a special mechanism responsible for the material behavior, thus neither explains the origins of that behavior. A distinction is made here: creep is a macroscopic rate-dependent process, whereas aging is a time-dependent process. For instance, the behavior of bituminous materials is well-described by a viscoelastic (rate-dependent) model, whereas aging is not rate-dependent but time-dependent. If the response to stress depends on time, it is the same at any instant of time, the behavior is viscoelastic (rate-dependent), and such material can creep (Findley et al. 1989). Curing concrete, however, will respond to stress differently at different times of curing; this is aging.

Dissolution of minerals, pressure solution, and bond formation are processes that were given prominence in earlier stages of aging studies. Pressure solution plays an important role during diagenesis of rocks. It is one of the primary mechanisms for cementation in porous sedimentary rocks and a mechanism for deformation of nonporous sedimentary and low-grade metamorphic rocks (Tada et al. 1987). Mineral dissolution may be an important factor at the engineering temporal scale in processes involving chemical reactions (Hu and Hueckel 2007) and with minerals particularly susceptible to dissolution, such as halite (rock salt; Tada and Siever 1986). However, recent studies (Baxter 1999; Baxter and Mitchell 2004) indicate that it is unlikely that this process plays a significant role in increasing penetration resistance after dynamic compaction of silica sand. The hypothesis advocated in this paper is more consistent with that suggested by Schmertmann (1987), and it indicates the change in the stress state as the chief reason for the time-delayed increase in penetration resistance after dynamic compaction.

“Particle rearrangement” and “structuration” are often cited as causing changes in granular assemblies leading to variation in properties (Mesri et al. 1990; Bowman and Soga 2003), and they have been used to explain the puzzling effect of delayed increase in cone penetration resistance in dynamically compacted sands. Rearrangement of particles and restructuring, however, are the results of an underlying process and cannot be considered as causes of this mystifying behavior. A hypothesis is presented in this paper as to what the dominant mechanism is, triggering the peculiar behavior in sand after dynamic compaction. The evidence available in the literature to support this hypothesis is discussed, discrete element simulations are used to complement this evidence, and energy considerations are carried out as a supporting argument.

**Change in the Stress State versus a Change in Material Properties**

**Multiscale Consideration of Sand**

For the purpose of this study, three scales are distinguished at which the sand needs to be considered. The microscale relates to interactions between individual grains; it includes the response of an individual contact to applied load. Surfaces of sand grains at this...
At this scale, the interaction involves fracture of asperities and crystalline debris on grain surfaces. A contact of two grains is illustrated in Fig. 3(d).

Assemblies of grains are described at the mesoscale. This scale includes load transfer through formation of force chains, and relative motion of grains. The development of the force chains between grains was illustrated early by Drescher and De Jong (1972) by using optically sensitive disks (elastooptics). This is the scale at which the soil fabric is formed and in which contraction and Reynold's dilatancy find their proper explanation. This is also where the concept of internal friction can be interpreted.

Finally, the macroscale is the level at which the response of the assemblies or clusters of grains is averaged. The notions of an average stress $\sigma_{ij}$ and average strain $\varepsilon_{ij}$ are introduced at the macroscale (see the appendix). Relations between the increments of the two give rise to constitutive equations. This is the scale at which engineering problems are solved, but the explanation of the behavior needs to be sought at micro- and mesoscales.

The stiffness of a granular assembly is a property affected directly by interactions between grains (microscale), with the contact stiffness of prime importance. This macroscopic stiffness is also dependent on the size of grains or the density of contacts (number of contacts per volume). Anisotropy of stiffness is then greatly affected by the fabric of sand. The strength of soil is affected predominantly at the mesoscale (fabric). Hence, it is hypothesized that the strength of sand increases after dynamic compaction as soon as the new fabric is formed, but the time-delayed increase in cone penetration resistance is a result of the gradual change in the stress state in the sand bed. The latter is a result of an increase in contact stiffness caused by time-delayed fracture of micromorphological features at contacts.

### Structural Shakedown versus Increase in Material Strength

A distinction is made here between the response of a structure to loading and the response of a loaded soil specimen (an element test). The latter is directly indicative of the soil mechanical properties (stiffness and strength), whereas the former needs to be interpreted as a boundary value problem. Shakedown is a particular response of an elastoplastic structure to cyclic loading, in which, during the first or the first few cycles, the response is elastic-plastic, but afterward, it becomes purely elastic. The explanation of this effect is in accumulation of irreversible strain in a structure. Because this plastic strain is incompatible with the elastic strain during unloading, a residual stress field remains in the structure upon unloading. If this residual stress is opposite to the stress induced by the cyclic load, the elastic load limit on the structure in the next cycle appears to increase because this cycle is superimposed over the existing stress of the opposite sign. There is, of course, a limit to the amplitude of the loading beyond which shakedown will not occur. The theorem predicting this limit (static shakedown theorem) was proved by Melan (1938).

If the residual stress in the structure is of the same sign as that caused by the applied load, the superposition of the two will lead to a decrease in the apparent elastic limit of the load, giving the impression of reduced strength. However, it is the limit load on the structure that is reduced and not the material strength. It is suggested that the decrease in the cone penetration resistance of sand after dynamic compaction is a result of the stress state altered by the dynamic compaction. The difference between the stress state before and immediately after dynamic compaction plays the same role in the sand bed as the residual stress does in a structure subjected to
cyclic loading. This stress difference causes an increase in the deviatoric stress in the sand bed (moves the stress state closer to failure), and, therefore, the resistance to cone penetration is reduced immediately after compaction.

Static Fatigue at Grain Contacts—Dominant Cause of Time Effects in Sand

Stress Corrosion Cracking or Static Fatigue

When two particles come into contact [Fig. 3(d)], it is the asperities and the small crystalline fragments that are first loaded to a considerable degree as the areas of the surfaces in contact are relatively small. The microscopic features of the contact morphology are prone to develop cracks in time, and this process is called stress corrosion cracking in this paper. It is suggested that the time and rate effects in sand are owed primarily to stress corrosion cracking at intergranular contacts. It is a delayed fracture process, accelerated by stress and environmental factors, such as moisture. This process is also referred to as static fatigue, at contacts. This hypothesis was suggested recently by Michalowski and Nadukuru (2010, 2011).

To illustrate the physical consequences of static fatigue, an experiment was conducted in which 3 grains mounted at a distance of 5 mm from one another (equilateral triangular pattern) were loaded with 0.65 N (per particle). Fig. 4 shows an asperity on the grain surface before loading, 15 min after application of the load, and after 1 week of loading. The damage (fractured asperity) is clearly visible in Fig. 4(b); most of this damage may have occurred immediately after the load was applied. Further microfracturing, seen in Fig. 4(c), occurred over the following week. This process occurs on intergranular contacts in sand, particularly those along the strong force chains. These discrete fracture events, however, do not occur simultaneously at all contacts, but when integrated at the macroscopic scale, they have an initial appearance of a rate effect.

Fracture of asperities at intergranular contacts brings grains closer together, a process called grain convergence in this paper. The consequence of grain convergence is an increase in contact stiffness, resulting in an increase in the elastic moduli at the macroscopic scale. This is a time-delayed process, as evidenced in experiments of Afifi and Woods (1971) and Daramola (1980). Not only was a consistent increase of the small-strain shear modulus recorded in sand subjected to prolonged stress, but the effect was found more distinct in sand with a smaller grain size. This is because the elastic modulus is affected by static fatigue at the contacts, and per given volume, there are more contacts in the soil with smaller grain size (more contacts per distance traveled by an elastic wave). The adjustments of grain positions caused by grain convergence are of the order of the asperity size, and they are likely to have small consequences on the fabric of the sand. This is why the strength of the sand, which is driven by its fabric, was not affected by static fatigue in Daramola’s triaxial testing (Fig. 2), whereas the stiffness was clearly influenced by time.

Static fatigue is not a new concept, but only recently was it used to explain mechanical behavior of sand by Lade and Karimpour (2010a, b), who considered static fatigue of grains (delayed grain fracture). Static fatigue has been considered in material science for decades (Charles 1958) as a time-dependent fracturing process in brittle materials. Scholz (1968a, 1972) reviewed experimental work on creep of rocks and concluded that, at low temperatures and pressure, the primary mechanism of creep of brittle rocks is delayed microfracturing. Scholz (1968b) also indicated that microfracturing releases elastic energy analogous to earthquakes. The delayed microfracturing (static fatigue) is then analogous to aftershocks. Microseismicity was considered more recently by Gudehus (2006) in the context of hypoplasticity.

Research has shown that the presence of moisture is more conducive to stress corrosion cracking than a dry environment. For
instance, when fiber optic cables are bent, the stresses produced by the bending moment lead to static fatigue, and the process was shown to be dependent on the presence of moisture (Cuallar et al. 1987).

Fracturing of brittle materials, such as concrete and rocks, leads to degradation of elastic properties and so does crushing of grains in granular materials (Yamamuro and Lade 1996). Grain crushing is a process at the mesoscale, leading to formation of a new (metamorphic) material. Static fatigue at contacts, however, occurs at smaller loads, and it affects the asperities and crystalline fragments (debris) at contacts between grains (micro scale). Static fatigue at contacts in sands subjected to long-term loads (such as gravity loads) will decay in time and will become eventually insignificant. When sand is disturbed, however, and new contacts are formed between grains with a rich surface morphology, stress corrosion cracking will become a dominant time-dependent mechanical process again. In this paper, it is argued that this process is the cause of the increase in the cone penetration resistance in the weeks and months after dynamic compaction.

Change in the Stress State after Dynamic Compaction

It is argued that the strength of sand after dynamic compaction increases as soon as the new fabric of the compacted sand is formed, whereas the delayed increase in the cone penetration resistance is caused not by an increase in strength but by a gradual change in the stress state in the compacted soil. Discussed first is the development of the stress state in freshly compacted sand, followed by discrete element simulations.

Disturbance by Dynamic Compaction and Postcompaction State

At the macroscopic scale, sand is typically described with some elastic-plastic continuum model. Under one-dimensional (1D) strain conditions, and in the elastic range of response, an increase in the horizontal effective average stress $\delta\sigma_h$ caused by variation $\delta\sigma_v$ in the vertical stress can be found from

$$\frac{\delta\sigma_h}{\delta\sigma_v} = \frac{\nu}{1 - \nu}$$

where $\nu =$ Poisson’s ratio. This is a direct consequence of Hook’s law for an isotropic material applied to a 1D deformation problem. The ratio of in situ stresses in soils, however, does not conform to Eq. (1) because the natural stress state is affected by the geologic history, whereas Eq. (1) only indicates the ratio of increments governed by the current value of Poisson’s ratio.

Compaction of sand requires a change in its packing so that the volume of pores can be reduced. When saturated sands are dynamically compacted, the contact forces between grains are lost (liquefaction), and the contacts themselves are reconfigured so that a tighter packing can be achieved. The effective stresses are reduced to zero during liquefaction, but the stress state is rebuilt upon dissipation of the excess pore water pressure. The vertical macroscopic stress is governed by gravity, but the horizontal stress does not have a comparable governing rule. When the effective vertical macroscopic stress increases from zero during liquefaction to a value that balances the buoyant weight of the soil at any depth, the horizontal stress will trail with its value such so that the stress state is just below the yield condition of the newly compacted sand. This horizontal stress is then likely to be significantly lower than the stress in the preliquefaction sand bed that was subjected to long-term gravity loads. To demonstrate that this conjecture is reasonable, discrete element simulations are performed.

Discrete Element Method Simulation

An assembly of over 11,000 grains was generated by using the Particle flow code PFC3D (Itasca Consulting Group, Inc. 2008) computer code [Fig. 5(a)]. These grains were predominantly spherical with radii between 2.3 and 3.7 cm. 25% of the grains were in the form of clumps, i.e., particles with a shape of two overlapping spheres with the centers offset by one radius [peanut-shape particles, shown darker in Fig. 5(a)]. The assembly was formed in a cube of 1.2 m in size, and the porosity of the assembly was
0.35. The particles were assumed smooth during deposition, and the stress state in the specimen was hydrostatic. Next, the diameter of all particles was reduced by fraction 0.0005 and the interparticle friction coefficient was increased to $\mu = 0.65$. After the system reached the state of equilibrium, the porosity was practically unchanged and equal to $n = 0.35$. The contact normal and tangential stiffness was set to $K_n = 4.0$ and $K_t = 1.6$ MN/m, respectively, the mass density of the grains was set to 2,650 kg/m$^3$, and gravity acceleration was taken as 9.81 m/s$^2$. The ratio of horizontal-to-vertical macroscopic stress in the prepared specimen was $k = 0.473$. A triaxial compression test of the specimen was then simulated with a confining pressure of 50 kPa. Interpreting the strength of the granular assembly as frictional with a linear envelope, the peak internal friction angle was found to be 33.5$^\circ$.

Discrete element method (DEM) simulations of liquefaction as a hydromechanics problem are cumbersome, and they are not attempted often. Two other methods were considered: (a) inducing liquefaction through a constant-volume cyclic strain process, and (b) a minute reduction in grain size causing an instant loss of intergranular contacts. Both methods lead to a reduction in the coordination number (number of contacts per particle) and loss of shear resistance interpreted as the onset of liquefaction, even though no pore fluid is considered. In the first method, the initial confining (or consolidation) stress is gradually reduced in a kinematically controlled constant-volume cyclic process, and the drop in the average stress (pressure) associated with this process is interpreted as a buildup of the pore water pressure. Successful simulation of this type can be found, for instance, in Ng and Dobry (1994). Although effective when simulating liquefaction of specimens, this method is less helpful when applied to boundary value problems with distributed loads, such as gravity. This is because, without pore fluid, the gravity load has to be sustained by the force chains requiring active contacts. Therefore, the second method is resorted to, in which instantaneous loss of contacts (onset of liquefaction) is simulated by a small reduction in grain size. Although not physically based, the method is effective and reasonable, particularly because the interest in this paper is in the postliquefaction process of effective stress buildup and not in the process that led to liquefaction.

A small reduction in the size of all grains was simulated (0.001 reduction in grain diameters), causing a sudden loss of all contacts. This reduction in grain size produced no meaningful reduction in the void ratio (simulation of compaction was not attempted). The purpose of this DEM simulation is to indicate the change in the stress state from the preliquefaction to postliquefaction state. After the instantaneous loss of contacts, the contacts were regained in the process driven by gravity. However, the stiffness of these new contacts was taken as half of the stiffness of previous contacts. This is because grains would rotate during liquefaction so that the newly formed contacts are shifted with respect to preliquefaction contacts. The new contacts have micromorphological features (Fig. 3) not previously subjected to stress corrosion cracking, hence the expected reduction in contact stiffness.

Microscopic morphology at the grain surface produces substantially lower contact stiffness than that calculated from the Hertzian theory and intact mineral elastic modulus. In modeling, this can be interpreted as grains covered with a layer of softer material, susceptible to stress corrosion cracking [Fig. 5(b)]. The time-dependent behavior of sand is greatly affected by this layer.

During formation of new contacts and the regaining of static equilibrium the specimen was confined by the rigid smooth vertical and bottom boundaries. After equilibrium was reached, the average horizontal-to-vertical macroscopic stress ratio was found to be $k = 0.339$, substantially lower than the preliquefaction value of 0.473. In calculations of this ratio, the horizontal stress was averaged over the vertical boundaries, and the vertical stress was taken as half of the stress at the bottom boundary (average of the stress at the top and bottom). The specimen was then subjected to a (numerical) triaxial test under a confining pressure of 50 kPa. The peak internal friction angle was determined to be 33.6$^\circ$, almost identical to the preliquefaction value. The average stress state in the specimen before and after liquefaction is illustrated in Fig. 5(c). It is evident that the stress state after simulated liquefaction has moved closer to failure, whereas the yield condition was not changed (no compaction was simulated).

Next, the normal and tangential stiffness of grain contacts were increased by 10%, simulating an increase in stiffness caused by static fatigue and convergence of grains. The horizontal-to-vertical stress ratio now increased to the value of 0.365; with another 10% increase in the contact stiffness, the horizontal-to-vertical stress ratio increased to 0.378. These increases are reported in Table 1. Contact stiffness was then increased in another eight steps up to the original value before liquefaction. After each step, the grain assembly was allowed to reach the state of static equilibrium. Ratio $k$ increased to 0.459 after the last simulated increment (Table 1). At this stage, the stress reached a state close to the preliquefaction level. The numerical triaxial compression test revealed the peak internal friction angle to be $\phi = 33.5^\circ$.

Subsequently, another cycle of liquefaction was simulated with the reduction of grain diameters, loss of intergranular contacts, and the reduction in contact stiffness to $K_n = 2.0$ and $K_t = 0.8$ MN/m, as new contacts after liquefaction were formed. Immediately after liquefaction, the stress ratio dropped down to $k = 0.338$, and the gradual increase in contact stiffness (in 10 steps) to $K_n = 4.0$ and $K_t = 1.6$ MN/m (considered to be caused by static fatigue), then led to an increase in the stress ratio up to $k = 0.444$. The peak internal friction angle from a numerical triaxial compression test (confining pressure of 50 kPa) was found to be $\phi = 33.5^\circ$. This simulation confirms that liquefaction causes a drop in the horizontal stress under 1D strain conditions, and the subsequent increase in horizontal stress can be attributed to an increase in contact stiffness caused by static fatigue and convergence of grains. However, the internal friction angle is not affected, as long as the sand is not compacted during the process.

One would expect the process simulated here to occur after any disturbance of sand that will cause the loss of contacts and formation of new contacts that have not been previously subjected to stress corrosion cracking. This is why, in the tests of Dowding and Hryciw (1986), the newly formed bed that was not subjected

<table>
<thead>
<tr>
<th>$K_n$(MN/m)</th>
<th>$K_t$(MN/m)</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>0.8</td>
<td>0.339</td>
</tr>
<tr>
<td>2.2</td>
<td>0.88</td>
<td>0.365</td>
</tr>
<tr>
<td>2.4</td>
<td>0.96</td>
<td>0.378</td>
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<tr>
<td>2.6</td>
<td>1.04</td>
<td>0.391</td>
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<tr>
<td>2.8</td>
<td>1.12</td>
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<tr>
<td>3.0</td>
<td>1.2</td>
<td>0.413</td>
</tr>
<tr>
<td>3.2</td>
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<td>0.424</td>
</tr>
<tr>
<td>3.4</td>
<td>1.36</td>
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<td>1.44</td>
<td>0.442</td>
</tr>
<tr>
<td>3.8</td>
<td>1.52</td>
<td>0.451</td>
</tr>
<tr>
<td>4.0</td>
<td>1.6</td>
<td>0.459</td>
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</table>
to blasting also exhibited some delayed increase in penetration resistance.

It should be stressed that DEM is a model, and although the results support the hypothesis advocated here, it is not by itself a proof of the hypothesis.

**Anisotropy of Sand Bed**

Because the process of fabric formation after liquefaction is affected by gravity, it is expected that the soil will have mechanical properties at the macroscopic scale characterized by cross-anisotropy. Of interest here are the elastic moduli and Poisson’s ratios, necessary to calculate the strain energy in the soil. The objective here is not to relate the topology of the packing to the macroscopic elastic constants (as is often the case in composite materials, Cowin 1985) but to numerically test the specimens created under gravity and determine the elastic macroscopic parameters given the contact properties. The constitutive relation for an elastic cross-anisotropic material takes the form

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\varepsilon_{xy} \\
\varepsilon_{yz} \\
\varepsilon_{xz}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{E_h} & \frac{\nu_{hh}}{E_h} & \frac{\nu_{hv}}{E_h} & 0 & 0 & 0 \\
-\frac{\nu_{hh}}{E_h} & \frac{1}{E_h} & -\frac{\nu_{hv}}{E_h} & 0 & 0 & 0 \\
-\frac{\nu_{hv}}{E_h} & \frac{\nu_{hv}}{E_h} & \frac{1}{E_v} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2G_{hh}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2G_{hh}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2G_{hh}}
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{xy} \\
\tau_{yz} \\
\tau_{xz}
\end{bmatrix}
\]

where the dash indicates stresses and strains at the macroscopic scale, as defined in Eqs. (21) and (22); and the material parameters are elastic moduli \( E_h \) and \( E_v \) in the horizontal (x, y) and vertical (z) directions, respectively, and Poisson’s ratios \( \nu_{hh}, \nu_{hv}, \) and \( \nu_{vh} \).

Under conditions of cross-anisotropy

\[
\frac{\nu_{hh}}{E_v} = \frac{\nu_{hv}}{E_h}
\]

The coefficient of lateral pressure increment, given in Eq. (1) for isotropic soil, now takes the form

\[
\frac{\delta \sigma_h}{\delta \sigma_v} = \frac{\nu_{hh}}{1 - \nu_{hh}} \frac{E_h}{E_v} = \frac{\nu_{hv}}{1 - \nu_{hh}}
\]

To validate the statement that the deposition of granular materials under gravity leads to cross-anisotropy in elastic behavior, two numerical tests on an assembly of particles in cubical specimens are performed, as shown in Fig. 6(a) and 6(b). The specimens were created in the same manner as the one in Fig. 5(a). These experiments will be used to determine the material constants in Eq. (2).

**Test 1 (Triaxial Elastic Compression)**

Numerical tests were performed on the specimen that was previously generated for the purpose of simulating liquefaction and static fatigue effects. Tests were performed at stages represented in Table 1 with \( K_s \) of 2.0, 3.0, and 4.0 MN/m. At each testing stage, the specimen was covered with a rigid boundary at the top, gravity was switched off, and the force chains within the specimen were allowed to reach a new static equilibrium state. Without gravity, the experiment can be considered an element test. The cube was initially loaded with stresses: \( \delta \sigma_x = \delta \sigma_y = \delta \sigma_z = 100 \, \text{kPa} \) and \( \delta \sigma_x = \delta \sigma_y = 200 \, \text{kPa} \). The force chains at that stage are illustrated in Fig. 6(c). Next, displacement was induced at the top horizontal boundary, whereas the stress on the vertical boundaries was kept constant at 100 kPa. After the vertical strain reached \( \delta \varepsilon_z = 1.873 \cdot 10^{-5} \), the specimen was unloaded; the numerically simulated process was linear and reversible. The increase in vertical stress was \( \delta \sigma_z = 382.6 \, \text{Pa} \) when the maximum of the vertical strain was reached, and the horizontal strain increments (needed to keep constant horizontal stress) were \( \delta \varepsilon_x = 1.917 \cdot 10^{-6} \) and \( \delta \varepsilon_y = 2.000 \cdot 10^{-6} \). This data was obtained for the specimen with contact stiffness of \( K_s = 2.0 \, \text{MN/m} \) and \( K_s = 0.8 \, \text{MN/m} \). The test was then repeated for the stages when \( K_s = 3.0 \) and \( K_s = 4.0 \); the respective numerical results for the latter stage were: \( \delta \varepsilon_x = 1.378 \cdot 10^{-5}, \delta \sigma_z = 424.7 \, \text{Pa}, \delta \varepsilon_z = 1.664 \cdot 10^{-6}, \) and \( \delta \varepsilon_y = 1.748 \cdot 10^{-6} \).

**Test 2 (1D Strain Test)**

The same numerical specimen as in Test 1 was used. After switching off gravity, a 1D strain test was performed with \( \delta \varepsilon_z = \delta \varepsilon_x = 0 \), up until \( \delta \varepsilon_y \) reached 1.958 \cdot 10^{-5}. The stress increments associated with this process were: \( \delta \sigma_x = 86.03 \, \text{Pa}, \delta \sigma_y = 395.4 \, \text{Pa}, \) and \( \delta \sigma_z = 102.1 \, \text{Pa} \). Upon unloading, the process was found to be reversible. The test was repeated for the stages in the middle and the last row in Table 1; the respective results for the latter were: \( \delta \varepsilon_z = 1.439 \cdot 10^{-5}, \delta \sigma_x = 97.78 \, \text{Pa}, \delta \sigma_y = 416.4 \, \text{Pa}, \) and \( \delta \sigma_z = 129.5 \, \text{Pa} \).
With the results from the two numerical tests, matrix Eq. (2) allows one to write four independent equations for five unknowns: \(E_h\), \(E_v\), \(\nu_{hh}\), \(\nu_{vh}\), and \(\nu_{hv}\). The fifth equation comes from the symmetry of transverse isotropy in Eq. (3). Calculated elastic constants are given in Table 2.

Further testing indicated that a grain assembly with constant contact stiffness does not conform to linear elasticity, and the response of the cube to loading depends on the current stress ratio. This is consistent with physical testing results of Belotti et al. (1996). The tests reported were performed with an initial vertical-to-horizontal stress ratio of 0.5. For a lower stress ratio, the difference in the vertical and horizontal Young’s moduli and Poisson’s ratios was found to be larger.

The discrete element simulations confirm that a granular material deposited under gravity has cross-anisotropic properties. Notice that anisotropy was not assumed a priori in the DEM model, and should the grain assembly behave isotropically, this would have been revealed in calculated elastic constants.

### Energy Argument for Stress Increase

Central to the thesis in this paper is the process of increase in horizontal macroscopic stress in sand following dynamic compaction. The energy balance equation is used to indicate that such balance requires the horizontal stress to be altered when the elastic stiffness of the sand increases (in 1D strain conditions). This consideration is exploratory; at this time the energy dissipated because of stress corrosion microfracturing (acoustic emission) as well as irreversible strains caused by grain convergence can only be roughly estimated.

Consider a force in a spring between two rigid walls as an analog of horizontal stress in the sand bed under 1D strain conditions [Fig. 7(a)]. The spring was forced between the walls, causing the force to increase from 0 to \(P_1\) along path \(OB_1\). If a change in the spring stiffness was now to occur from \(K_1\) to \(K_2\) without any expenditure of energy, the force would need to increase to \(P_2\) along path \(B_1B_2\) [Fig. 7(b)]. The value of \(P_2\) is uniquely determined from the balance of elastic energy (shaded areas are equal to one another). However, this is clearly a paradox, as no energy was dissipated, yet irreversible displacement \(\delta u_{\text{irrev}}\) remains after removing one of the walls (unloading along path \(B_2C\)). Introduction of a dissipative energy term in a balance equation (needed for the change in the elastic spring constant and irreversible deformation) will cause the calculated force in the spring to increase to a lesser extent (and unload along the dashed line), not increase at all, or even drop. It is argued that, in a soil bed subjected to dynamic compaction, horizontal stresses increase to some extent in the post-compaction period.

### Strain Energy

Throughout the remainder of this section, the balance of energy in a soil column is considered, Fig. 8(a), to indicate that a change in horizontal stresses must occur if the elastic constants of the soil are to increase.

The elastic strain energy in a unit volume of soil, in terms of macroscopic stresses and strains, is equal to

\[
W_e = \frac{1}{2} \sigma_{ij} \varepsilon_{ij}, \quad i, j = 1, 2, 3
\]  

There is ample evidence for the small-strain elastic properties of sand deposited under gravity to be cross-anisotropic (e.g., Bellotti et al. 1996; Kuwano and Jardine 2002). The presence of cross-anisotropy was also found in numerical simulations described in the previous section. Therefore, the expression sought is that for the strain energy accounting for cross-anisotropy.

The coaxiality of the principal directions of the macroscopic stress and strain tensors in the soil bed in 1D strain condition is enforced by the gravity direction perpendicular to the plane of the soil bed.
transverse isotropy, hence the summation in Eq. (5) needs to be performed only over the respective principal components:

\[
W_e = \frac{\sigma_1^2}{2E_h} + \frac{\sigma_2^2}{2E_v} + \frac{\sigma_3^2}{2E_v} - \frac{\nu_{hv}}{E_h} \sigma_3 \sigma_y + \frac{\sigma_3 \sigma_x}{E_v}
\]

\[
- \frac{1}{2} \left( \frac{\nu_{hv}}{E_h} \sigma_3 + \sigma_3 \sigma_y + \sigma_3 \sigma_x \right)
\]

(6)

This equation assumes a more familiar form for isotropic material \((x,y,z)\) are principal directions):

\[
W_e = \frac{1}{2E} \left( \sigma_1^2 + \sigma_2^2 + \sigma_3^2 \right) - \frac{\nu_{hv}}{E} \sigma_3 \sigma_y + \sigma_3 \sigma_x + \sigma_3 \sigma_x
\]

(7)

Once the pore water pressure dissipates after dynamic compaction, the soil assumes the stress state characterized by the horizontal-to-vertical ratio

\[
k_1 = \frac{\sigma_h}{\sigma_v}
\]

(8)

Introducing \(\sigma_z = \sigma_v \) and \(\sigma_x = \sigma_h \) and using Eq. (8)

\[
W_e = \sigma_3^2 \left[ \frac{1}{2E_v} + \frac{k_2^2}{E_h} \left( 1 - \nu_{hv} \right) - k_1 \left( \frac{\nu_{hv}}{E_h} + \frac{\nu_{hv}}{E_v} \right) \right]
\]

(9)

After a change in macroscopic elastic constants has occurred (attributed to static fatigue), the expression for the strain energy in a volume element becomes

\[
W'_e = \sigma_3^2 \left[ \frac{1}{2E_v} + \frac{k_2^2}{E_h} \left( 1 - \nu_{hv} \right) - k_2 \left( \frac{\nu_{hv}}{E_h} + \frac{\nu_{hv}}{E_v} \right) \right]
\]

(10)

with \(k_2\) a new horizontal-to-vertical stress ratio. The prime denotes the parameters after they have been altered by the process of static fatigue; both Young’s moduli and Poisson’s ratios were assumed to have changed.

### Work of Gravity Forces

There are two causes for vertical displacement of sand during the time following dynamic compaction. The first one is the convergence of grains caused by static fatigue, and the second reason is the change in elastic properties.

Studies of grain surfaces indicate that the size of asperities involved in static fatigue is of the order of 10^{-1} to 10 \(\mu\)m (Fig. 3). Grain convergence caused by the fracture and collapse of these microscopic features is likely to be of the order of 1 \(\mu\)m per contact in the months after compaction. To assess an order of magnitude of the strain at the macroscopic scale, take a 1-m column of grains with 2,000 contacts; this will yield approximately 2 mm displacement, or an average strain of 0.2%. Because static fatigue is dependent on the load at contacts, it is reasonable to assume the increment of strain caused by grain convergence to be dependent on depth:

\[
\Delta \varepsilon_z = \xi \gamma \varepsilon_z
\]

(11)

Parameter \(\xi\) characterizes the convergence strain, and it is a function of time because the static fatigue is caused by delayed microfracturing process. Because this strain is likely to be of the order \(10^{-3}\) (sand column consideration) and possibly larger for finer sand, parameter \(\xi\) is likely to be of the order \(10^{-6}\) to \(10^{-5}\) (kPa^{-1}) weeks and months after dynamic compaction. Vertical displacements in a soil bed [Fig. 8(a)] caused by the convergence strain are

\[
\Delta \varepsilon_z(z) = \int_z^H \Delta \varepsilon_z dz = \frac{\gamma \xi}{2} \left( H^2 - z^2 \right)
\]

(12)

The second cause for increment of strains in the soil bed after dynamic compaction is the change in elastic properties. This increment can be calculated as \(\Delta \varepsilon_z = \varepsilon_z' - \varepsilon_z\), where \(\varepsilon_z\) is the initial strain and \(\varepsilon_z'\) is the strain after an increase in elastic moduli. Using Eqs. (2) and (3), and \(\sigma_z = \sigma_v\), \(\sigma_x = \sigma_h\), one obtains

\[
\Delta \varepsilon_z = \frac{1}{E_h} \left( 1 - k_2 \frac{\nu_{hv}}{E_h} + 2k_1 \frac{\nu_{hv}}{E_h} \right)
\]

(13)

with prime denoting the material properties changed because of static fatigue after a dynamic disturbance; \(k_1\) is the horizontal-to-vertical stress ratio immediately after the disturbance; and \(k_2\) is stress ratio associated with increased (primed) elastic properties. Consider a soil bed with a layer of soil subjected to compaction, as shown in Fig. 8(a). Taking \(\sigma_z = \gamma\) \((\gamma\) is the buoyant unit weight of soil\), the vertical displacement increment in the soil bed caused by the change in the elastic properties can be found as

\[
\Delta u(z) = \int_z^H \Delta \varepsilon_z dz
\]

\[
= \gamma \left( \frac{1}{2} \left( H^2 - z^2 \right) \right) \left( \frac{1}{E_h} \right) \left( - \frac{1}{E_h} \right) - 2k_2 \frac{\nu_{hv}}{E_h} + 2k_1 \frac{\nu_{hv}}{E_h}
\]

(14)

The increment of vertical displacement in Eq. (14) associated with an increase in elastic moduli is negative (upward), leading to negative work of gravity forces, thus increasing the potential energy. However, this tendency to upward movement is small, and it is counteracted by the displacement caused by the strain attributed to grain convergence.

The total work of the soil weight (in a soil column) can now be written as

\[
W_t = \int_z^H \gamma (\Delta u' + \Delta u) dz + \gamma h (\Delta u_{z=h} + \Delta u_{z=h})
\]

(15)

The first term is the work of gravity forces in the compacted layer, Fig. 8(a), produced by both the change in elastic moduli and the convergence of grains, and the second term includes the work of the weight of the layer above the compacted zone. For simplicity, it was assumed that the elastic moduli change uniformly within the compacted layer. Another simplifying assumption was made that the unit weight of the soil within the compacted layer is the same as that of the layer above it.

### Energy Balance

The energy balance equation for a column of soil with a unit area cross section takes the form

\[
\int_z^H W_e dz + W_t - D = \int_z^H W_t dz
\]

with the left-hand side describing the initial strain energy (immediately after compaction), work of gravity forces in the time after compaction, and dissipated work; the right-hand side represents the elastic strain energy after the increase in soil stiffness. For the lack of data regarding energy dissipation, it is assumed to be fraction \(\chi\) of the work done by gravity forces on displacements caused by convergence of grains:

\[
D = \chi \left( \int_z^H \gamma \Delta u' dz + \gamma h \Delta u_{z=h} \right)
\]

(17)

Presumably, part of this energy is released through acoustic emission because of fracture of asperities at contacts between grains, and dissipated because of fracture, sliding, and rolling of crystalline...
fragments in the contact zone. After substituting Eqs. (9), (10), (15), and (17) into Eq. (16) and taking \( \sigma_c = \gamma_c \), one obtains a closed-form solution for stress ratio \( k_2 \) in an “aged” soil bed:

\[
k_2 = \frac{\sqrt{a}}{b} \tag{18}
\]

where

\[
a = \frac{E_h}{E_v} (1 - \nu_{hh}) k_1^2 - \frac{1}{2} \left( \frac{E_h - E_v}{E_v} \right) + \xi (1 - \chi) E_h \tag{19}
\]

\[
b = 1 - \nu_{hh} \tag{20}
\]

The last term in coefficient \( a \) contains two well-defined but somewhat vaguely assessed constants: \( \xi \) defining the strain due to convergence of grains [Eq. (11)], and \( \chi \) defining energy dissipation associated with the process of stress corrosion cracking [Eq. (17)]. The former is of the order of \( 10^{-6} \) to \( 10^{-5} \) kPa\(^{-1}\), as argued earlier, and the latter is guessed to be of the order of \( 10^{-1} \). A convenient consequence of all terms in Eq. (16) being of the same order in \( z \) is the result independent of the thickness and depth of the compacted layer. The major assumptions in deriving the stress ratio in Eq. (18) were linear elasticity, hence independence of the elastic properties of the stress level (depth), constant (buoyant) unit weight of soil \( \gamma \), and the change in elastic moduli independent of depth. An increase in the horizontal stress depends on the anticipated increase in the elastic moduli and Poisson’s ratio \( \nu_{hh} \) but not \( \nu_{hh} \) or \( \nu_{hv} \).

The DEM simulations indicated that a 50% increase in intergranular contact stiffness produced an increase in vertical and horizontal elastic moduli of 22 and 24% (Table 2), and this change produced an increase in the horizontal-to-vertical stress ratio of approximately 22% under 1D strain conditions. The increase in macroscopic elastic moduli was substantially higher when the contact stiffness was doubled (51 and 41% for \( E_v \) and \( E_h \), respectively), and the stress ratio increased by 35%. The anisotropic elastic moduli \( E_v \) and \( E_h \) and Poisson’s ratio \( \nu_{hh} \) of the grain assembly obtained from DEM simulations (Table 2) will now be used as data for calculations of the change in the horizontal-to-vertical stress ratio from Eq. (18).

No data is available to assess coefficients \( \xi \) and \( \chi \) quantitatively. It was argued earlier, however, that the order of magnitude of \( \xi \) in Eq. (11) is \( 10^{-6} \) to \( 10^{-5} \) kPa\(^{-1}\), and it increases with time as the stress state already close to failure.

Concluding Remarks

Sands liquefied during dynamic compaction regain the effective stress state with the horizontal stress lower than that before liquefaction. This change in the stress state was confirmed in DEM simulations. The low horizontal stress, together with the vertical stress governed by gravity, produce the deviatoric stress that moves the stress state closer to the yield condition, even though the sand strength may have increased because of compaction. The reduction in the cone penetration resistance immediately after dynamic compaction is a consequence of the penetration load superimposed on the stress state already close to failure.

Discrete element simulations presented in the paper indicated that an increase in the horizontal stress in a sand bed can be caused by an increase in the intergranular contact stiffness. This increase in contact stiffness is owed to the delayed process of stress corrosion cracking of the micromorphologic features on grain surfaces at the contacts, which brings grains closer together (grain convergence). Consequently, the macroscopic stiffness of sand increases, and under 1D strain conditions, the horizontal stress in the sand bed increases. It was demonstrated in this paper that a change in horizontal stress is necessary for balance of energy. Because of this increase in horizontal stress, the in situ stress state moves further away from the yield condition, hence the increase in the cone penetration resistance. A hypothesis of the change in the stress state as responsible for the increase in the penetration resistance was considered by Schmertmann (1987), and it was also alluded to by Mesri et al. (1990), and a mechanism by which such process can occur is described in this paper.

Intergranular contacts subjected to static fatigue change their stiffness over time and so they hold information about previous loading history. When the sand is disturbed, the memory of the previous history is lost, and the process of static fatigue (or stress corrosion cracking) will start at the newly formed contacts without previous memory.

It has already been emphasized that effects, often referred to as “aging,” are “ubiquitous among freshly deposited and/or densified deposits of silica sands” (Mitchell 2008). These effects have important consequences on structures back-filled with these soils. It was demonstrated that the delayed increase in contact stiffness will affect the stress state in the soil bed. Consequently, one should expect a delayed increase in the load on a retaining wall produced by a sand backfill, time-delayed increase of the bearing capacity of driven piles in sand, or a change in the stress distribution in a backfill placed in a trench over a pipeline. Static fatigue is likely to be the predominant mechanism responsible for the rate effects in the process of sand compression.

The hypothesis advocated in this paper allows for logical explanation of time effects in sand and more specifically, time-delayed increase in cone penetration resistance after dynamic compaction. The microscopic observations indicate that the microfracturing of morphological features on grain surfaces may indeed play a primary role in time effects. The DEM and energy arguments support the hypothesis, but both are based on mechanical models, and physical testing is needed to gain further experimental evidence.

Appendix. Macroscopic Stresses and Strains

The average (macroscopic) stress tensor \( \sigma_{ij} \) in granular material can be calculated in representative volume \( V \) from (see, e.g., Drescher and De Jong 1972):

\[
\sigma_{ij} = \frac{1}{V} \int_{V} \sigma_{ij} \, dV
\]
\[
\sigma_{ij} = \frac{1}{V} \int_S \tau_{ij} dS, \quad i,j = 1,2,3
\]  
(21)

where \(\tau_{ij}\) = force at coordinate \(x_i\) on boundary \(S\) of volume \(V\). The average strain tensor is defined as

\[
\varepsilon_{ij} = -\frac{1}{2} (\dot{u}_{ij} + \dot{u}_{ji})
\]  
(22)

where the displacement gradient is calculated as (comma denotes differentiation)

\[
\dot{u}_{ij} = \frac{1}{V} \int_V u_{ij} dV = \frac{1}{V} \int_S u_i n_j dS
\]  
(23)

with \(n_j\) being the unit outward vector normal to boundary \(S\).

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