Critical Pool Level and Stability of Slopes in Granular Soils

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Abstract: The influence of pore-water pressure and the pool water pressure on stability of submerged slopes was investigated using the kinematic approach of limit analysis. For soils with some cohesive component of strength, the critical pool level is slightly below half of the slope height, whereas for slopes built of purely granular soils the critical pool level is not well defined. The most critical mechanism of failure for submerged granular slopes was found to have the failure surface intersecting the face of the slope, with one intersection point above, and the other one below the pool level. The solution to the stability problem was found to be independent of the length scale (slope height), and equally critical mechanisms of failure can be triggered "locally" with any water level in the pool. The safety factor associated with these mechanisms is lower than the well-known factor defined by a planar failure surface approaching the slope face.

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Introduction

A method used increasingly more often for analysis of stability of soil masses is based on the kinematic approach of limit analysis. The collapse mechanisms in this analysis are comprised of rigid blocks and continually deforming regions that allow the soil to move down the slope, causing a landslide. Hydraulic conditions may exacerbate the state of the slope, and trigger a landslide. A peculiar result was observed recently in numerical simulations: the most critical pool level of a partially submerged slope is not when the slope is fully submerged, but rather when the submergence is partial. This was observed by Lane and Griffiths (1997) in stability analyses that employed both the finite-element analysis and the traditional limit equilibrium method. Subsequent contributions are found in Griffiths and Lane (1999), Bromhead et al. (1999), Lane and Griffiths (2000), and Viratjandr and Michalowski (2006). It has been well documented that the failure of slopes built of dry granular soils, such as sand, occurs in regions characterized by large areas, but of relatively shallow depth, with the failure surface approaching the slope face. Such slides are often considered maintenance problems rather than failures. Partially submerged slopes, however, behave differently, and the mechanism of failure is not necessarily shallow. During shearing, the soil density decreases because of the dilatancy (volumetric strain) that is manifested in the increase in the void ratio of the soil. If water is present in the voids of a free draining soil, then the pore-water pressure does work on the dilation ("expansion") of the skeleton. This work has an adverse effect on stability, and may be a chief factor leading to landslides in submerged slopes.

The premise upon which the limit analysis is based will be reviewed first, and the influence of partial submergence on triggering landslides in granular soils, such as sands and gravels, will be investigated. It will be demonstrated that equally critical multiple mechanisms (failure surfaces) are possible in partially submerged granular slopes.

Kinematic Approach of Limit Analysis

The kinematic approach in soil mechanics is well established, and the new aspects of applications have been summarized recently in Michalowski (2005). With the assumptions of the convexity of the soil yield condition and the normality of flow, one can prove that an active failure load calculated from the energy rate balance equation is an upper bound estimate to the true failure load. Alternatively, this approach allows one to calculate an upper-bound estimate to a critical height of the slope, the lower-bound estimate of a material property needed to avoid failure, etc. In this technical note, this theorem will be used to examine the influence of partial submergence on the stability of slopes.

The kinematic theorem of limit analysis states that in any kinematically admissible mechanism, the rate of internal work during failure is not less than the work rate of true external loads. This can be mathematically expressed as

$$\int_V \sigma_{ij}^k \dot{\varepsilon}_{ij}^k dV \geqslant \int_S T_i v_i dS + \int_V X_i v_i^k dV - \int_V u \dot{\varepsilon}_{ii}^k dV - \int_S u n_i v_i dS \tag{1}$$

where v_l^k and $\dot{\varepsilon}_{ij}^k$=kinematically admissible velocity field and its strain rate, respectively; T_i=traction on boundary S; X_i=distributed load, such as the soil weight; u=pore-water pressure; and σ_{ij}^k=stress state related to the kinematically admissible velocity field (σ_{ij}^k is not necessarily in equilibrium). This theorem is written here including the submergence and the pore-water effects (last two terms).

Water pressure on the submerged part of boundary S and pore-water pressure in the failure mechanism volume V is denoted by

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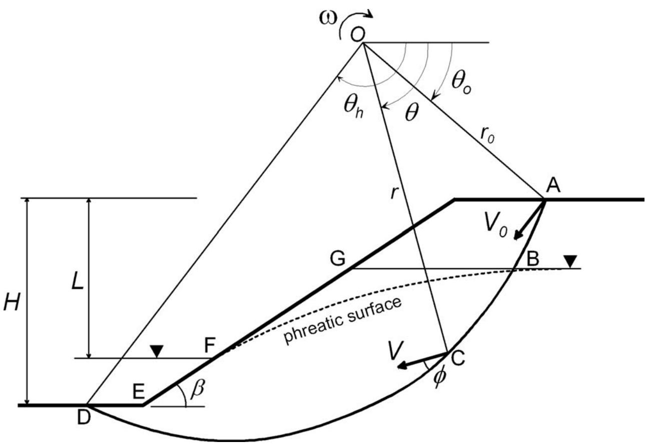

Fig. 1. Rotational collapse mechanism of a slope

u. Water is considered an external load here, and the work rate of the pressure on boundary S due to submergence is reflected in the last term of inequality (1); n_i=outward unit vector perpendicular to S. The second to last term in this inequality is the rate of work of the pore-water pressure u on the volumetric strain rate (dilatancy) of the soil. The minus sign comes from the expansion-negative convention (compressive pressure does positive work on skeleton expansion). The sum of these last two terms has been proved to be equivalent to the seepage and buoyancy effects (Michalowski 1995), and they both need to be included in the analysis of submerged slopes.

Rigid-Rotation Failure Mechanism

The rigid rotation failure pattern, depicted in Fig. 1, was proved earlier to be the most critical mechanism for "dry" slopes with well-defined crest and toe (Chen 1975), and it is adopted here to simulate collapse of submerged slopes. The soil slides along the log-spiral failure surface $ABCD$, and the work dissipation rate and the work of the pore pressure needs to be integrated along this surface. The work rate of the pore-water pressure requires that the pressure along the failure surface be known first. The pore-water pressure is estimated approximately by introducing a region with (fictitious) vertical equipotentials below segment FG of the slope, and constant potentials to the left and to the right of that region.

The log-spiral surface is described by

$$r = r_0^{(\theta-\theta_0)\tan\phi} \tag{2}$$

and the magnitude of velocity discontinuity vector **v** along this surface varies according to

$$v = v_0^{(\theta-\theta_0)\tan\phi} \tag{3}$$

Angles θ_0 and θ, as well as r_0 and v_0, are marked in Fig. 1, and ϕ=internal friction angle.

Partial Submergence and Drawdown

Calculations of the rate of work dissipation and the work rate of the soil weight for the mechanism in Fig. 1 can be found elsewhere (for instance, Chen 1975), but calculations of the last two terms in Eq. (1) require some comment. The second to last term in Eq. (1) is the work of the water pressure on the dilative component of the velocity discontinuity vector **v** sin ϕ, and it only needs to be evaluated along surface BCD, as the soil inside the rotating mass does not deform. The last term in Eq. (1) is the work of the water pressure on boundary DEF. The details of calculations are described in Viratjandr and Michalowski (2006).

It is convenient to introduce stability factor $c/\gamma H$ to characterize the slope (Taylor 1948) with c=cohesion; γ=soil unit weight (saturated); and H=slope height. Once all the terms in Eq. (1) are calculated, a lower bound to the stability factor $c/\gamma H$ can be derived in the following form:

$$\frac{c}{\gamma H} = \frac{2\tan\phi\left(f_1 - f_2 - f_3 - f_4 + \frac{\gamma_w}{\gamma} f_5\right)}{e^{2(\theta_h-\theta_0)\tan\phi} - 1}\frac{r_0}{H} \tag{4}$$

where coefficients f_i=functions of geometry of the slope and the failure mechanism, with f_1 through f_4 originating from calculations of the work of the weight of the soil, and f_5 being dependent of the work of the water pressure (both surface and pore water). Functions f_1 through f_4 as well as ratio r_0/H can be found elsewhere (Chen 1975; Michalowski 1995). Function f_5 does not have a convenient analytical form, and had to be evaluated numerically [as in Viratjandr and Michalowski (2006)]. An approximation was made in neglecting the difference in the unit weight of the soil above and below the water table when calculating the work of the soil weight in Eq. (1), leading to Eq. (4).

The stability factor in Eq. (4) is the lower bound to the "true" value, and the maximum of this factor is sought in calculations, with angles θ_0, θ_h and the position of point D (Fig. 1) being variable.

A measure used to characterize slopes is the safety factor, defined as the ratio of the shear strength parameters to those necessary only to maintain limit equilibrium (c_d and ϕ_d)

$$F = \frac{c}{c_d} = \frac{\tan\phi}{\tan\phi_d} \tag{5}$$

The expression in Eq. (4) can be transformed to calculate the factor of safety. Once c and $\tan\phi$ are replaced with c/F and $\tan\phi/F$, the upper bound to the factor of safety can be easily derived (Michalowski 1995)

$$F = \frac{2(\theta_h - \theta_0)\tan\phi}{\ln\left[1 + 2\frac{\gamma H}{c}\frac{r_0}{H}\left(f_1 - f_2 - f_3 - f_4 + \frac{\gamma_w}{\gamma} f_5\right)\tan\phi\right]} \tag{6}$$

The process of calculating F is iterative, because f_1 through f_5 are all functions of ϕ_d as defined in Eq. (5). To investigate the influence of the submergence on the possible loss of stability, three regimes of water drawdown are considered. These are indicated in Fig. 2.

The water configuration in Fig. 2(a) is associated with a rapid drawdown regime where the level of submergence drops down quickly, but the level of water in the slope has not yet had time to drop. The second one is a slow draining regime where the level of submergence and the level of water in the soil are approximately the same throughout the drawdown process. The last draining process is characterized with a constant drop in the water level from that in the slope to the submergence table, and it is termed a *constant gradient* regime. As shown next, a 1:2 slope characterized by $c/\gamma H$=0.05 and ϕ=40° appears to be safe under all three regimes.

The expression in Eq. (6) was used to calculate the safety factor for a submerged slope subjected to the three different re-

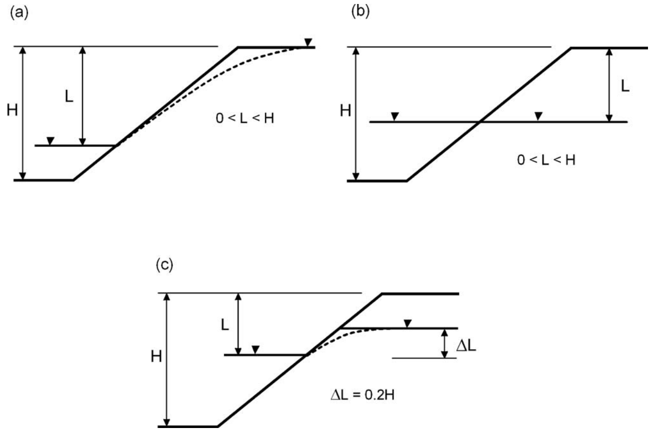

Fig. 2. Drawdown regimes: (a) rapid drawdown; (b) slow drawdown; and (c) constant gradient in water levels

gimes (no influence of the water drawing process on the variation of the soil strength was considered). The results are plotted in Fig. 3. Not surprisingly, the rapid water drawdown led to the lowest safety factor at the end of the process. However, the results appear to be surprising for the other two regimes. The most critical location of the water table is at about a third of the slope height. The minimum of the safety factor on the plot of F versus water level L/H indicates a *critical pool level*. The occurrence of the critical pool level was detected earlier by Lane and Griffiths (1997), who used both the limit equilibrium method and FEM [also Griffiths and Lane (1999)] and Viratjandr and Michalowski (2006), who used kinematic limit analysis. A similar effect is present for the *constant gradient* regime.

Slopes in Granular Soils

For purely frictional soils (such as sand or gravel), the rate of work dissipation during shearing is zero in limit analysis calculations. This is a direct consequence of the Mohr–Coulomb yield

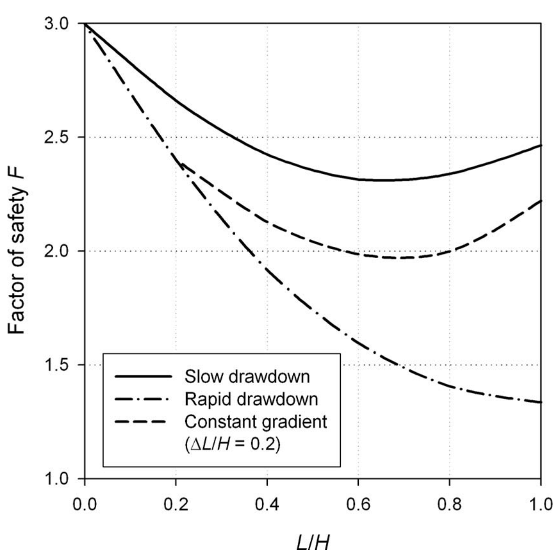

Fig. 3. Variation of the safety factor of a submerged slope subjected to different drawdown regimes; slope 1:2, $\phi=40°$, $c/\gamma H=0.05$ (after Viratjandr and Michalowski 2006)

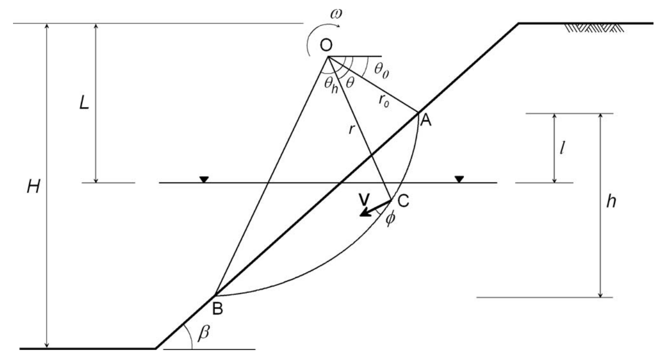

Fig. 4. Failure surface for a submerged granular slope

condition and the normality flow rule. With the left-hand side of inequality (1) equal to zero, following the same steps as in the previous section, one arrives at

$$\frac{\gamma_w}{\gamma}=-\frac{f_1-f_2-f_3-f_4}{f_5} \tag{7}$$

which also can be obtained directly from Eq. (4) after substituting $c=0$. The utility of this equation is not immediately obvious; γ_w/γ in Eq. (7) is the upper bound to the ratio of the pore fluid unit weight to the unit weight of the (saturated) soil at which the slope loses its stability; the pore-water pressure is considered an external load, and this external load would increase with an increase in the unit weight of the pore fluid. Because γ_w/γ is known for a given slope, it is not a very practical interpretation of this equation. However, one can use Eq. (7) to find the lower bound to the internal friction angle necessary to prevent failure of the slope when ratio γ_w/γ is given. Because functions f_i in Eq. (7) depend on geometry of the failure mechanism (including the ratio of the height of the nonsubmerged portion of the failing mass to the total height of the mechanism) and the internal friction angle (ϕ), the procedure must be iterative, where the geometric parameters are varied in the search for maximum ϕ (best lower bound). Denoting the best lower-bound solution as ϕ^m and the true internal friction angle as ϕ, the factor of safety can be calculated as the ratio $\tan\phi/\tan\phi^m$. The safety factor so calculated is its upper-bound estimate. The solution to maximum ϕ revealed an interesting characteristic: the most critical mechanism is a log-spiral failure surface that intersects the slope as shown in Fig. 4. A somewhat different outcome was recently obtained by Baker et al. (2005), who used a different technique (a slice method) to estimate the safety of submerged granular slopes. Their failure surface was always associated with either the crest or the toe of the slope; likely, an artifact of the method used.

The most critical mechanism is not associated with the planar surface approaching the slope inclination, as in the classical approach (leading to safety factor $F=\tan\phi/\tan\beta$). Rather, it is a deeper failure surface, defined with respect to the water level, but not with respect to the crest or the toe of the slope. In other words, the solution is not defined by L/H, but rather l/h (see Fig. 4). For this particular mechanism, coefficients f_2 and f_4 in Eq. (7) are both zero, and the only two independent parameters defining the geometry of the mechanism are θ_0 and l/h [θ_h is no longer independent, and l/h enters Eq. (7) through coefficient f_5 which, in dimensionless manner, includes the influence of the water pressure on the submerged part of the slope surface, in addition to the pore-water pressure influence; Viratjandr and Michalowski (2006)].

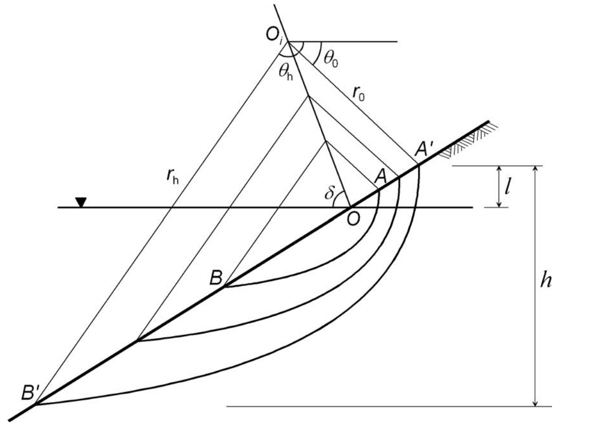

Fig. 5. Equivalent failure surfaces for a submerged slope

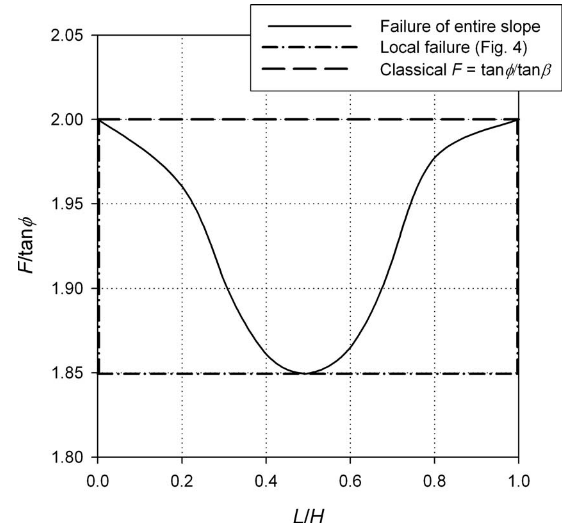

Fig. 6. Variation of a safety factor for a granular 1:2 slope during a slow drawdown

Because h is not related to the slope height (except that $h \leqslant H$), the solution to ϕ needed to maintain limit equilibrium is independent of the slope height. This implies that the geometry of the failure mechanism is linked to the water table in the pool, but not its particular level with respect to the slope height. The solution has no *characteristic length*, and the mechanisms of any size can be equally critical, as long as they do not interfere with the slope crest or the toe.

The kinematic approach of limit analysis yields a lower bound to the strength parameter needed to maintain limit equilibrium (avoid failure); hence, this parameter (here ϕ) needs to be maximized in an optimization procedure where geometry of the mechanism is varied. If the failure surface AB in Fig. 5 assures the maximum solution to ϕ from Eq. (7) (for given γ_w/γ), then any geometrically similar failure surface, for instance $A'B'$, will yield an identical solution to ϕ. Hence, for slopes built of soils with some cohesive component of strength, there is a well-defined critical pool level (see Fig. 3), but it is not so for the slopes built of purely frictional soils.

The results of calculations for a 1:2 "granular slope" are shown in Fig. 6; for convenience, $F/\tan\phi$ is plotted as a function of the level of water L/H. The traditional infinite slope analysis yields the limit state when $\phi=\beta$, independent of the level of water in the pool [the hydraulic conditions are consistent with those in Fig. 2(b)]. This $F/\tan\phi$ is depicted by a horizontal dashed line at $F/\tan\phi=2$ (1:2 slope inclination). If the entire slope were to collapse (toe failure, $h=H$), then the factor of safety would decrease with the drop of the water level in the pool, to reach the minimum when the water level is at about midheight of the slope (the solid line in Fig. 6). For the given 1:2 slope, the solid line was obtained from Eq. (7) where ϕ was maximized with given $\gamma_w/\gamma=0.6$, for a series of L/H. As coefficients f_i are functions of ϕ, the calculations were iterative.

This result can be deceptive, because in the search for the most critical failure surface, it was assumed a priori that the collapse would include the entire slope, no matter how high or low the pool level is. It was demonstrated earlier that the solution to the problem is scale independent, and any failure mechanism depicted in Fig. 5 is equally critical. Consequently, if the mechanism of failure is not "analytically forced" to include the entire slope, but it is allowed to form a smaller local collapse mechanism passing through the water table, the minimum safety factor is achieved at any pool level, as depicted by the dash-dot line in Fig. 6. The algorithm that produced the value of $F/\tan\phi=1.85$ was based again on Eq. (7), but the mechanisms of the type shown in Fig. 5 were allowed to form. For a fully submerged slope ($L/H=0$) and for a fully drained pool ($L/H=1$), the safety factor F assumes its "traditional" value of $\tan\phi/\tan\beta$ (for 1:2 slope, $F/\tan\phi=2.0$), but it assumes the minimum of 1.85 (for this particular case) everywhere in between.

In a recent paper, Baker et al. (2005) concluded that the critical pool level for granular slopes is about the midheight of the slope. Such a result was obtained here only when the failure mechanism was "forced" to include the entire slope height (solid line in Fig. 6). The investigation described in this technical note indicates that equally critical failure mechanisms can form at any level of the pool (excluding full submergence and a fully drained pool), and there is no well-defined critical pool level.

Granular soils exhibit some apparent cohesion in the zone of water capillary rise. Therefore, the size of the smallest failure region (Fig. 5) is not expected to be smaller than the zone affected by water capillary rise. The safety factor will then drop down from $\tan\phi/\tan\beta$ to its minimum level (Fig. 6) at some finite gradient. This effect, however, was not part of the analysis.

Calculations were performed for partially submerged slopes [Fig. 2(b)] to assess the combination of internal friction angle ϕ and slope inclination angle β at failure, for $\gamma_w/\gamma=0.6$. For ease of use, the results are presented in Fig. 7 as $F/\tan\phi$ versus β. The classical solution $F=\tan\phi/\tan\beta$ is depicted by the solid line, whereas the dashed line indicates the solution with a log-spiral failure surface, as shown in Fig. 4. Clearly, the latter is a more critical mechanism than the plane surface approaching the slope.

Conclusions

The influence of pore-water and pool pressure on stability of submerged slopes was investigated using the kinematic approach of limit analysis. The stability problem formulated in terms of limit analysis allows one to evaluate the pool level associated with the slope being most susceptible to collapse. For soils with some cohesive component of strength, this critical pool level is below half the slope height (about 1/3 of the slope height for a 1:2 slope with $\phi=40°$ and $c/\gamma H=0.05$). This has been confirmed by analyses using three distinctly different methods (Griffiths and Lane

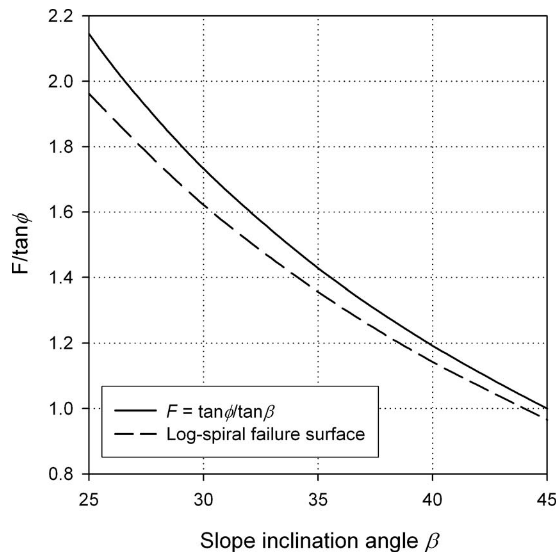

Fig. 7. Safety factor for granular slopes as a function of slope inclination and internal friction angle

1999; Bromhead et al. 1999; Viratjandr and Michalowski 2006). Slopes built of purely granular soils, however, do not have a well-defined critical pool level.

The most critical failure mechanism for a partially submerged granular slope was found to be a log-spiral surface intersecting the face of the slope. This is contrary to the traditional perception that, for slopes built of granular materials, the most critical mechanism is a "shallow" collapse with the failure surface parallel to the slope. The analysis indicated that the solution to the most critical log-spiral failure surface is independent of the length scale; i.e., a geometrically similar family of failure surfaces can be found, all forming equally critical mechanisms characterized by the same safety factor. Apparent cohesion due to capillary water rise may prevent failure mechanisms that are relatively small in size.

The difference in the "traditional" safety factor for a partially submerged slope formed of granular soil $(\tan\phi/\tan\beta)$ and that for a log-spiral failure is relatively small (several percent, Fig. 7), but the important distinction between the two is that, in the former case, the slope (of finite height) undergoes a shallow or "surfacial" failure, often considered a maintenance problem, whereas the latter leads to a deeper collapse.

Acknowledgments

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