Physics 126 Winter 2009  
Lecture #27  
Detecting Gravitational Waves  

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Final Exam

- Exam Time: 7:30 – 9:30 pm Friday April 24
- Calculator allowed
- 4 notecards (3”×5”) or sheet of paper (8.5”×11”)
- Exam covers chapters 18-32.5
- Practice exams posted on Ctools

- Reviews:  
  Tues 6:00-8:00 p.m. 340 West Hall (Narayanan)  
  Thur 5:00-7:00 p.m. 340 West Hall (Krisch)

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Gravitational waves are said to be “ripples in the fabric of space”

They are waves of space itself, created by massive, cataclysmic phenomena in the Universe:
- Exploding supermassive stars
- Collisions of neutron stars and black holes
- The Big Bang
- Rotating “bumpy” neutron stars

Courtesy: National Center for Supercomputing Applications
Space deformation

“Rubber-sheet model” of space:

Einstein’s Field Equation:

\[ G_{\mu\nu} = 8 \pi T_{\mu\nu} \]

Oscillating masses make waves!

Courtesy: LIGO Laboratory
Making waves

Now imagine two very compact stars (neutron stars or black holes) in a tight binary orbiting system:

Space is “swirled” by the orbiting stars, creating a ripple that propagates to distant regions of the universe (to us we hope!)
Are there really such star systems out there?

**YES!**

In 1974 Joseph Taylor and Russell Hulse discovered a binary system with two neutron stars, one of which is a pulsar.

- Observed 17-Hz pulsar
  - PSR 1913+16

Orbital Period (“year”) is 7.75 hours
Digression: What is a Pulsar?

Under a variety of conditions, a star can become too massive to be supported by atomic-level forces.

Its core undergoes collapse as protons combine with electrons to form neutrons (emitting neutrinos in the process).

The escaping neutrinos explode the outer shell of the stellar material away, creating a bright “Super-nova”.

The core left behind is like a gigantic atomic nucleus with stupendous density [all of the Sun’s mass in a ball of radius 10 km!]

$10^{14}$ g/cc ➔ teaspoonful of neutron star weighs a billion tons!

$\rightarrow$ Like a nucleus with $A \sim 10^{57}$ !!!

The shrunken core cannot lose its angular momentum and speeds up during the collapse.
What is a Pulsar?

Magnetic fields are trapped and intensified in the collapse process, leading to strong magnetic north and south poles.

Charged particles are in turn trapped by the magnetic fields and bounce back and forth between the poles, just as on the Earth (cf. Aurora Borealis), but the intensities are far higher.

“Hot spots” of radiation emission created at poles.

If magnetic poles not aligned with rotation axes (just like Earth!), then radiation beams act like lighthouse beacons continuously sweeping the sky.

Courtesy: NASA
What is a Pulsar?

Most known pulsars are seen via their radio-wave emission, but some also emit x-rays:

![Images show x-ray emission from Crab nebula when pulsar is “on” (beaming at us) vs “off”]

Because the neutron star is like a giant flywheel, its rotational speed is extremely stable.

This stability leads to **highly periodic** radio and/or x-ray pulses.

Some pulsars are more stable than atomic clocks on Earth, allowing precise measurements of their orbits in binary systems.
Making waves

The Taylor-Hulse binary system’s orbit shrinks about 3 mm every revolution

→ Coalescence in about 300 million years

So why is the orbit decaying?

Gravitational wave emission → Energy loss!

Weisberg et al, 1981
Making waves

Why are we so sure we understand the decay of this binary system?

By measuring the precise timing of pulsar signals, one can infer the present orbital parameters

Einstein’s Theory of General Relativity then predicts the rate of decay from gravitational radiation energy loss

Did Einstein get it right?
Graph at right shows the change in orbital period (seconds) over 25 years of observations.

Smooth curve is absolute prediction from General Relativity (no free parameters!)

Dots are measured data.
Can we detect this orbital system’s waves here on Earth?

NO

We can hope to see only waves with frequencies greater than \( \sim 10 \text{ Hz} \) on the Earth’s surface.

The characteristic frequency for this system is \( \sim 1/(4 \text{ hours}) \sim 70 \text{ \( \mu \)Hz} \)
Making waves

Well, what if we waited around for 300 million years?
What might we “see”?  

A Chirp!

Graphs show waveform for 4 different 1-second intervals near the end of the inspiral, a.k.a., “death spiral” (in arbitrary but consistent units)

Analogous to ball in a funnel
Making waves

Last nine seconds of inspiral
Nature of Gravitational Waves

Gravitational waves = “ripples of space”
(or “ripples in the space-time fabric”)

Perturbations propagate in similar way to light:
- Same speed! (graviton and photon are both massless)
- Two independent polarizations

Example:
- Ring of test masses responding to wave propagating along z
Nature of Gravitational Waves

How is the strength of a gravitational wave described?

By fractional change in distance, i.e., strain

Denote time-dependent dimensionless strain displacement (tiny!) by \( h(t) \):

\[
\Delta L(t) \sim h(t) \times L
\]
Gravitational Waves from “Out There”

Imagine two neutron stars:
- Each with mass equal to 1.4 solar masses
- In circular orbit of radius 20 km (imminent coalescence)
- Resulting orbital frequency is 400 Hz (!)
- Resulting GW frequency is 800 Hz

Plugging the numbers into Einstein’s formula gives:

\[ h \approx \frac{10^{-21}}{(r/15\text{Mpc})} \]
How small is $10^{-21}$?

$$0.000000000000000000001$$

or

$$\frac{1}{\text{million} \times \text{million} \times \text{million} \times \text{thousand}}$$

or

$$\frac{\text{diameter of an atom}}{\text{distance from Earth to Sun}}$$

→ Experimental challenge!
Gravitational Waves from “Out There”

What else might we see?

Super-novae
(requires asymmetry in these massive explosions)

Thought to be process leading to neutron stars

Numerical simulation shown at right

What might it sound like?

Courtesy: Dr. Tony Mezzacappa -- Oak Ridge National Laboratory
Gravitational Waves from “Out There”

What else might we see?

Cosmic GW background (“stochastic”) – Big Bang remnant?

Fluctuations in cosmic microwave radiation background

Gravitation wave analog unlikely to be detectable, but some exotic theories do predict an effect

Courtesy: NASA WMAP Project
Gravitational Waves from “Out There”

What else might we see?

**Spinning neutron stars (e.g., pulsars)**

Need some slight asymmetry:
- “Mountain” on neutron star (mm high) or
- Wobble in the spin

*Crab spins at 29.8 Hz, giving a GW frequency = 59.6 Hz*

→ Stereo hum !

*Sound of “Middle A” pulsar (440 Hz):* 📸
Why look for Gravitational Radiation?

Because it’s there! (presumably)

To test General Relativity:
- Quadrupolar radiation? Travels at speed of light?
- Unique probe of strong-field gravity

To gain a different view of Universe:
- Sources cannot be obscured by dust
- Detectable sources are some of the most interesting and least understood in the Universe
- Opens up entirely new non-electromagnetic spectrum
What will the sky look like?
What will the sky look like?

Visible

Infrared
What will the sky look like?
What will the sky look like?
What will the sky look like?

Visible

Infrared

COBE

γ-ray

GRBs
What will the sky look like?
Gravitational Wave Detection

Several methods have been tried over the years to detect gravitational waves directly:

Low frequencies:
- Tracking distant spacecraft signals – search for modulation
- Tracking distant pulsar timing – search for modulation

Higher frequencies: (terrestrial experiments)
- Energy deposited in suspended resonant bar
- Disturbance of laser interferometer
Suspended Resonant Bars

The response of a suspended cylindrical bar to gravity wave passage is analogous to that of a two masses on a lightly damped spring:

A gravitational wave impulse perturbs the bar, i.e., “rings the bell”
Suspended Resonant Bars

Many suspended bars have been built and operated in the last 40 years.

Inherently “narrow band”, i.e., they sense a narrow part of the gravitational wave spectrum, e.g., a few Hz around 900 Hz.

Not well suited to reconstructing waveforms like chirps (but “xylophone” would help!)

Joseph Weber founded the GW detection field – shown here with an early bar.
Suspended Resonant Bars

Modern bars operated at low temperatures inside cryostats to reduce thermal noise

Nautilus detector in Rome

Explorer detector at CERN

Allegro detector at LSU
Suspended Interferometers

Why use an interferometer?

Interference at detector depends on difference in lengths of two arms

Similar to Michelson-Morley Experiment!
Suspended Interferometers

Gravitational waves cause length differences!

\[ \Delta L(t) \sim h(t) \times L \]
Suspended Interferometers

Why Suspended?

Suspended mirrors are in near “horizontal free-fall” and better isolated from ground motion.

Interferometers have broad-band response (~10 Hz to few kHz)

Well suited to reconstructing waveforms like inspiral chirps:

The bigger L is, the better the precision on h(t)
Suspended Interferometers

But can one really measure $h \sim 10^{-21}$ ???

1 Watt of wavelength ($\lambda$) = 1.0 $\mu$m laser light in 1-meter interferometer

- $5 \times 10^{18}$ photons / second
- Fractional fluctuation during half period of 1 kHz grav. wave $\sim 10^{-8}$
- Intensity fluctuation $\sim 10^{-8} = 8\pi (\Delta L / \lambda)$
- Precision on $\Delta L$ is roughly $10^{-8} \times 1.0 \mu$m / ($8\pi$) $\sim 10^{-15}$ meters
- Smaller than proton radius!

Can get to $\Delta L/L \sim 10^{-21}$ via:

- Long $L$ (kms)
- Higher laser power
- Multiple laser bounces in arms
Laser Interferometer Gravitational-wave Observatories (LIGO)

Observation of nearly simultaneous signals 3000 km apart rules out terrestrial artifacts

4-kilometer arms!
LIGO Detector Facilities

Vacuum System

- Stainless-steel tubes (1.24 m diameter, ~10^{-8} torr)
- Gate valves for optics isolation
- Protected by concrete enclosure
LIGO Detector Facilities

**LASER**
- Infrared (1064 nm, 10-W) Nd-YAG laser from Lightwave (now commercial product!)
- Elaborate intensity & frequency stabilization system, including feedback from main interferometer

**Optics**
- High-quality fused silica (25-cm diameter)
- Suspended by single steel wire
- Actuation of alignment / position via magnets & coils
LIGO Detector Facilities

Seismic Isolation

- Multi-stage (mass & springs) optical table support gives $10^6$ suppression
- Pendulum suspension gives additional $1/f^2$ suppression above $\sim 1$ Hz
LIGO Astrophysical Searches

Many searches underway:

- Inspiraling neutron star and black hole binaries
- None seen out to Virgo cluster of galaxies
- Coincident “glitches” in all three interferometers
- Nothing yet...
- Coincident glitches with known gamma ray bursts
- No GRB’s close enough yet

Courtesy: NASA
LIGO Astrophysical Searches

Cosmic gravitational wave background

Showed contribution to universe energy density $< 10^{-5}$

Known pulsars

Have shown Crab’s energy loss is NOT dominated by gravitational radiation.
LIGO Astrophysical Searches

**Unknown pulsars!** ← Michigan group focus

Seems easy: Just look for single spike in spectrum (pure tone)

→ Ideal "middle A" pulsar (220 Hz rotation → 440 Hz signal)

But there are two serious and related difficulties:

- **Weak signals** (\( h < \sim 10^{-24} \))
  → Must observe over months (years?)

- **Doppler shifts** that become large for large observing times
Doppler shifts seem tiny in $\Delta f / f$:

- Frequency modulation from earth’s rotation ($v_{\text{ROT}} / c \sim 10^{-6}$)
- Frequency modulation from earth’s orbital motion ($v_{\text{ORB}} / c \sim 10^{-4}$)

But let’s try some numbers:

For a source at 500 Hz tracked for six months, the resolution in frequency (width of perfect spike) is $1/(6 \text{ months}) = 63 \text{ nHz}$

The maximum Doppler shift is $2 \times 10^{-4} \times 500 \text{ Hz} = 0.10 \text{ Hz}$

→ “Single spike” is spread out over $0.10/(6.3 \times 10^{-8}) \sim 1.6$ million freq. bins!

And every point on the sky has a slightly different Doppler correction!
It gets worse…

The pulsar is likely spinning down from energy loss due to electromagnetic and gravitational wave emission

→ More Doppler shifting – and unknown!

The pulsar may be in a binary system with unknown binary orbital parameters

The pulsar’s detectable signal strength varies with Earth’s motion
→ Amplitude modulation

Analogous to using a slowly rotating radio/TV antenna
Good gravitational wave reception:

Interferometer arms aligned with wave polarization

Bad gravitational wave reception:

Interferometer arms rotated 45°
But two substantial benefits from modulations:

1) Reality of signal confirmed by need for corrections
2) Corrections give precise direction of source

Carrying out a 1-year search at full intrinsic detector sensitivity is beyond the total computing capacity on Earth (but see comment at end)

Must use hierarchical method

- Coarse search followed by finer search for candidates
- Less sensitive but computationally tractable

Ongoing search by Michigan graduate students Vladimir Dergachev and Evan Goetz
Vladimir’s successful reconstruction of artificially injected pulsar signal in recent data:

“Hot spot” with $h_{PULSAR} \sim 10^{-23}$

→ Critical first step in hierarchical search

Stay tuned! Or…Lend LIGO your CPU!
Einstein@Home

LIGO has collaborated with the seti@home group at Berkeley to allow anyone with a Windows / Macintosh / Linux computer to help search for pulsars in LIGO data.

Uses same software infrastructure as in search for extra-terrestrial intelligence (radio signals).

Tens of thousands of users and computers!

Sign up at http://einsteinathome.org/  !!!