Radioactivity

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Identifying Nuclei

atomic mass number

\[ A \]

Number of protons and neutrons

atomic number

\[ Z \]

Number of protons

\[ N \]

Number of neutrons

Nuclei that contain the same number of protons but a different number of neutrons are known as **isotopes**.
The radius \( (r) \) of the nucleus increases with the mass number \( (A) \), and the approximate radius is given by

\[
r \approx \left(1.2 \times 10^{-15} \, \text{m}\right) A^{1/3}
\]
The Curve of Binding Energy

[Graph showing the curve of binding energy with labeled nuclei and a note: Most tightly bound nucleus]
Isotopes and Nuclear Stability

- Some nuclei are less bound than similar “nearby” nuclei
- They can change to these and release energy
- Possible changes are limited by conservation laws
- Reverse processes too, you can build less bound nuclei if you put in energy
- Conservation laws:
  - Conservation of energy
  - Conservation of charge
  - Conservation of linear and angular momentum
  - Conservation of nucleon number
  - Conservation of “lepton” number
Radioactivity

Alpha particles (α): $^4\text{He}^{++}$ nuclei
Beta particles (β): electrons/positrons
Gamma particles (γ): photons from nuclear transitions
Alpha Particles

Produced in decay of HEAVY nuclei:

\[ {}^{226}\text{Ra} \rightarrow {}^{222}\text{Rn} + {}^{4}\text{He} \]

Conservation of CHARGE (Z)
\[ 88 = 86 + 2 \]

Conservation of Nucleons (A)
\[ 226 = 222 + 4 \]

Conservation of ENERGY
\[ m(226\text{Ra})c^2 = m(222\text{Rn})c^2 + m(4\text{He})c^2 - \Delta \]

Other alpha decays:
\[ {}^{238}\text{U} \rightarrow {}^{234}\text{Th} + \alpha \]
\[ {}^{232}\text{Th} \rightarrow {}^{228}\text{Ra} + \alpha \]

Moves heavy (unstable) nuclei to left toward greater binding energy per nucleon (reducing Z and A)
Alpha Particles

Alphas lose energy in matter by IONIZATION
• Highly ionizing because Z=2 and
• Because usually slow-moving
→ Short range in matter

Can damage tissue if α source ingested
Beta Particles

Electrons and Positrons are Beta-particles
\[ m = 9.1 \times 10^{-31} \text{ kg}; \quad mc^2 = 0.511 \text{ MeV} \]
\[ q = \pm 1.6 \times 10^{-19} \text{ Coulombs} \]

Neutron Rich Isotopes $\beta^-$ decay: $^{234}\text{Th} \rightarrow ^{234}\text{Pa} + e^- + \nu$

Conservation of charge ($Z$): $90 = 91 - 1$

Conservation of Nucleons ($A$): $234 = 234 + 0$

Conservation of energy: $m(234\text{Th})c^2 = m(234\text{Pa})c^2 + m(e)c^2 - \Delta$

Betas are PENETRATING (similar to gamma rays)
Longer Range than Alpha Particles
Gamma Rays (Photons)

Produced in nuclear transitions from excited states

Example:

\[ ^{234}\text{Th} \rightarrow ^{234}\text{Pa}^* + e^- + \nu \]  
Beta decay

\[ ^{234}\text{Pa}^* \rightarrow ^{234}\text{Pa} + \gamma \]  
Gamma decay

In radioactive isotopes, excited states of nuclei can be populated by β-decay

Any photon with \( E_\gamma > \) few 100 keV is called a γ-ray
Radioactive Decay Series
Concept Test #1

Which of the following decays is **forbidden** by a conservation law?

1) $^{234}\text{Pa}_{(Z=91)} \rightarrow ^{234}\text{Pa}_{(Z=91)} + \gamma$

2) $^{234}\text{Th}_{(Z=90)} \rightarrow ^{234}\text{Pa}_{(Z=91)} + e^- + \nu$

3) $^{226}\text{Fr}_{(Z=87)} \rightarrow ^{230}\text{Ac}_{(Z=89)} + ^4\text{He}_{(Z=2)}$

4) $^{226}\text{Ra}_{(Z=88)} \rightarrow ^{222}\text{Rn}_{(Z=86)} + ^4\text{He}_{(Z=2)}$

226 $\neq$ 230 + 4
87 $\neq$ 89 + 2
Concept Test #2

Which of the following beta decays is **forbidden** by a conservation law?

1) \( ^{234}\text{Th}_{(Z=90)} \rightarrow ^{234}\text{Pa}_{(Z=91)} + e^- + \nu \)

2) \( ^1\text{H}_{(Z=1)} \rightarrow ^1\text{n}_{(Z=0)} + e^+ + \nu \)

3) \( ^1\text{n}_{(Z=0)} \rightarrow ^1\text{H}_{(Z=1)} + e^- + \nu \)

4) \( ^{234}\text{Pa}_{(Z=91)} \rightarrow ^{234}\text{Th}_{(Z=90)} + e^- + \nu \)

\[91 \neq 90 - 1\]
Half life $T_{1/2}$

\[ N = N_0 e^{-\lambda t} = N_0 \left[ \frac{1}{2} \right]^{(t/T_{1/2})} \]

\[ T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \]

Mean life tau:

\[ \tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} \]

Measured half-lives due to $\alpha, \beta$ decay vary from ms to $10^{20}$ y
Exponential behavior common:

Radioactivity; attrition in a population; discharging capacitor

When $\frac{\Delta Q}{\Delta t} \sim -Q$, the solution is always exponential decay

$\ln\left(e^{-t/\tau}\right) = -t / \tau$

Logarithmic scale (semi-log scale)
Concept Test #3

- Approximately how many half-lives are required for the number of radioactive nuclei to decrease to one thousandth of the initial number?

1) 5
2) 10
3) 15
4) 20
5) 25

**Shortcut:**

\[ 2^{10} = 1024 \rightarrow \left( \frac{1}{2} \right)^{10} \approx \frac{1}{1000} \]

**Slightly longer:**

\[ \left( \frac{1}{2} \right)^N = \frac{1}{1000} \rightarrow -N \ln(2) = -\ln(1000) \]

\[ N = \frac{\ln(1000)}{\ln(2)} = 9.97 \approx 10 \]
This female skull recently found on the Island of Flores is only 18,000 years old, so young that the species must have evolved in parallel with our *homo sapiens* ancestors. The Flores-man skull's age was determined by Carbon dating:

What is the basis of Carbon dating?

A. Analysis of charred, carbonized bones
B. *Measurement of the ratio of radioactive* $^{14}\text{C}$ *to stable* $^{12}\text{C}$
C. *Measurement of the abundance of the element carbon*
D. *Usually dinner and a movie*
Radiocarbon Dating

\[ T_{1/2}^{(14\text{C})} = 5730 \text{ years} \]
\[ N_0^{(14\text{C})} = 1.2 \times 10^{-12} \frac{N^{(12\text{C})}}{} \]
(constant production by cosmic rays)

When an organism (animal or vegetable) dies, the \(^{14}\text{C}\) clock starts "ticking"

\[ \frac{N^{(14\text{C})}}{N^{(12\text{C})}} = 1.2 \times 10^{-12} \left(\frac{1}{2}\right)^{t/5730} \Rightarrow t = 8267 \{ -27.45 - \ln\left[\frac{N^{(14\text{C})}}{N^{(12\text{C})}}\right] \} \text{ yr} \]
The age of an object, once living, determined by $^{14}\text{C}$ dating is given by:

$$t = 8267\text{y}\{-27.45 - \ln\left[\frac{N(^{14}\text{C})}{N(^{12}\text{C})}\right]\} \text{ yr}$$

How does the age of an object depend on the ratio of abundances?

A. *The greater the $^{14}\text{C}$ abundances, the older the object*
B. *The object’s age does not depend on the $^{14}\text{C}$ abundance*
C. *As the ratio of $^{14}\text{C}$ decreases, the age increases LOGARITHMICALLY*
D. *The age depends EXPONENTIALLY on the ratio of abundances*
Activity

\[ A = -\frac{\Delta N}{\Delta t} = \lambda N \]

\[ N = N_0 e^{-\lambda t} \]

\[ T_{1/2} = \ln 2 / \lambda = 0.693 / \lambda \]

A measured in Becquerels or Curies

1 Bq = 1 decay/second

1 Ci = $3.7 \times 10^{10}$ Bq
Radiation detection

Charged particles (alphas, betas) detected by way of **direct** ionization

Photons detected by **induced** ionization

Common instrument: **Geiger counter**

→ Exploits “avalanche effect” of ionized electrons
Gamma Ray (Photon) Interactions

Photon interactions:

- Photoelectric effect (ionizing)
  
  incident ➔ photoelectron

- Compton Scattering (ionizing)
  
  incident ➔ Compton electron
  ➔ Lower energy gamma

- Pair Production (for $E_\gamma > 1.022$ MeV)
  
  incident ➔ $e^+$ (antiparticle)
  ➔ $e^-$ (conserves charge)