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Lecture #18
The Special Theory of Relativity

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Pillars of Classical Physics

- **Congratulations!** Your classical physics education is complete!
- **Mechanics:** Newton’s Laws—how forces determine motion.
  - Space is a passive stage. Time is universal and absolute.
- **Electrodynamics:** Maxwell’s equations—how charges and currents interact with electric and magnetic fields.
- Both theories extraordinarily successful
- **Now the bad news...**

These two theories are mutually incompatible!
Albert Einstein (1879-1955)

- When you sit with a nice girl for two hours, it seems like two minutes. When you sit on a hot stove for two minutes, it seems like two hours. That's relativity.
- Common sense is the collection of prejudices acquired by age eighteen.
- People like you and I, though mortal of course like everyone else, do not grow old no matter how long we live...[We] never cease to stand like curious children before the great mystery into which we were born.
- I think and think for months and years, and ninety-nine times, the conclusion is false. The hundredth time I am right.
Newton’s First Law

1st Law - the Law of Inertia:

Every body continues in its state of rest, or in uniform motion in a straight line, unless it is compelled to change that state by outside forces impressed upon it.
Inertial Frames of Reference

- **Frame of reference**: a coordinate system in which you measure things (like position, time, energy, velocity, electric field...)
  - has an origin, and an orientation of the coordinate axes
- **Inertial frame of reference**: a frame of reference that is either at rest or is moving in a straight line at a constant speed
  - a frame in which Newton’s first law holds
- **Non-inertial frame of reference**: a frame of reference that is undergoing accelerated motion
  - first law does not hold
  - Examples: car rounding a curve, airplane taking off
Inertial Frames Are Special

- In mechanics, Newton’s 2nd and 3rd laws apply in all inertial frames.
- In other words, different inertial observers use the same laws to describe mechanics.
- This is known as the principle of relativity (Not Einstein’s idea; it was first articulated by Galileo).
- Does NOT say that different observers necessarily get the same numerical answer.

An event is a physical “happening” that occurs at a certain place and time.
You are driving down the road at 70 mph. In your hand, you hold a baseball. To you it looks like the baseball's kinetic energy is zero. To your friend standing beside the road, it looks like its kinetic energy is about 80 Joules. Which of you is right?

1) Your friend -- he is always right and you are wrong.
2) You are right as always, and your friend is wrong, as usual.
3) You are each correct.

Absolute values of energy are observer dependent: laws describe only changes in energy! (Work = ΔKE)
An example: relativity cart

- Ball thrown straight up from moving platform
  - Observer at rest in room: parabolic path
  - Observer at rest on the cart: straight up and down
  - BOTH: motion is described by Newton’s laws

\[
x = x_0 + v_{0x} t
\]
\[
y = y_0 + v_{0y} t - \frac{1}{2} gt^2
\]

- Disagree about \( x, y, y_0, x_0, v_{0x}, \) and \( v_{0y} \): different observers have different answers
- Agree completely about the laws
A Question

- Different inertial observers use the same laws to describe mechanics.
- Does it apply to laws describing other phenomena? E.g. electricity and magnetism?

\[
\begin{align*}
\nabla \cdot E &= \frac{1}{\varepsilon_0} \rho \\
\nabla \cdot B &= 0 \\
\nabla \times E &=-\frac{\partial B}{\partial t} \\
\nabla \times B &= \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}
\end{align*}
\]
The Principle of Relativity and Light

- Maxwell’s equations contain the fundamental constants $\varepsilon_0$ and $\mu_0$. Different inertial observers use these equations to measure $\mu_0$ and $\varepsilon_0$.
- According to the principle of relativity, they should all get the same answer.
- Maxwell’s equations relate these constants to the speed of light:
  \[ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \]
- So, if the principle of relativity applies to Maxwell’s equations, \textit{all observers will measure this same speed of light (proved by The Michelson-Morley Experiment)}
- That’s crazy, isn’t it?
Postulates of the Special Theory

- The special theory of relativity is based on two postulates formulated by Albert Einstein:

1) All inertial frames of reference are equivalent.

2) The speed of light in a vacuum, measured in any inertial reference frame, always has the same value of $c$, no matter how fast the source of light and the observer are moving relative to one another.
A rocket ship traveling \textbf{left to right} and near the speed of light relative to the Earth observes two lights flashing on Earth. The figure above shows the two lights on Earth flashing together (simultaneous). If the rocket observer is directly between them when they flash the observer

1) also determines that they flash at the same time.
2) sees the one on the right flash before the one on the left.
3) sees the one on the left flash before the one on the right.

Two events which are observed to occur simultaneously at different points in one frame of reference will not occur simultaneously according to an observer in a second frame of reference which is in motion relative to the first one.
Moving Clocks Tick Slowly

Both see the same speed of light $c$!

\[ \Delta t_0 = \frac{2D}{c} \]

\[ \Delta t = \frac{2\sqrt{D^2 + \left(\frac{v\Delta t}{2}\right)^2}}{c} \]
Time Dilation

- Time is measured differently in frames of reference moving relative to one another

Rearrange:

\[ \Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]

Proper time interval

\[ \gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} > 1 \]

\[ \Delta t = \gamma \Delta t_0 > \Delta t_0 \]
What does it mean?

- **Time dilation**: the time that passes while the light travels between the mirrors is actually longer for the Earth observer; the moving clock appears to run slower.

- Either relativity is wrong, or time is not the same for all observers!

- How do we find out what is true? Conduct experiments!
Traveling vs. Stationary Clocks

- Put an atomic clock on a plane and leave one on the ground
- Clock on the plane should see less time go by
- Maryland physicists did this in 1972
- Clocks on planes fell behind just as predicted!
Cosmic Rays

- Energetic protons strike the top of the atmosphere
- Produce short lived ‘muons’: lifetime is only $2.2 \times 10^{-6}$ s if sitting in the lab
- Even at close to the speed of light, they should travel $c \Delta t = 660$ m
- Instead they reach the ground, 10’s of km below!
- Decay takes longer for the moving particle!

Muons live longer, so would you!
How about length?

(a) Spaceship Moving at the 10 % the Speed of Light

(b) Spaceship Moving at the 86.5 % the Speed of Light

(c) Spaceship Moving at the 99 % the Speed of Light

(d) Spaceship Moving at the 99.99 % the Speed of Light

http://www.physicsclassroom.com/mmedia/specrel/lc.html
Length Contraction

- Objects (or distances between them) appear shortened along the direction of motion.
- $L_0$ = “rest length” of an object.
- Length as seen by an observer moving at speed $v$ relative to the object:

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = \frac{L_0}{\gamma} < L_0$$

- Does NOT affect directions perpendicular to the direction of motion.
Concept Test #3

An observer A traveling in a spacecraft measures the length of a piece of wood to be 1.0 meter. The spacecraft is moving at 0.80c relative to an observer B. Determine the length of the wood as observed by observer B.

1) 0.5 m
2) 0.6 m
3) 1.5 m
4) 1.6 m

\[
L = L_0 \sqrt{1 - \frac{v^2}{c^2}}
\]

\[
L = (1 \text{ m}) \sqrt{1 - \left(\frac{0.8c}{c}\right)^2} = 0.6 \text{ m}
\]
Garage Paradox

- A bus is the same length as a garage. A reckless driver is driving close to the speed of light towards the garage.
- Will the bus fit in the garage?

- Bus moving relative to garage looks shorter to garage attendant – fits.
- Garage moving relative to bus looks shorter to driver – doesn’t fit.

- It depends on who is measuring!
- How can this be?
Garage Paradox

- To garage attendant:
  - Front door opens
  - Front door closes
  - Back door opens
  - Back door closes

- To driver:
  - Front door opens
  - Back door opens
  - Front door closes
  - Back door closes

- Events occur at different times in different reference frames!
Most basic consequence of relativity: space and time are not independent.

Instead, they are coupled, time is a fourth dimension.

Moving through space affects time.

There is no absolute space and time, no independent stage on which physics plays out....
A key point to take away…

- These space-time effects emerge from the principle of relativity.
- These facts are required if the laws of physics are to be universal.
- That they are *true* reinforces our belief that our physical laws *are* universal.....