Physics 126 Winter 2009
Lecture #12

Energy and Polarization of EM Waves

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Electromagnetic Waves

- Magnetic field + electric field
- Magnetic field is perpendicular to electric field
- Magnetic field and electric field are both perpendicular to direction of travel
- Transverse waves
- Does NOT require a material medium (unlike sound waves)
- Sources: sun, fire, light bulb, etc...
How does an EM wave propagate?

1) If a charged particle **accelerates** (moves faster, slower or changes direction), it produces both an electric field (because the particle is charged) and a magnetic field (because the particle is moving).

2) Because the motion of the particle is changing, the electric field is changing and the magnetic field is changing.

3) The changing electric field creates a new magnetic field and the changing magnetic field produces a new electric field.

4) The collapsing and regeneration of the electric and magnetic fields is what allows EM radiation to propagate.
Energy Carried by Electromagnetic Waves

- Electromagnetic waves, like water waves, carry energy

  - energy per unit volume associated with an electric field is:
    \[ u_E = \frac{1}{2} \varepsilon_0 E^2 \]

  - energy per unit volume associated with a magnetic field is given by:
    \[ u_B = \frac{B^2}{2\mu_0} \]
Total Instantaneous Energy Density

\[
u_B = \frac{B^2}{2\mu_0} = \frac{(E/c)^2}{2\mu_0} = \frac{\mu_0\varepsilon_0}{2\mu_0} E^2 = \frac{1}{2} \varepsilon_0 E^2
\]

\[
u_B = \nu_E = \frac{1}{2} \varepsilon_0 E^2 = \frac{B^2}{2\mu_0}
\]

\[
B = \frac{E}{c} \\
c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}
\]

\[
u = \frac{\text{Total energy}}{\text{Volume}} = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2 = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}
\]
Concept Test #1

Which one of the following statements concerning the energy carried by an electromagnetic wave is true?

1) The energy is carried only by the electric field.
2) More energy is carried by the electric field than by the magnetic field.
3) The energy is carried equally by the electric and magnetic fields.
4) More energy is carried by the magnetic field than by the electric field.
5) The energy is carried only by the magnetic field.
Total Average Energy Density

\[ E = E_y = E_{\text{max}} \sin(kx - \omega t) \]
\[ B = B_z = B_{\text{max}} \sin(kx - \omega t) \]
where \[ k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f, \quad \text{and} \quad f \lambda = \frac{\omega}{k} = c \]

RMS values of the fields:

\[ E_{\text{rms}} = \frac{E_{\text{max}}}{\sqrt{2}} \]
\[ B_{\text{rms}} = \frac{B_{\text{max}}}{\sqrt{2}} \]

\[ \bar{u} = \frac{1}{2} \epsilon_0 E_{\text{rms}}^2 + \frac{1}{2 \mu_0} B_{\text{rms}}^2 = \epsilon_0 E_{\text{rms}}^2 = \frac{B_{\text{rms}}^2}{\mu_0} \]
Intensity = Power/Area

\[ S = \frac{P}{A} = \frac{\text{Total energy}}{tA} = \frac{uctA}{tA} = cu \]
At a distance of 4.00 m from a 100.0-W light bulb, what is the energy density? Assume that all of the electric power consumed goes into EM radiation and that the radiation is equal in all directions.

1) $1.2 \times 10^{-9}$ J/m$^3$
2) $1.5 \times 10^{-9}$ J/m$^3$
3) $1.7 \times 10^{-9}$ J/m$^3$
4) $2.0 \times 10^{-9}$ J/m$^3$

\[
S = \frac{P}{A} = cu
\]

\[
\Rightarrow u = \frac{P}{Ac} = \frac{P}{4\pi r^2c}
\]

\[
\Rightarrow u = \frac{100.0 \text{ W}}{4\pi(4 \text{ m})^2(3 \times 10^8 \text{ m/s})} = 1.66 \times 10^{-9} \text{ J/m}^3
\]
The Doppler Effect in EM Waves

Electromagnetic waves also can exhibit a Doppler effect, but it differs for two reasons:

- Sound waves require a medium, whereas electromagnetic waves do not.
- For sound, it is the motion relative to the medium that is important. For electromagnetic waves, only the relative motion of the source and observer is important.

\[ f_o = f_s \left( 1 \pm \frac{v_{\text{rel}}}{c} \right) \quad \text{if} \quad v_{\text{rel}} \ll c \]

\( v_{\text{rel}} = \) speed of source and the observer relative to one another
An Example: Radar Speed Traps

The radar gun of a police car emits an electromagnetic wave with a frequency of $8.0 \times 10^9$ Hz. The approach is essentially head on. The wave from the gun reflects from the speeding car and returns to the police car, where onboard equipment measures its frequency to be greater than the emitted wave by 2100 Hz. Find the speed of the car with respect to the highway. The police car is parked.
frequency “observed” by speeding car

\[ f_o = f_s \left( 1 + \frac{v_{rel}}{c} \right) \]

\[ \Rightarrow f_o - f_s = f_s \left( \frac{v_{rel}}{c} \right) \ldots \ldots (1) \]

frequency observed by police car

\[ f_o' = f_o \left( 1 + \frac{v_{rel}}{c} \right) \]

\[ \Rightarrow f_o' - f_o = f_o \left( \frac{v_{rel}}{c} \right) \ldots \ldots (2) \]

\[ (2) + (1) : f_o' - f_s = \Delta f = f_o \left( \frac{v_{rel}}{c} \right) + f_s \left( \frac{v_{rel}}{c} \right) \]

\[ \Rightarrow \Delta f = f_s \left( 1 + \frac{v_{rel}}{c} \right) \left( \frac{v_{rel}}{c} \right) + f_s \left( \frac{v_{rel}}{c} \right) \]
Solution con’t

\[
\Delta f = f_s \left( 1 + \frac{v_{\text{rel}}}{c} \right) \left( \frac{v_{\text{rel}}}{c} \right) + f_s \left( \frac{v_{\text{rel}}}{c} \right) = f_s \left( \frac{2v_{\text{rel}}}{c} + \left( \frac{v_{\text{rel}}}{c} \right)^2 \right)
\]

\[
\Delta f \approx f_s \frac{2v_{\text{rel}}}{c}
\]

very small compared to the first term

\[
v_{\text{rel}} \approx \left( \frac{\Delta f}{2f_s} \right) c = \left[ \frac{2100 \text{ Hz}}{2 \left( 8.0 \times 10^8 \text{ Hz} \right)} \right] \left( 3.0 \times 10^8 \text{ m/s} \right) = 39 \text{ m/s}
\]
A Linearly Polarized Transverse Wave

- Linearly polarized wave on a rope
- Cannot pass through a slit that is perpendicular to the vibration
Polarized EM Waves

- An electromagnetic wave in which the electric vector is vibrating in only one direction is said be linearly polarized.
- Ordinary light is not polarized, which means that the electric vector is vibrating in many directions at the same time.
Polarizing Material

- To polarize light, one can use material such as polaroid or natural polarizers (crystals etc.) that removes all the electric field vectors except those with a component in one particular direction.

\[ S = \frac{1}{2} S_0 \]
Malus’ Law

- After the polarizer, only $E_y$ can pass the analyzer
- $E_y = E_0 \cos \theta$, $S = \varepsilon_0 E^2$

![Diagram of light passing through polarizer and analyzer](image)

$$\overline{S} = \overline{S}_o \cos^2 \theta$$

intensity after analyzer

intensity before analyzer
An Example

\[ S_1 = \frac{1}{2} S_0 \]

\[ S_2 = S_1 \cos^2 60^\circ \]

\[ S_3 = S_2 \cos^2 30^\circ \]
A linearly polarized beam of light is incident upon a group of three polarizing sheets which are arranged so that the transmission axis of each sheet is rotated by 45° with respect to the preceding sheet as shown. What fraction of the incident intensity is transmitted?

1) 1/8
2) 3/8
3) 3/4
4) 1/4
5) 1/2
Polaroid Sunglasses

http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/polabs.html