Ph126 Spring 2008
Lecture #4
Capacitors and Dielectrics

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Recall where we are:

- Point charge makes an electric field
  \[ E = \frac{k|Q|}{r^2} \]

- Force on a test charge \( q \):
  \[ \vec{F} = q\vec{E} \]

- Point charge \( Q \) generates potential:
  \[ V = \frac{kQ}{r} \]

- Potential energy of a charge \( q \) in potential \( V \):
  \[ E_{PE} = qV \]
“A positive charge naturally moves from high potential to low potential, but a negative charge naturally moves from low potential to high potential.” Is the above statement a true statement?

- Yes
- No

All charges naturally move from **high potential energy** to **low potential energy**.
Electric Field vs. Electric Potential

\[ E = -\frac{\Delta V}{\Delta s} \]

component of \( E \) along \( \Delta s \)

- An example:

\[ E = -\frac{V_B - V_A}{\Delta s} \]

EPE = 0 here!
“If the electric field is zero throughout a given region, the potential must be zero in this region.” Is this a true statement?

- Yes
- No

If $E = 0$, then $\Delta V = 0 \rightarrow V = \text{constant}$
E and V of a charged conducting sphere

- When a conductor is at equilibrium, the electric field inside it is constrained to be zero.
- Voltage inside a conductor at equilibrium is constrained to be constant at the value it reaches at the surface of the conductor.
Parallel-Plate Capacitors

- Made by placing two thin plate conductors close together with charge +Q, -Q respectively.

- \( \sigma = \frac{Q}{A} \)
- \( E = \frac{\sigma}{\varepsilon_0} = \text{constant} \)
- \( F = qE \)
- \( V_A = Eh \)
- \( V = \text{potential difference between two plates:} \)
  \[ V = V_+ - V_- = Ed \]
Capacitance

- Linear relation between Q and V:

\[ Q = CV \]

- Capacitance tells how much charge stored per volt

- Unit of capacitance: Coulombs/Volt or “Farads”
Capacitance of a parallel-plate capacitor

\[ C = \frac{Q}{V} \]

\[ Q = \sigma A \]

\[ V = Ed \]

\[ E = \frac{\sigma}{\varepsilon_0} \]

\[ C = \frac{A \varepsilon_0}{d} \]

For a parallel-plate capacitor, the capacitance is proportional only to geometric factors (the area of the plates and the separation distance between them) and the natural constant, \( \varepsilon_0 \rightarrow C \) is fixed out of the factory!
Dielectrics & Capacitors

- Most capacitors have an insulating (or DIELECTRIC) material between the plates
- E field is partly cancelled
- $\kappa$ = “dielectric constant” of material
- Increase $\varepsilon_0 \rightarrow \kappa \varepsilon_0$
- Parallel plate capacitor filled with dielectric:

\[
\kappa = \frac{E_0}{E} \quad \quad \quad C = \frac{\kappa \varepsilon_0 A}{d} = C_0
\]
Dielectric Constants

- Capacitance is increased with dielectric

\[ C = \frac{\kappa \varepsilon_0 A}{d} > C_0 \]

- Store more charge \( Q \) per volt \( V \)

Table 19.1: Dielectric Constants of Some Common Substances

<table>
<thead>
<tr>
<th>Substance</th>
<th>Dielectric Constant, ( \kappa )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1</td>
</tr>
<tr>
<td>Air</td>
<td>1.000 54</td>
</tr>
<tr>
<td>Teflon</td>
<td>2.1</td>
</tr>
<tr>
<td>Benzene</td>
<td>2.28</td>
</tr>
<tr>
<td>Paper (royal gray)</td>
<td>3.3</td>
</tr>
<tr>
<td>Ruby mica</td>
<td>5.4</td>
</tr>
<tr>
<td>Neoprene rubber</td>
<td>6.7</td>
</tr>
<tr>
<td>Methyl alcohol</td>
<td>33.6</td>
</tr>
<tr>
<td>Water</td>
<td>80.4</td>
</tr>
</tbody>
</table>

\( ^a \)Near room temperature.
Permittivity

- Permittivity is a physical quantity that describes how an electric field is affected by a dielectric medium.

- Vacuum permittivity (also called permittivity of free space):

\[ \varepsilon_0 = 8.8541878176 \times 10^{-12} \text{ F/m (C}^2/\text{Jm)} \]

\[ \varepsilon = \kappa \varepsilon_0 \]
An empty capacitor is connected to a battery and charged up. The capacitor is then disconnected from the battery, and a slab of dielectric material is inserted between the plates. The voltage across the plates

1) increases.
2) remain the same.
3) decrease.

Hint: The battery’s job is to provide charges!

First: $C_0 = \frac{\varepsilon_0 A}{d} = \frac{Q}{V}$

Q is fixed when disconnecting the battery.
Then C increased $\to$ V must decrease!
Energy stored in a capacitor

- Battery does work in transferring an increment of charge from one plate to the other plate.
- Energy is stored in the \( E \) field between the plates.

\[
W = \text{Energy} = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C}
\]
A parallel plate capacitor has a charge \(+Q\) on one plate and \(-Q\) on the other. A total energy \(W\) is stored in this arrangement. If I now pull the plates farther apart, what can I say about the new stored energy \(W'\)?

1) \(W' = W\)
2) \(W' > W\)
3) \(W' < W\)
4) \(W' = 0\)

Energy stored = \(W\)  
Energy stored = \(W'\)

\[\text{Energy} = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C}\]

\(Q\) is fixed and \(V\) increases, so \(W' > W\).
Energy Density (Energy per Volume)

\[
\text{Energy density} = \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2} \kappa \varepsilon_o E^2
\]
Energy Density (Energy per Volume)

Energy density characterizes a point in space

→ Energy is required to create an electric field

This expression was derived for a uniform static E-field in a capacitor, but it applies to non-uniform dynamic fields too

Half of the energy carried by light (e.g., from Sun to Earth) is contained in the electric field (other half is in magnetic field)