Physics 126 Winter 2009
Lecture #2

Electric Fields and Gauss’ Law

Prof. Keith Riles
kriles@umich.edu
### Last Lecture

- **Fundamental unit of charge:**
  \[ e = 1.602 \times 10^{-19} \text{ C} \]

- **Charge a conductor:**
  - By conduction
  - By induction

- **The electric charge on a conductor is entirely on the surface and is distributed according to the shape of the object**

- **Coulomb’s Law**

  \[
  F = \frac{k|q_1||q_2|}{r^2}
  \]

  - Attractive \( \rightarrow \) charges of opposite sign
  - Repulsive \( \rightarrow \) charges have same sign
  - Non-contact force
  - Similar to gravitational force
Lightning

- Turbulence in the cloud causes the charges to separate (- down, + up)
- Some of the negative charges on the ground are pushed down away from the surface
- A streamer of negative charges approaches the ground → lightning

http://regentsprep.org/Regents/physics/phys03/alightnin/
Electric Force = Spooky Force

- Electric force is non-contact force
- Einstein: “Spooky action-at-a-distance”
- How does $Q$ know $q$ is there?
- Has troubled physicists since Newton

How does $q$ know I am here?

How does $Q$ know I am here?
A Charge’s Territory: Electric Field

- Each charge has its “territory” ⇒ electric field
  - Big charge ⇒ big territory
  - Closer to the charge ⇒ strong territory

Oh! I am in Q’s territory. I better run!
Magnitude of Electric Fields

- Q produces electric field, $E$, around it
- At a point $p$ that is $r$ away from Q, the strength (magnitude) of $E$ is

$$E = \frac{k|Q|}{r^2}$$
Test Charge

- Now bring in a charge, q
- q is called test charge
- q experiences a force from Q:

\[ F = \frac{k|Q||q|}{r^2} \]

\[ F = |q|E \]
Direction of Electric Fields

- Electric field is a vector

\[ \vec{F} = q \vec{E} \]

In the same direction as \( \vec{F} \)
Drawing Field Lines

- Electric field lines are always directed away from positive charges and toward negative charges.
- “Lines of force” on + charge at each point.
Note for electric field lines

- # of lines $\propto$ magnitude of charge
- Density of lines $\propto$ electric field strength
- No two field lines can cross
- Lines always start on + sources and end on – sinks (or at infinity)
- The electric field vector $\mathbf{E}$ is tangent to the field lines
- Multiple charges done the same way, using vector addition
- The electric field is three dimensional
- Electric field lines are not material objects
- Finite number of lines can be misleading
- The electric field is continuous and exists at every point
Examples of Field Lines
Concept Test #1

Which of the pictures represents a possible arrangement of electric field lines?

A. A
B. B
C. C
D. D
Parallel Plate Capacitor

- Charge +Q on top plate, -Q on bottom
- \( \sigma = \frac{Q}{A} \) = surface charge density
- \( E = \frac{\sigma}{\varepsilon_0} \) (direction: from + to -)
- \( \varepsilon_0 \) : permittivity of free space = \( 8.85 \times 10^{-12} \) C²N⁻¹m⁻²
- If there is a test charge q between the plates, q experiences \( F = |q|E \).
At equilibrium under electrostatic condition:
• Any excess charge resides on the surface
• Electric field inside is everywhere zero
Shielding

- Shielding: the conductor shields any charge within it from electric fields created outside the conductor (Faraday cage)
- Field lines enter $\perp$ to surface

A large spark jumps to the car’s body and then jumps the car’s front tire. The person inside is unharmed!
I went into the cube and lived in it, and using lighted candles, electrometers, and all other tests of electrical states, I could not find the least influence upon them… though all the time the outside of the cube was very powerfully charged, and large sparks and brushes were darting off from every part of its outer surface.

--Michael Faraday (~1830)
What happens if I place a charge \( +q \) inside a hollow conductor? Which of the following represents the correct arrangement of electric field?

- Option A
- Option B
- Option C
- Option D
Electric Flux

- For uniform E, electric flux $\Phi_E = E A \cos \phi$

- $\phi$ is the angle between E field and the NORMAL to the surface

\[ \Phi_E = E A \]

\[ A_\perp = A \cos \phi \text{ so } \Phi_E = EA_\perp = EA \cos \phi \]
Electric Flux continued

- What about the case when $E$ is not uniform and the surface is not flat?

We divide the surface into small elements and add the flux through each.

$$
\Phi_E = E_1 \Delta A_1 \cos \phi_1 + E_2 \Delta A_2 \cos \phi_2 + E_3 \Delta A_3 \cos \phi_3 + \cdots
$$
Gauss’ Law

- The total electric flux through a closed surface with the charge $q_{\text{inside}}$ the surface is

$$\Phi_E = \frac{q_{\text{inside}}}{\varepsilon_0}$$
An Example

- Starting with Gauss’s law, calculate the electric field due to an isolated point charge $q$.

- We choose a Gaussian surface that is a sphere of radius $r$ centered on the point charge.

\[ \Phi_E = \frac{q_{\text{inside}}}{\varepsilon_0} = EA \cos 0^\circ \]

\[ E = \frac{q}{\varepsilon_0 A} = \frac{q}{\varepsilon_0 4\pi r^2} \]

\[ E = \frac{kq}{r^2} \]

Same as before!
Implication of Gauss’s law

For a spherical distribution, the field OUTSIDE is the same as for A POINT CHARGE at the center.

\[ E = k \frac{Q}{r^2} \]  
for all three cases!
Surface curvature affects field strength

\[ k \frac{Q}{(r_1)^2} < k \frac{Q}{(r_2)^2} \]
Lightning Rod (Ben Franklin)

The presence of a lightning rod allows for the gradual release of static charge from a storm cloud. This prevents the sudden and explosive discharge which is characteristic of a lightning strike.

http://www.glenbrook.k12.il.us/GBSSCI/PHYS/CLASS/estatics/u8l4e.html