Wavelength selection in the crown splash

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The impact of a drop onto a liquid layer produces a splash that results from the ejection and
dissolution of one or more liquid sheets, which expand radially from the point of impact. In the
crown splash parameter regime, secondary droplets appear at fairly regularly spaced intervals along
the rim of the sheet. By performing many experiments for the same parameter values, we measure
the spectrum of small-amplitude perturbations growing on the rim. We show that for a range of
parameters in the crown splash regime, the generation of secondary droplets results from a
Rayleigh–Plateau instability of the rim, whose shape is almost cylindrical. In our theoretical
calculation, we include the time dependence of the base state. The remaining irregularity of the
pattern is explained by the finite width of the Rayleigh-Plateau dispersion relation. Alternative
mechanisms, such as the Rayleigh–Taylor instability, can be excluded for the experimental

I. INTRODUCTION

The impact of a drop with a thin film of the same liquid
produces a spray of secondary droplets that results from the
emission, expansion, and breakup of one or more sheetlike
jets. Splashes are essential to diverse physical processes and
applications such as gas transfer across the air-sea interface,1
cooling,2 coatings,3 and combustion.4 The spatial pattern and
size distribution of secondary droplets are in general highly
complex,5 vary qualitatively with experimental conditions,6,7
and have yet to be understood.8

Figure 1 shows the end stage of a crown splash in which
the rim of a sheetlike jet breaks into secondary droplets dis-
tributed almost uniformly along its perimeter. The name of
the splash derives from the resemblance of this final stage
to a crown with equally spaced tines, as exemplified in
Edgerton’s iconic photograph Milk Coronet.9 The events cul-
minating in a crown splash begin with a smooth cylindrical
sheetlet jet extending outward and upward. The leading
edge of this jet (i.e., the rim) is pulled by surface tension
toward the sheet,10,11 and grows in diameter as it entrains
fluid from the sheet. Next, the rim develops a symmetry-
breaking corrugation, and in a much later nonlinear phase of
the original instability, the rim’s crests sharpen into jets
which pinch off to form secondary droplets.

Due to the high speed and small scale structure of a
splash there are few quantitative time-resolved observations
of crown formation.12–22 This is true in particular for the
earliest stages of the growth of the rim, during which pertur-
bations of the rim are extremely small. Basic questions re-
garding the origin and evolution of splashes remain
unanswered.8 Foremost among these is: what mechanism
leads to the generation of secondary droplets?

From the earliest investigations, the leading suspect for
the mechanism responsible for secondary droplets was the
Rayleigh–Plateau instability,23 which causes cylindrical jets
to break into droplets. Worthington24 experimented with tori
of mercury on a solid surface to address this question, but
was unable to directly compare the number of ejected dro-
plets with Plateau’s theory. Yarin and Weiss14 cast doubt on
the relevance of this mechanism based on timescale argu-
ments, and noted that there is a discrepancy between the
expected and measured number of ejected droplets. Fullana
and Zaleski25 argued that the Rayleigh–Plateau instability is
slowed prohibitively by the increase of the rim’s radius with
time. Rieber and Frohn26 found support for the Rayleigh–
Plateau mechanism in computer simulations but only for
large initial perturbations. Bremond and Villermaux27 ana-
yzed rim instabilities for the lamella produced by impacting
jets, and found the Rayleigh–Plateau instability to be the
dominant mechanism.

Mechanisms other than Rayleigh–Plateau have been sug-
gested for the generation of secondary droplets. Yarin and
Weiss14 proposed a nonlinear amplification mechanism.
Gueyffier and Zaleski28 argued for the Richtmyer–Meshkov
instability. Krechetnikov and Homys22 argued for a combi-
nation of the Richtmyer–Meshkov and Rayleigh–Taylor
instabilities.

We think there are two reasons why the symmetry break-
ing instability of a splash has been interpreted in so many,
seemingly contradictory ways. First, depending on param-
eters, there exist many different splash morphologies,5 which
may result from different instabilities. Second, there is a lack
of quantitative experimental data that characterizes both the
symmetry breaking as well as the base state on which the
instability grows. Here we focus on a parameter regime in
which the crown splash grows from an initially smooth sheet
formed on impact into a thin layer of the same fluid at moderate speed. We image the instability that appears on the leading edge of the ejected sheet at multiple times onward from \(\approx 100,000 \mu s\) after impact for multiple impact speeds and layer depths. From our images we extract the spectrum of the instability, and compare this to the various mechanisms that are cited in the literature. Our results for peak position and width of the spectrum are in excellent agreement with Rayleigh–Plateau mechanism.

The paper is formatted as follows. Section II describes our experimental setup and procedures. Section III A describes our observations of the morphology of splashes. We show that highly regular crown splashes in thin layers occur only for low Reynolds number. Section III B describes our experimental measurements of the instability spectra from still images of the rim’s corrugation. For the majority of experiments the spectra at any given time after impact were produced from a single image; for our most precise experiments we averaged over ten spectra obtained at equal times. Section IV describes a comparison of our data with a calculation based on the Rayleigh–Plateau instability. For our most precise measurements with averaged spectra the measured and calculated peak wavelengths agree within 5%. Section V describes a comparison of our experiments with other proposed mechanism such as Rayleigh–Taylor and Ritchmyer–Meshkov. Our measurements exclude these mechanisms. Section VI discusses and summarizes our results in the context of other splashing studies.

**II. EXPERIMENTAL METHODS**

Our experiments identified the parametric regime for crown splashes and measured the evolution of these splashes. A 10 cm diameter \(\lambda/10\) glass optical flat, coated with an indium-tin oxide film to prevent the destabilization of the thin liquid films due static charge build up, was placed in a \(15 \times 15\) cm container with a glass bottom. Fluid was added to the container until the optical flat was submersed, forming a film of height \(h\) above the optical flat, as shown in Fig. 2.

The orientation of the container was then adjusted so that the flat lay parallel to the fluid’s surface to within \(3 \times 10^{-4}\) rad. The depth of the layer was varied between 100 and 400 \(\mu m\) depending on the experiment. A single drop of diameter \(D\) was released from a gravity fed 30 gauge hypodermic needle at fixed height above the liquid layer. The interval between drops was longer than 10 s, which ensured that the liquid layer fully relaxed between impact events. The drop struck the liquid layer with a velocity \(U\) normal to the surface.

The dimensionless parameters for describing droplet impact in the absence of a surrounding gas are the Weber number \(We=(\rho DU^2/\gamma)\), the Reynolds number \(Re=(DU/\nu)\), the Froude number \(Fr=(U^2/gD)\), and the dimensionless fluid depth \(H^+=h/D\), where \(g\) is the acceleration due to gravity, and \(\rho, \gamma,\nu\) are the density, surface tension, and kinematic viscosity of the fluid, respectively. Past studies ignored the ambient gas and gravity on the basis that densities and viscosities of liquids are much higher those of the gas, and that the timescale for gravitational effects is long compared to the duration of a splash. We follow this practice here, and concentrate on the \(We, Re,\) and \(H^+\). Nonetheless, we note that the recent work of Xu et al.\(^{29}\) found a significant influence of the ambient air on drop impact on a dry solid. We believe the effect of air to be much weaker in our experiment, since there is no moving contact line.

FIG. 1. Crown splash (silicone oil: \(Re=966, We=874, H^+=0.2\)).

FIG. 2. (Color online) Apparatus for observing the evolution of crown splashes. A drop forms and detaches from a hypodermic needle held a fixed height above a thin layer of the same liquid. The needle is gravity fed from a reservoir. As the drop falls toward the liquid layer, it interrupts a laser sheet focused onto a photodetector which initiates a countdown on a delay generator. After a preprogrammed time, the delay generator fires a flash onto a diffusion screen, and a back-lit image of the impact event is recorded from below through the transparent substrate or from the side.
Our data on the morphology of splashes were obtained with a high speed camera (Phantom 5.0 or 7.3) viewing the impact from the side. Our data on the evolution of the rim were obtained from images simultaneously recorded through the glass substrates and from the side using digital cameras (Nikon D80 with a 90 mm f/2.8 macro and Canon 20D with a 100 mm f/2.8 macro lens) and a single 600 ns, 6 J pulse from a spark flash (Palflash 501, Pulse Photonics Ltd.). The flash was triggered after a preprogrammed delay interval initiated by the drop cutting a laser sheet focused onto a photodiode. The triggering event was reproducible to within ±5 µs. By varying the delay time, the evolution of the impact was recreated from a composite of still images. Examples of typical bottom and side images are shown in Figs. 3 and 4. The virtue of this technique is that it produces much higher spatial and temporal resolution than can be achieved with existing high speed video cameras. The speed of the drop at impact was measured with the high speed video camera.

Our measurements of the crown splash were performed with a 5 cSt (ρ=0.918 g/cm³, γ=19.7 dynes/cm) or 10 cSt (ρ=0.935 g/cm³, γ=20.6 dynes/cm) silicone oil purchased from Clearco. Our experiments were performed for the parameters listed in Table I.

For each parameter set we took data at least every 100 µs after the rim emerges from beneath the drop, prior to which the rim is not visible. For parameter set number 1 (Re=1060, We=760, H′=0.20) we collected much more data than in other parameter sets in order to reduce fluctuations.

III. EXPERIMENTAL RESULTS

A. Splash morphologies

By observing splashes for a range of Weber and Reynolds numbers and fixed H′=0.2, we classified the morphologies. The parametric dependence of these morphologies is shown in Fig. 5. Crown splashes appear only in the regime labeled crown droplets. A regular crown is also observed in the parameter range below this regime, but these crowns do not

![FIG. 3. Crown splash (Re=894, We=722, H′=0.1) from below at t=1.85 ms and t=3.15 ms after impact showing the one-to-one correspondence between instability wavelength and the number of droplets. The arrows define rim radius as seen from below r_0 and rim’s radial distance from the impact center R.](image)

![FIG. 4. (left) Images illustrating the crown sheet at 250 and 2100 µs after impact. (right) Cartoon of sheet cross-section. At early times, the leading edge of the sheet is almost horizontal. The shape changes as the rim retracts relative to the fluid (though still moving away from the center in the laboratory frame) and entrains the fluid in the sheet. Eventually the entire flat portion of the sheet is entrained in the rim. The kink in the late time sheet is a wave generated when the rim reaches the vertical section of the sheet. Scale bar equals 2 mm.](image)

![FIG. 5. (Color online) Qualitative character of impact for H′=0.2. No splash (black small circles), crown droplets with (open circles) and without (squares) microdroplets, and microdroplets without crown droplets (diamonds). The filled squares indicate the parameter set for all experiments at H′=0.2 reported here. Spatially periodic crown splashes form exclusively in the crown droplets regime.](image)
not form secondary droplets because the growth of the corrugation pattern is slower and thus the sheet retraction occurs before droplets can pinch-off. Outside of this domain, splashes are more irregular and complicated as shown by the examples in Fig. 6.

**B. Crown splashes**

From our bottom-view images we extract the corrugation of the outer edge of the splash to within ±6 \( \mu \)m. A selection of these data are shown in Fig. 7. For each parameter set, we capture this corrugation every 100 \( \mu \)s and compute the power spectrum of these data. From the spectra, we extract the peak wavelength (i.e., the most energetic mode) by fitting the peak to a Gaussian. For parameter set number 1, we capture the edge profile with much greater time resolution (up to every 10 \( \mu \)s during the first 700 \( \mu \)s) and in addition repeated the measurement at 500 \( \mu \)s intervals ten times. From the latter data, we computed the spectrum for each image, and averaged these results to arrive at the spectra shown in Fig. 8. The peaks of these spectra, obtained by fitting the upper parts of the peaks to a Gaussian, are plotted in Fig. 11. The data from the inner edge yield the same position which would move the rim out of the depth of field. The radii of the shallow depth of field of our optical system, and were only possible over a short segment of the rim because of various investigators to the Rayleigh–Plateau instability, the Rayleigh–Taylor instability, the Kelvin–Helmholtz instability, the Richtmyer–Meshkov instability, locations. These domains grow and merge, consistent with the growth of the unstable modes from random noise. At later times each corrugation sharpens and begins to form a droplet, as demonstrated by the example in Fig. 3. The number of corrugations, and hence the number of proto-droplets, is set by the ratio of the rim circumference to the peak wavelength. The actual number of emitted droplets is smaller because of mergers between adjacent incipient droplets. Therefore, the number of secondary droplets is bounded from above by the most unstable wavelength of the instability.

From our bottom-view images, we also extracted the average radius of the rim as seen from below \( r_s^B \), and the horizontal distance of the centerline of the rim from the impact center \( R \). We processed the images to extract the position of the inner and outer edges of the rim. The radii of the inner \( r_{inner} \) and outer \( r_{outer} \) edge were determined by fitting to a circle, and from these we obtained the rim radius \( r^B \) and the radial position of the rim \( R(t) = \frac{1}{2}(r_{outer}^B + r_{inner}^B) \). Figure 3 illustrates the physical features corresponding to these parameters, and Fig. 10 shows their time dependence for parameter set number 1.

From our side-view images, we measured the average radius of the rim as seen from the side \( r_s^B \). Measurements were only possible over a short segment of the rim because of the shallow depth of field of our optical system, and were difficult to obtain due to the small changes in the impact position which would move the rim out of the depth of field. These data are shown by the squares in Fig. 10. Our side view is to good approximation orthogonal to the bottom view, and hence the effective radius of the rim \( r_e \) was taken as the geometric mean of \( r_s^B \) and \( r^B \). The geometric mean is indicated by the dashed line in Fig. 10.

The corrugation of the rim following impact has been attributed by various investigators to the Rayleigh–Plateau instability, the Rayleigh–Taylor instability, the Kelvin–Helmholtz instability, the Richtmyer–Meshkov instability,

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**Fig. 6.** Morphologies of splashes outside of the crown splash regime: (a) water (\( Re = 6044 \), \( We = 254 \), \( H^2 = 0.2 \)) produces a highly irregular crown sheet and splash, (b) glycerol/water mixture (\( Re = 2566 \), \( We = 314 \), \( H^2 = 0.2 \)) produces droplets immediately upon impact, long before the crown sheet forms, (c) silicone oil (\( Re = 1392 \), \( We = 1266 \), \( H^2 = 0.2 \)) produces a trapped torus of air, and (d) isopropanol for \( Re = 1354 \), \( We = 406 \), \( H^2 = 0.2 \) produces a wavy crown sheet and continuously ejects droplets as the crown grows upward.

**Fig. 7.** (Color online) Profiles of the outer rim corrugation at successive times (from bottom to top: 115, 365, 605, 1105, 1605, and 2085 \( \mu \)s after impact), starting from a smooth profile and growing to a highly corrugated state. Note that the corrugation nucleates at different places and these domains merge at later times. The noise on the early time profiles are digitization artifacts: the camera pixel size is 6.0 \( \mu \)m.
cylinder is unstable to perturbations whose wavelengths are greater than $2\pi$ times the radius of the cylinder. This so-called Rayleigh–Plateau instability accounts for the decay of cylindrical jets into droplets. If the jet is subject to a broad spectrum of initial perturbations, the typical drop size is set by the wavelength of the perturbation that has the highest growth rate. A characteristic feature of the instability is that this “Rayleigh” mode is a multiple of the radius of the cylinder. In our calculation, we assume that the attachment of the sheet to the rim is negligible. We also treat the cylinder as a cylinder. In our calculation, we assume that the attachment of the sheet to the rim is negligible. We also treat the cylinder as a cylindrical jets into droplets. If the jet is subject to a broad spectrum of initial perturbations, the typical drop size is set by the wavelength of the perturbation that has the highest growth rate. A characteristic feature of the instability is that this “Rayleigh” mode is a multiple of the radius of the cylinder. In our calculation, we assume that the attachment of the sheet to the rim is negligible. We also treat the cylinder as a cylinder.

To perform a linear stability analysis, the local rim radius $r(\theta, t)$ is written as

$$r(\theta, t) = r_0(t) + \epsilon(\theta, t),$$

where $\theta$ is the angle as seen from the center, and $\epsilon$ is a small perturbation to the mean radius. The overwhelming majority of past studies focused on the growth of a single mode. Instead, we consider a spectrum of modes in order to characterize the randomness of the perturbations growing on the rim. Thus as an initial perturbation we take the spectral decomposition

$$\epsilon(\theta, 0) = \sum_{m=-N}^{N} a_m e^{im\theta}.$$  

In our model, the initial amplitude of each mode $a_m$ is set equal to the same constant $a_0$ for all values of $m$. This is equivalent to assuming that the initial spectrum is flat.

We start from the classical result for the growth rate of perturbations on a cylinder of constant radius with no flow inside the cylinder. The growth rate $\sigma_m$ of the $m$th mode is determined implicitly by

$$2x^2(x^2 + y^2) \frac{I_m''(x)}{I_m(x)} \left[ 1 - \frac{2xy}{x^2 + y^2} \frac{I_1(y)}{I_1'(y)} \right] = x^4 + y^4$$

$$= \frac{\gamma r_w x I_1(x)}{\rho \nu^2 I_0(x)} (1 - x^2).$$

Here $x = k_m r_o$, $k_m = m/R$ is the wavenumber of the $m$th mode, $y^2 = x^2 + \sigma_m r_o^2 / \nu$, and $I_n$ are modified Bessel functions of the first kind. As explained in Eggers and Villermaux, we expand on this result to include (a) the growth of the rim as it entrains the sheet and (b) the stretching of the rim as the sheet expands. According to Eq. (106) of Eggers and Villermaux, the time evolution of the perturbation in the presence of (a) and (b) is given by

$$\frac{d \ln \epsilon(\theta, t)}{dt} = -\frac{s}{2} + \sigma_m,$$  

where $s$ is the stretch rate due to the expansion of the rim. In other words, longitudinal stretching always decreases the amplitude of the perturbation, since it causes fluid elements to contract in the radial direction. The growth rate $\sigma_m$ is calculated from the classical theory with no stretching or flow into the cylinder [i.e., Eq. (3)]. This approximation is justified by the fact that the base state changes according to a
stable wavenumber “Rayleigh mode” is amplified more strongly than the wavenumbers surrounding it, and thus dominates the spectrum more and more. The theoretical peak wavenumber is obtained by fitting the calculated spectra with a Gaussian. The peak wavenumber at any given instant \( t \), defined as the mode with the most power, is determined by not only the instantaneously most rapidly growing mode but also by the history of the other modes. Thus the peak wavenumber is given by the maximum of \( \exp(\sum dt' \sigma_m(t')) \). A comparison of these data with experiments is shown in Fig. 11. The calculation reproduces the measured wavelength to within \( \pm 10\% \) for all our experiments and to within \( \pm 5\% \) for our most precise experiments at \( Re=1060 \), \( We=760 \), and \( H^* = 0.20 \). It bears emphasizing that the calculation of the peak position and width depends only on the experimental determined variables \( r_s(t) \) and \( R(t) \), i.e., there are no adjustable parameters.

Our results also account for irregularities in the pattern. Under the action of the Rayleigh–Plateau instability, perturbations away from the most unstable wavelength are amplified as well, albeit at a smaller rate. As shown in Fig. 8, the width of the spectrum evolved by our equations from initial white noise spectrum is equivalent to that of our data. Thus the irregularity is governed by the width of the central peak.

Since the stretch rate is \( \dot{s} = \dot{R}/R \), Eq. (4) can be integrated in time to give

\[
\ln \left( \frac{\epsilon(\theta, t)}{\epsilon(\theta, 0)} \right) = -\frac{1}{2} \ln \left( \frac{R(t)}{R(0)} \right) + \int_0^t dt' \sigma_m.
\]

Using the initial perturbation (2), we then obtain

\[
r(\theta, t) = r_0 + \sqrt{\frac{R(0)}{R(t)}} \sum_{m=-N}^{N} a_m \times \exp \left\{ i m \theta + \int_0^t dt' \sigma_m(k_m = m/R(t), r_0(t)) \right\}.
\]

The spectra obtained from our calculation are compared in Fig. 8 to those obtained from our most precise experiments. We find excellent agreement in both the position and width of the central peak. Note that the peak position moves to lower wavenumbers as time goes on, since the original perturbation is stretched out as \( R(t) \) increases and \( r_s(t) \) swells. The peak width decreases in time as the most unstable wavenumber “Rayleigh mode” is amplified more.
FIG. 11. Ratio of measured to theoretically expected peak wavelength vs time for the Rayleigh–Plateau model for all experiments: number 1 (hexagons), number 2 (circles), number 3 (stars), number 4 (diamonds), number 5 (squares), and number 6 (triangles). Note that the most accurate data are for parameter set number 1.

V. OTHER MECHANISMS

A. Rayleigh–Taylor instability

Mechanisms based on the Rayleigh–Taylor instability posit that the generation of secondary droplets arises from the deceleration of the sheet. Krechetnikov and Homsy\textsuperscript{22} have argued for this mechanism based on their experiments conducted with milk. Here we show that the Rayleigh–Taylor mechanism predicts wavelengths significantly greater than those observed experimentally. This prediction is based on direct measurements of the acceleration of the rim.

The driving force for the Rayleigh–Taylor instability comes from the deceleration of a fluid surface relative to a less dense external medium, in our case air (whose density can be neglected). It is opposed by surface tension, which favors a flat interface. As a result, the characteristic length scale governing this instability is the effective “capillary length”

\[
\ell_a = \sqrt{\frac{\gamma}{\rho a}} \tag{7}
\]

based on the deceleration \(a\). If \(\ell_a\) is greater than the rim radius \(r_0\), inertial effects are small at the length scales relevant to our study.

We compute the Bond number \(Bo^{-1}=\ell_a^2/r_0^2\) using the deceleration measured by fitting the vertical and horizontal components of position and differentiating the result twice with respect to time. The deceleration and the Bond number are plotted in Fig. 12. These data show that over the time interval for which we observe growth of perturbations along the rim, \(\ell_a\) is indeed far greater than \(r_0\), confirming that the effect of acceleration is unimportant.

The same conclusion can also be drawn directly from the dispersion relation for the Rayleigh–Taylor instability of a flat interface, for which the growth rate \(\sigma_m\) is defined implicitly by Chandrasekhar\textsuperscript{30}

\[
\sigma_m^2 = ak\left(1 - \frac{k^2\gamma}{a\rho}\right) - 4k^2\nu\left(\sigma_m + k\nu(\sqrt{k^2 + \sigma_m^2})\right) \tag{8}
\]

The cutoff wavenumber, above which no amplification takes place, is \(1/\ell_a\). The most amplified wavenumber is slightly smaller (corresponding to a longer wavelength), but its precise value depends on the viscosity. Using the acceleration shown in Fig. 12, the peak wavelength as predicted by Eq. (8) is plotted in Fig. 9. At \(t=250\ \mu s\) the peak wavelengths coincide, but thereafter the Rayleigh–Taylor prediction is in increasing disagreement with the experimental results, consistent with our expectations based on the Bond number.

The rim of the sheet is curved, not flat, as assumed in the derivation of Eq. (8). However, as argued by Krechetnikov\textsuperscript{32} this only weakens the Rayleigh–Taylor instability, and shifts the range of unstable wavenumbers to even smaller values, or equivalently to higher wavelengths. In other words, the dashed line shown in Fig. 9 is only a lower bound for the true peak wavelength of the Rayleigh–Taylor instability. The physical reason is that if the heavy fluid is bounded by a convex interface, the mass of fluid being accelerated is less than that for a flat interface. Since the Rayleigh–Taylor in-
stability is inertia-driven, it becomes less effective. To make up for the loss of mass, the wavelength in the direction along the rim has to be even greater than expected on the basis of Eq. (8).

Even more recently, the same point has been expanded upon in Krechetnikov\textsuperscript{33} by performing a linear stability calculation about a fluid cylinder of circular cross section, which is accelerated in a direction normal to its axis. It is claimed that the interplay between the Rayleigh–Plateau and Rayleigh–Taylor instabilities leads to a change in the dispersion relation. The problem with the calculation of Krechetnikov\textsuperscript{33} is that a hydrostatic pressure gradient will build up inside the cylinder, which deforms its equilibrium state from a circular cross section to a new deformed base state. The linear stability calculation thus is not performed about the true equilibrium state of the system, which leads to inconsistencies. For example, the dispersion relation reported in Krechetnikov\textsuperscript{33} shows instability at zero wavenumber, corresponding to an exponential time dependence of the base state. But as argued above, the fluid cylinder will merely relax to a new equilibrium state when accelerated. We hasten to add, though, that none of these effects are of importance for the present experiments, since the Bond number is small.

### B. Richtmyer–Meshkov instability

Gueyffier and Zaleski\textsuperscript{28} present simulations in which the length of fingers on the rim grow linear in time. They interpret this result as evidence for a Richtmyer–Meshkov instability triggered by the impulsive impact of the drop on the substrate. Our experiment addresses a much earlier regime, during which the amplitude of perturbations is much smaller than the width of the rim. We find exponential growth for this initial regime, whereas a Richtmyer–Meshkov instability is expected to grow linearly in time. Krechetnikov and Homsy\textsuperscript{22} also argue for the Richtmyer–Meshkov instability, based on the experimental observation that the instability occurs in the very early stages of impact, but their experiments were conducted with a non-Newtonian fluid and for $Re \approx 3000$, $We \approx 1000$ which, if the fluid was Newtonian, would correspond to the highly irregular splashes we observe in the crown droplet/microdroplet regime.

### C. Nonlinear instabilities

Yarin and Weiss\textsuperscript{14} proposed a nonlinear amplification mechanism governed by the eikonal equation. This mechanism does not select a particular wavelength but rather sharpens pre-existing perturbations of finite amplitude. Our measurements of the power spectrum show a clear wavelength and thus are inconsistent with this prediction.

### VI. DISCUSSION AND CONCLUSION

Other investigators reached the same conclusions as ours. Bremond and Villermaux\textsuperscript{27} found that the Rayleigh–Plateau instability accounts for the fragmentation of the lamella produced by colliding jets. Our results are also consistent with numerical simulations of Rieber and Frohn.\textsuperscript{26}

Our results do not agree with those of Yarin and Weiss\textsuperscript{14} who found a discrepancy between the expected and measured number of ejected droplets. We suspect that differences in the experimental conditions account for the disagreement as their experiments were conducted at a high repetition rate (up to 15 000 Hz) in which the liquid layer was highly perturbed by the previous impact. Our results are also at odds with the work of Fullana and Zaleski,\textsuperscript{25} who use numerical simulations as well as theoretical arguments very similar to ours to conclude that the Rayleigh–Plateau instability does not produce sufficient growth to lead to the breakup of the rim. However, no justification is given for their choice of parameter values. In addition, the total length of the sheet simulated by Fullana and Zaleski\textsuperscript{25} was only 40 times the thickness of the sheet, and thus permitted to follow the rim evolution only over a limited period of time. Our calculation, by contrast, is based on parameter values obtained directly from experiment, and leads to order one perturbations of the rim over the time the sheet retracts. Thus Fullana and Zaleski’s conclusions are invalidated for the experimental parameter regime studied by us.

In conclusion, we study the crown splash regime of the impact of a drop onto a thin film. This permits us to follow the growth of the symmetry-breaking instabilities from very small values through significant perturbations of the rim,
leading to the formation of droplets. For a wide range of parameters in this regime, we show (i) that there is a well-defined wavelength selection process, (ii) that the amplification of the selected modes is consistent with a linear instability, and (iii) that the maximum number of secondary droplets is determined by the most unstable wavelength. Our measurements of the most unstable wavelength are in excellent agreement with a model based on the Rayleigh–Plateau instability. Hence, the number of incipient secondary droplets is proportional to the circumference of the splash sheet divided by the most unstable wavelength (and therefore proportional to the radius of the rim), and the irregularity of the crown is set by amplification of random noise by the spectrum of growth rates of the Rayleigh–Plateau instability.

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