Diffusion Tensor Imaging

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Outline
Problem to be solved (attacked)
Physics of diffusion
Diffusion weighted imaging
Diffusion Tensors
Imaging methodology
Examples
Advanced techniques

Problem
Anatomic imaging (T1, T2, FLAIR, MT, etc)
MR5 (spectroscopic imaging)
Activation imaging (BOLD, ASL)
Problem
Dynamics of white-matter
Activation gives temporally correlated regions of grey matter

Unfortunately, we can't put tracers in humans.
Physics of Diffusion (Brownian Motion)

Particles (dust, molecules, atom) in liquid/gaseous state randomly move about.

Described by Brown and Einstein

What determines the properties of this random motion?
What determines the properties of this random motion?
Mass
Size (a.k.a. radius)
Energy (temperature → velocity)
Stokes-Einstein Equation gives diffusion coefficient $D$

\[ D = \frac{kT}{6\pi \eta r} \]  
(water versus honey)

Facts about diffusion:
no net displacement of particles
but do have root mean square displacement  \[ R^2 = 2D\Delta \]
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no net displacement of particles
but do have root mean square displacement
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Evolution time \( \Delta \)

Diffusion Weighted Imaging (DWI)

We need to be able to detect random motion and to image this random motion.
However, this random motion is on a microscopic scale compared to imaging resolution.
Coherent Phase  Incoherent Phase

\[ \Sigma, = I \]
\[ \Sigma, = 0 \]

\[ \omega = \gamma B_0 \]
\[ B(x) = B_0 + Gx \]
\[ \omega(x) = \gamma (B_0 + Gx) \]
\[ \phi(x) = \omega(x)\delta, \ \delta = \text{time} \]
Phase accrual due to magnetic field gradient, with no diffusion.

Time

Space (x)
Phase accrual due to magnetic field gradient, with no diffusion.

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Thursday, August 11, 2011
No diffusion and BUT changing phase

Diffusion but NO changing phase

Diffusion AND changing phase
Diffusion (3x, 4x mixing) AND changing phase

Diffusion (?) AND changing phase

Diffusion (Dx small, Dy huge) AND changing phase
Diffusion (??) AND changing phase

Diffusion (Dx huge, Dy small) AND changing phase

No diffusion and BUT changing phase
Artifacts

Unlike high resolution imaging, very susceptible to shot-to-shot (excitation) error
VERY susceptible to bulk motion, cardiac pulsatility
SPIRAL or EPI, but then low-resolution
Parallel imaging (SENSE), or multi-shot with phase correction

Simple Diffusion Weighted Imaging

$S_0$, $S(b)$, ADC

$b = 800 \text{ s/mm}^2$
...but what if there is something that impedes free diffusion?

...or even impedes in one direction but not another

We must be able to then describe diffusion according to directions.

Simplest is allowing three directions to be independent of each other.
Thus the diffusion tensor (matrix)

$$D \rightarrow \hat{D} = \begin{pmatrix}
D_{xx} & D_{xy} & D_{xz} \\
D_{yx} & D_{yy} & D_{yz} \\
D_{zx} & D_{zy} & D_{zz}
\end{pmatrix}$$

$$S(b) = S_0 e^{-bD} \rightarrow S(\hat{b}) = S_0 e^{-\hat{b}\hat{D}}$$
Imaging

Must now take diffusion weighted images with magnetic gradients along different directions.

7 Unknowns...must make at least 7 measurements.

Operationally like a time series, but each volume has “diffusion weighting”. Very, very sensitive to movement.

33 direction dataset and 1 b=0 image

\[ S(b_1) = S_0 e^{-b_1 \Delta} \]
\[ S(b_2) = S_0 e^{-b_2 \Delta} \]
\[ S(b_3) = S_0 e^{-b_3 \Delta} \]
\[ \vdots \]
\[ S(b_n) = S_0 e^{-b_n \Delta} \]
Application of linear algebra

\[ \hat{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix} \]

Application of linear algebra

\[ \hat{\lambda} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \]

Application of linear algebra

\[ \hat{\lambda} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \]

Eigenvalues \( \lambda_1, \lambda_2, \lambda_3 \)

Eigenvectors \( \epsilon_1, \epsilon_2, \epsilon_3 \)
What do these mean?

Eigenvalues give you an indication of how freely or bounded the diffusion is.

Eigenvectors inform you of the principle directions.
How can we summarize this highly complex data?

Fractional Anisotropy

\[ FA = \sqrt{3} \left( \frac{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}{\lambda_1^2 + \lambda_2^2 + \lambda_3^2} \right) \]

FA=0, isotropic
FA≠0, fully anisotropic
Examples

Calculate FA

Examples

Calculate FA → Co-Register $T_1/T_2$

Examples

Calculate FA → Co-Register $T_1/T_2$

Normalize to MNI
Examples

Calculate FA → Co-Register T₁/T₂
Normalize to MNI

Examples

Calculate FA → Co-Register T₁/T₂
Normalize to MNI
† Tests

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We can also encode the 1st eigenvector (associated w/ largest eigenvalue) with color.

red = L/R

green = anterior/posterior
We can also encode the 1st eigenvector (associated w/ largest eigenvalue) with color.

red = L/R

green = anterior/posterior

blue = inferior/superior
Issues

Crossing fibers - Single DTI lacks description
Noise - effect on eigenvalues
Gold standard, quantifying connectivity, number of directions, diffusion time , etc.
Conclusion

Exploitation of water diffusion leads to mapping of white matter

DTI along with variety of mathematical techniques becoming very accessible

Leads to a better understanding of underlying neuroanatomy

Application to fundamental neuroscience as well as clinical science/practice.

Thanks