A TEACHING STATEMENT

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My own interest in math stems mainly from a few teachers I have been fortunate enough to have in both formal and informal settings. I have consequently been interested in education for a long time. For me, the overarching goal of education is to create citizens of a free community, able to think about, participate in, and form their own judgments of the world in which they live. In the next few pages I outline some concrete ways this translates into the lecture hall and classroom while teaching math.

Perhaps the most important thing a student should get from a math class is enough experience and fluency with a particular set of mathematical objects that she or he can confidently reach her or his own conclusions about them. I find even at a high level the most effective way to bring about students who confidently reach their own conclusions in math is to begin by discussing mathematical concepts which are primitive or basic – very much within the existing experience of the learner – and only later build up to a discussion of more abstract formulas or logical objects. This approach was advocated by John Dewey, who labeled it the “chronological method.” In this approach, to take an example, one defines what a “group” is only after having discussed a rich array of specific groups, like translations and rotations of the plane or the units of \( \mathbb{Z}/n\mathbb{Z} \), and made each of them interesting in its own right. To take another example, when I have covered volumes of rotations in introductory calculus courses, I have always begun discussions not with recalling an abstract formula that was once proved (with the proof often forgotten) but rather with a discussion that gets at why such formulas are really true – by breaking volume being considered up into infinitesimally thin wafers or washers, and asking students what the volume of the individual wafers or washers were, and from this point essentially re-deriving the formula which students are sometimes told to memorize by rote. This approach has the distinct advantage of remaining with students much longer than simply recalling an abstract formula would.

Also very important in teaching math is to convey to a student the reasons a particular topic is worth studying. This can be done most especially by connecting the topic to a student’s own pre-existing interests, and also by conveying the charm inherent in a certain area. The goal is to provide an intrinsic motivation for learning; without this it is impossible to learn or retain any subject very well, and with it learning proceeds much more quickly and enjoyably. Thus when lecturing on Fourier series to upper level math majors, before coming to Fejér’s theorem, I began with some historical discussion and the question, motivated by concrete examples, of whether functions can always be recovered just from having the information provided by their Fourier coefficients. Getting an answer to this was in fact Fejér’s actual motive for proving his theorem, and to know this made for a more interesting lecture than simply delivering a theorem then a proof. I have also tried to connect topics being learned about to pertinent real world examples; to take an example, in a differential equations course taken by a wide variety of undergraduate majors at Michigan, during a time when a flu epidemic was hitting many American campuses I motivated studying the solutions of systems of ordinary differential equations by discussing a differential equation model for the spread of diseases. It was an intriguing exercise for everyone – myself included – to see according to this model what percentage of a campus population would eventually come down with flu as certain parameters of the model varied.

Another way to demonstrate the charm of math to students – who sometimes don’t come in sharing my infatuation with it – is to situate the subject in a wider historical and human context. I
have sometimes taken time in calculus courses to mention how Newton used the concepts of calculus to give in only a few lines a full description of the motion of the planets, which had taken Ptolemy hundreds of epicycles. For some students, especially those interested in the humanities, discussions like this can be very meaningful, and student feedback has been positive.

Of course not all students share my own interests, and so while conveying what I find intriguing about mathematics is important, it is equally important to discover what interests and abilities students come in with and relate the subject at hand to these. This can be done at the beginning of a course by getting to know the students and their majors. I have also maintained online discussion boards during courses partially for this purpose.

Even when students have an intrinsic motivation to learn about a topic, work still must be put into developing a real understanding. For this reason, it is also important to give students an opportunity to reflect on material at their own pace, or indeed to take their own initiative in asking new questions about the material. Several of the classes I have taught at Michigan have been structured around inquiry based learning, where lecturing is kept to a minimum and students learn material mostly by working in small groups on problems posed during the class (before class students are typically asked to complete a reading). The insights developed this way from a student’s own activity will always lead to a deeper and longer lasting understanding than information simply being absorbed passively. Indeed, even in larger lecture classes, I have frequently broken up my lecture to ask students to convince their neighbor that some claim I’ve made is true or false or to explain a step in a proof to each other. Feedback for these experiments has been overwhelmingly positive, even from top students and even with large class sizes. It allowed students who were struggling a chance to interact with students who understood the material, and it allowed students who understood the material well to cement their knowledge. In many cases a student who has just learned a topic is better equipped to resolve some confusion than a professor who may have dealt with similar confusions some time ago but has since forgotten about them.

In encouraging the classroom to be a more active place, students will sometimes react with surprising creativity, coming up with their own problems that can be posed to the class, and this is always an especially effective learning experience. To take a very recent example, while covering polar coordinates in a calculus class, a student wondered what is the equation for a line in polar coordinates, and this was a very instructive exercise to work through.

Creativity of this sort should be encouraged, as its possibility is one of the great charms of math. My own interest in math was piqued by a teacher in elementary school who each week gave the class a problem which could not be solved using the ordinary methods. (We had the entire week to solve it.) In several classes I have taught, I have emulated these methods, giving the class somewhat more difficult but optional problems each week.

In education at its best, a student’s growth does not cease with the end of a course. In teaching a class, we should regard the student, in the words of Bertrand and Dora Russell, “as a gardener regards a young tree, as something with an intrinsic nature which will develop into an admirable form given proper soil and air and light.” A course is really a full success for a student if not only has he or she understood the material that was covered, but also has been motivated to continue learning about it afterward or even to strike out a path of his or her own in it.