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# Principles of Optimal Design

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## Modeling and Computation

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**THIRD EDITION**

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**CAMBRIDGE**  
UNIVERSITY PRESS

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UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

4843/24, 2nd Floor, Ansari Road, Daryaganj, Delhi - 110002, India

79 Anson Road, #06-04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9781107132672](http://www.cambridge.org/9781107132672)

DOI: 10.1017/9781316451038

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First edition 1988

Second edition 2000

Third edition 2017

Printed in the United States of America by Sheridan Books, Inc.

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloging-in-Publication data*

ISBN 978-1-107-13267-2 Hardback

Additional resources for this publication at [www.cambridge.org/optimaldesign](http://www.cambridge.org/optimaldesign)

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## Principles of Optimal Design

*Modeling and Computation, third edition*

Design optimization is a standard concept in engineering design, and in other disciplines which utilize mathematical decision-making methods. This textbook focuses on the close relationship between a design problem's mathematical model and the solution-driven methods which optimize it. Along with extensive material on modeling problems, this book also features useful techniques for checking whether a model is suitable for computational treatment. Throughout, key concepts are discussed in the context of why and when a particular algorithm may be successful, and a large number of examples demonstrate the theory or method right after it is presented. This book also contains step-by-step instructions for executing a design optimization project – from building the problem statement to interpreting the computer results. All chapters contain exercises from which instructors can easily build quizzes, and a chapter on “principles and practice” offers the reader tips and guidance based on the authors' vast research and instruction experience.

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