A performance based computational method for assembly design of reciprocal architectural systems with 2D elements

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ABSTRACT

In this research a computational method is developed to study the form-finding process of non-standard reciprocal systems with 2D elements based on the current methods on the morphology of reciprocal systems with 1D elements. The developed form-finding methods will be used in a performance based form exploration process for geometrical and structural performance enhancement. The proposed computational framework will explore new potentials for variations in the assembly design of these systems through the introduction of new geometric parameters both at the component level and the assembly level within the form exploration process. The proposed method integrates parametric assembly design with structural analysis in a stochastic optimization process to explore the design space while minimizing the total weight of the structure. The results of the form exploration process will be stored for the post processing phase in which the solution space is explored to study the variation of the emerging assemblies. In this paper the proposed method is explained and implemented via two case studies towards the further exploration of the concept.

Author Keywords

Performance based design, Reciprocal systems, Associative assembly design, Form-Finding, Finite element analysis

1 INTRODUCTION

The principle of reciprocity is based on the use of loadbearing-elements which support one another along their spans rather than at their ends, and which compose a spatial configuration with no clear structural hierarchy (Pugnale and Sassone [14], see (Figure 1).

From a historic stand point, primitive reciprocal systems were used in early construction methods as reciprocal frames both in the East and the West. In Europe, structural reciprocity has mainly been used, at least until 20th Century, to span distances longer than the length of available timber beams, so called, short beams [3].

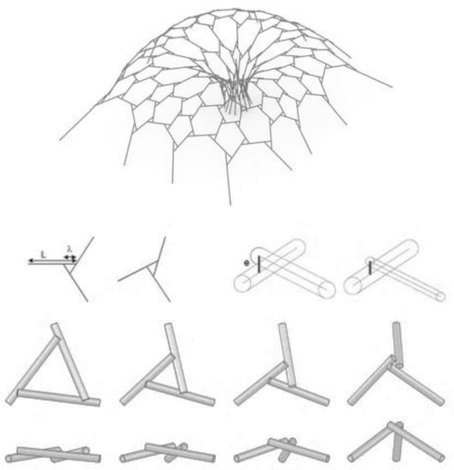


Figure 1. An example of a reciprocal system with 1D elements [9].

In eastern culture interest in reciprocal structures derives mainly from the use of interwoven strips of bamboo for the realization of baskets, an old tradition that has been transferred to the building scale.

From primitive reciprocal structures to modern freeform spatial reciprocal configurations significant research has been done on the form finding, analysis and fabrication of these systems. However, the majority of the research in this field focuses on the study of reciprocal systems with 1D elements leaving the potentials of 2D and 3D element assemblies unexplored. Based on the current literature, the foundational research can be categorized into the two following groups.

1.1 Research on the morphology and geometry of reciprocal spatial structures

The main literature in this category focuses on physical prototyping and experimental study of reciprocal systems through geometrical variations. Which includes studies on different elemental configurations through physical prototyping to be used in reciprocal systems [1], and morphological study of reciprocal systems with planar elements through prototyping and physical studies [8, 13] and also the structural study of different configurations of spherical and fractal reciprocal systems with 1D elements [5,15].

1.2 Research on form-finding and morphogenesis of such systems using computational tools

The core of the research in this category focuses on computational methods for the form-finding and analysis of reciprocal systems. Which includes form finding process based on surface tessellation and constraint formulation for reciprocal cells coupled with dynamic relaxation [4, 7, 10, 16, 19]. Song et al. developed a tool for form finding of reciprocal systems based on two-dimensional pattern creation and conformal mapping on three dimensional forms [17, 18].

This current research focuses on the morphology of reciprocal systems with 2D elements and proposes a performance oriented method for the design of reciprocal systems with planar elements, which is relatively an unexplored ground. Building on the existing literature on the morphological study of reciprocal systems with planar elements [2], these systems can be classified in five categories. a) Reciprocal systems with individual planar elements, b) reciprocal systems with planar elements consisting of groups of linear elements, c) reciprocal systems with planar configurations with bending capacity at connections, d) reciprocal systems with truss like combinations of elements, and e) reciprocal systems with hybrid modules integrating two or more of the previous configurations (Figure 2).

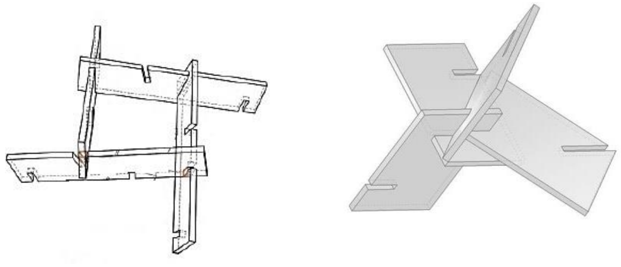


Figure 2. Reciprocal assembly with planar elements (left), group of planar elements in a 3D assembly [13].

The five categories explained above are entirely based on physical prototyping rather than computational modeling, nevertheless it is a valuable classification of these systems based on their configurational behavior, and a good starting point for computational modelling and form finding. The change from configurations with 1D elements to 2D and 3D elements effects both the form finding process as well as the structural behavior of these systems, which requires a different simulation and form finding process in each case [2,11].

This current research uses the stated categorization focusing on the translation of form finding methods for 1D elements towards the development of computational form finding method for reciprocal systems with 2D members.

2 METHODOLIGY

The unique characteristics of reciprocal systems, such as self-equilibrium, modular assembly, inherent three dimensionality and potential for generative growth, qualify these systems as sources of ideation for innovative assembly design and performance integration. However, the key to take advantage of these potentials is a consistent design framework which not only can accommodate geometric form exploration and performance feedback but also provides flexibility to change the conventional reciprocal concept towards the emergence of better performing assemblies. To realize this goal, a computational method for the assembly design of reciprocal systems with planar elements is introduced and implemented in two case studies. This method integrates parametric assembly design with FE analysis and a performance feedback loop in a from exploration process which explores the design space while minimizing the total weight of the structure, Figure 3.

Through the introduction of a rotational parameter for planar elements in the assembly of the second case study new typologies of the structure are explored through the form exploration process.

The rotational parameter compromises the structural behavior however it opens opportunities towards the development of new assembly logics.

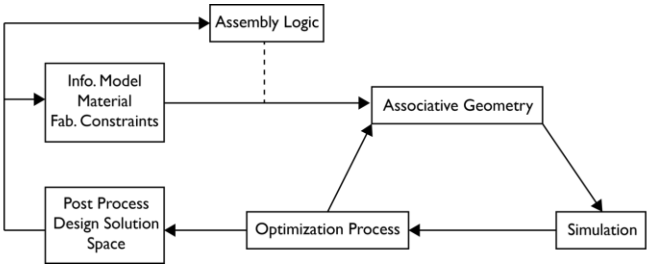


Figure 3. Form exploration workflow and design considerations.

2.1 Geometry Definition and Parametric Modeling

The first case study is a flat reciprocal structure with a structural depth in the mid-span comprised of four membered reciprocal modules (Figure 4). The 2D conformal pattern mapping method for reciprocal systems with 1D elements is used to model the associative parametric geometry [17,18]. A 2D parametric pattern of the structure was created in the XY plane and this pattern was mapped on a surface with a parametric depth in the mid span. Subsequently, the mapped members where extruded in the Z direction to create the 2D planar elements (Figure 4).

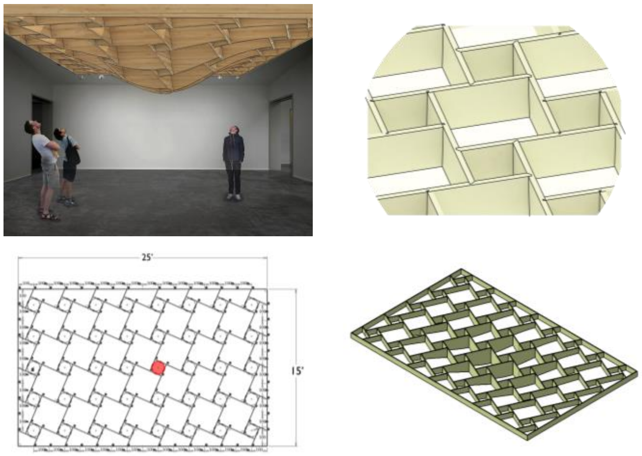


Figure 4. First case study, 2D parametric pattern, local and global geometry.

This parametric model has four controlling parameters. a) the reciprocal parameter which controls the opening of the reciprocal modules based on their engagement length. b) the thickness parameter which controls the thickness of the elements, c) The structural depth parameter which controls the depth of the members by controlling the mid-span depth and d) the depth on the edge (Figure 5). This parametric model will be used in a form exploration process with performance feedback and a database of the results to study the variation of form through the form finding process.

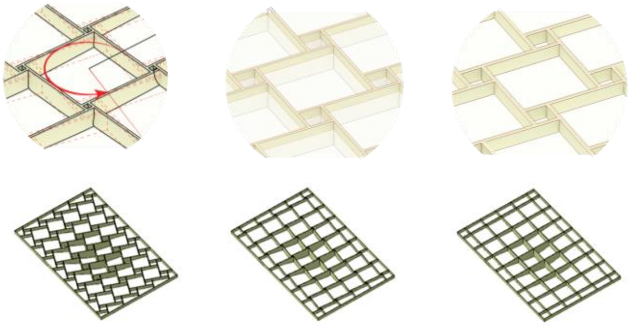


Figure 5. Design parameters, reciprocal parameter (left), Depth parameter (middle), Thickness parameter (right) , geometric variations based on the reciprocal changes.

The second case study uses the same 2D pattern with uniform depth, in this case a rotational parameter is introduced to the model where planar elements rotate around their longitudinal axis based on an angular parameter. This angular parameter is an important agent which transforms the reciprocal geometry allowing the assessment of non-orthogonal typologies of modules within the form exploration process (Figure 6).

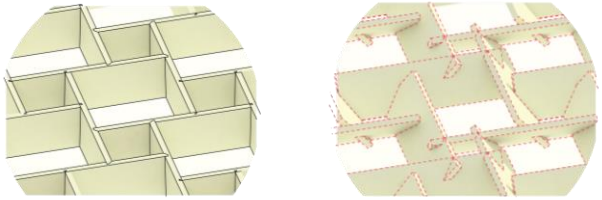


Figure 6. Rotational parameter and transformation of the reciprocal to non-orthogonal configuration.

2.2 Simulation Model

Structural models of the two case studies are created with fixed boundary conditions on four edges. A 30 psf snow load and a 15 psf cladding loads are applied to the structures in addition to the self-weight. Northern red oak wood material properties are used for the analysis for both case studies (Figure 7 and Figure 8). The common method for the analysis of reciprocal structures is a simplified 1D elemental representation with simple pin connections which does not accommodate three dimensionalities of modules and partial moment distribution of the notched connections.

In this study a three-dimensional finite element analysis is applied for the structural analysis of the reciprocal structure (Figure 7 and Figure 8). A fine 3D finite element mesh accommodates the three-dimensional geometry of the rotated connections and also correct representation of stress concentrations and guarantees a more accurate structural analysis of the structure. Moreover, the 3D finite element analysis of the assembly accommodates detailed connection design and analysis which is an integral part of design for fabrication (Figure 7).

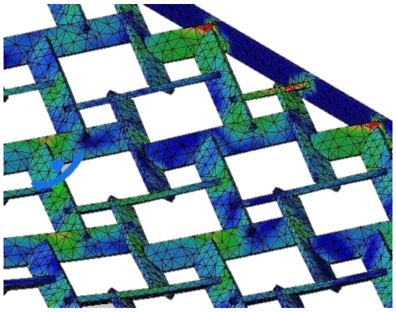


Figure 7. Detailed 3D Finite element mesh and analysis results.

The analysis results (maximum stress and maximum deflection) is fed back into the optimization process which informs the design parameter changes for the next iteration.

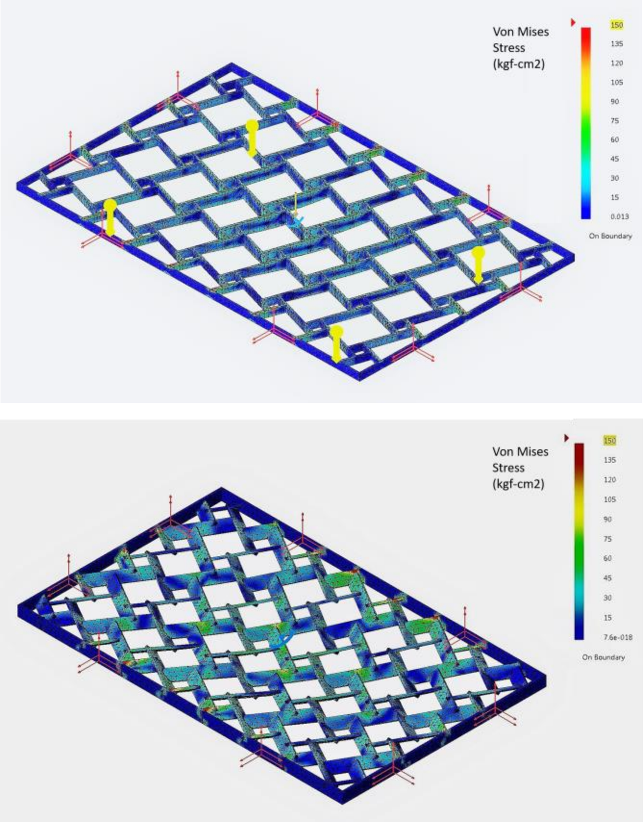


Figure 8. Structural model and Fe analysis results.

2.3 Form Exploration

Computational optimization methods for form exploration are primarily suited for well-defined design problems, and the choice of method is often a tradeoff between computing time and the nature of the solution space. However, in architectural design, the definition of a parametric model and boundary conditions and solution domains together with the understanding of how the optimization project actually performs the search for the suitable shape, is more important than reaching an optimal result [6]. In this regard population based form exploration methods which incorporate a database of solutions have become more popular in the form exploration processes. In the current study, CATIA (a software package developed by Dassault Systemes for CAD, CAM and CAE) has been used for the analysis and optimization process. The Product Engineering Optimizer (PEO) workbench, is used to integrate parametric modeling and FE simulation with feedback of results into the optimization process. The optimization algorithm changes the design parameters stochastically towards convergence to the optimal solution. Moreover, the simulation data in each step of the optimization process is stored for post processing and exploration of the design space.

Simulated Annealing (SA) is chosen among the available optimization algorithms in the PEO workbench. The stochastic nature of SA can accommodate the nonlinearity of the proposed optimization problem and improve the exploration of the design space. The results of the form exploration for 400 iterations are stored in a database for post processing. The optimization formulation for the case studies are given in Table 1 and Table 2.

Minimization Target Function	Total Mass (kg)
Constraints	Max Von Mises < 300 (kg/cm2) *Max Displacement < 2 (cm)*
Variable Bounds	*0.5 (cm) < Reciprocal < 70 (cm)* *1 (cm) < Mid-span depth < 20 (cm)* *5 (cm) < Edge Thickness < 30 (cm)* *1 (cm) < Thickness< 8 (cm)*

Table 1. Optimization formulation definition for the first case study.

Minimization Target Function	Total Mass (kg)
Constraints	Max Von Mises < 300 (kg/cm2) *Max Displacement < 2 (cm)*
Variable Bounds	*0.5 (cm) < Reciprocal < 70 (cm)* *5 (cm) < depth < 50 (cm)* *1 (cm) < Thickness< 8 (cm)* *1 (deg.) < Rotation < 70 (deg.)*

Table 2. Optimization formulation definition for the second case study.

Through the iterative optimization process design parameters are changed based on the performance feedback towards the minimization of the total mass of the structure and a range of design solutions are explored and stored for post processing towards the further study of geometric configurations with corresponding structural and geometric performances.

3 RESULTS AND DISCUSSION

In this section, we study the numerical results of the optimization with a focus on the geometric variations and changes in the design parameters of the reciprocal assembly for each case study.

The results of the optimization process for the first case study are shown in (Figure 9), which shows mass minimization of the reciprocal structure with respect to the optimization constraints. The minimization process converges at around 400 iterations.

Some of the critical design solutions are shown in the process of form exploration to demonstrate the geometrical variations.

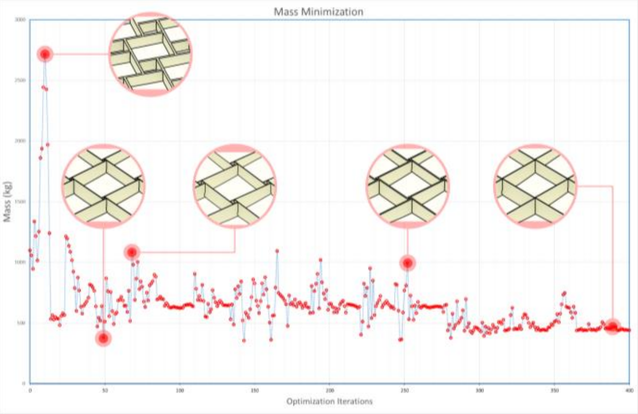
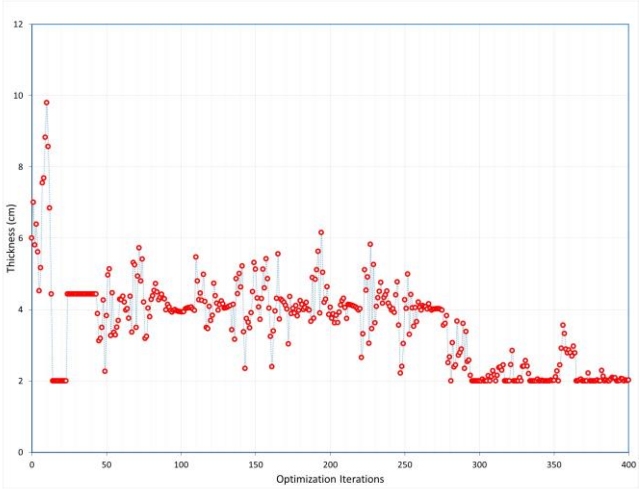


Figure 9. Optimization results for the first case study.

In this case study the numerical results indicate that the local optimum solution has the minimum engagement length (Figure 9). This case study is specifically interesting as it demonstrates the transition of the discrete reciprocal geometry with larger engagement lengths to a more continuous configuration close to a gridshell as the engagement length decreases towards zero in the process of optimization. This transition shows the behavioral connection between these two types of structural systems, Moreover, theoretically this transition from a discrete geometry of a reciprocal system to a continuous geometry of a gridshell is a proof of convergence to the global minimum for the optimization process as existence of a more continuous load path in the geometry increases the loadbearing efficiency of the system.

(Figure 10) shows the design variables variations through the process of optimization as it shows the interaction of depth, thickness and reciprocal parameters towards the optimal combination.



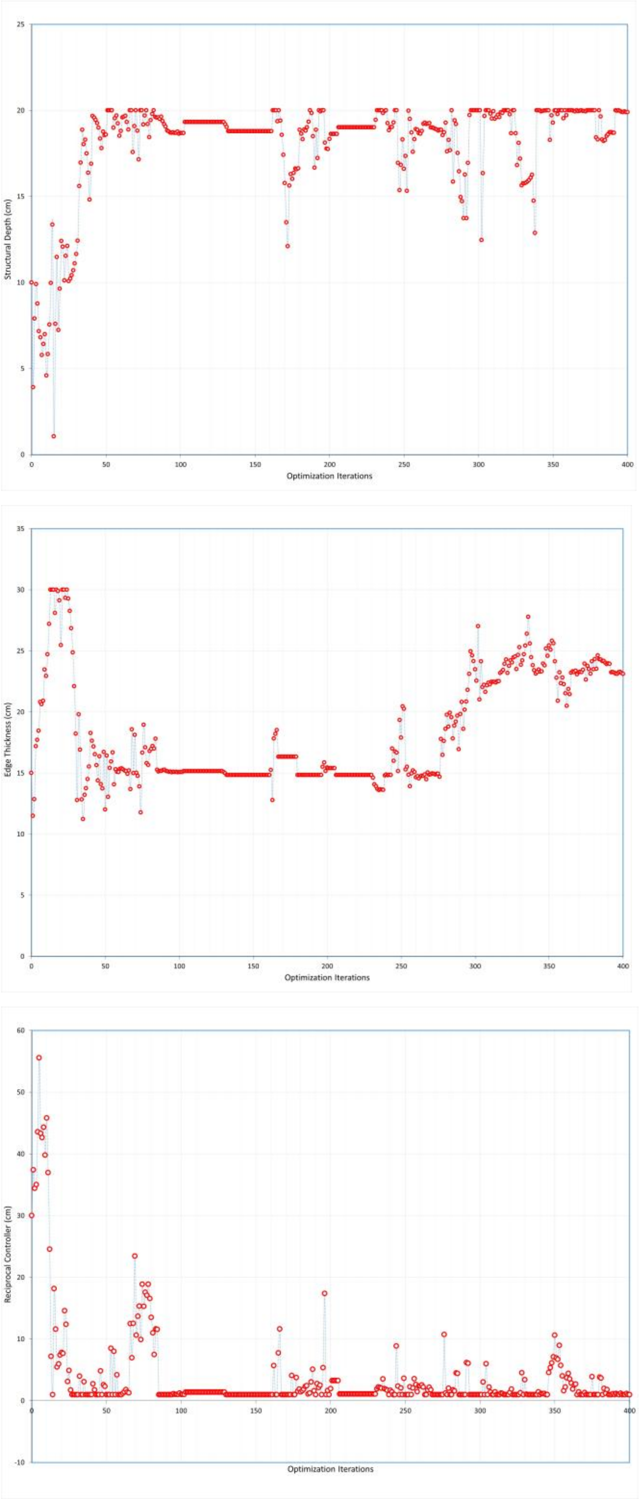


Figure 10. Design parameters through the optimization process.

Subsequently to testing the convergence of the process through the first case study, this process can be used to, incorporate a rotational parameter which transforms the modular assembly of the reciprocal system. The transformation of the geometry can be studied through the optimization process as is shown in (Figure 11).

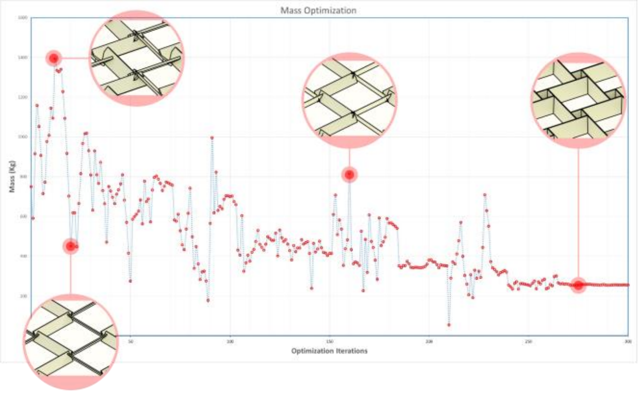


Figure 11. Optimization results for the second case study and geometric transformation.

As stated earlier, the focus of the second case study is to change the standard assembly of reciprocal systems, this change has been implemented through the introduction of a new geometric parameter which rotates the reciprocal elements around their longitudinal axis. The rotational parameter changes the orthogonal configuration and also the aperture of each cell which effects the structural performance of the system.

The numerical results show that the optimization process converges to an orthogonal reciprocal configuration as the rotation parameter is approaching zero, which corresponds to the fact that a larger rotation decreases the structural depth and consequently, the loadbearing capacity, However, this rotational parameter controls the aperture of the reciprocal cells, which as an example can be used as a design parameter for the shading performance of the structure. In this regard the post-process of the optimization results can provide a selected pallet of structurally well performing design solutions which provides a range of shading performance. Moreover, monitoring the geometrical variations in the optimization process, which produces non-orthogonal configurations, can be a source of inspiration for new assembly logics for non-standard reciprocal systems design.

4 CONCLUSIONS

In the current research a computational method is introduced for the design and form finding of reciprocal systems with planar elements. The method integrates parametric modelling with FE analysis through a stochastic search algorithm with performance feedback. The method is implemented for form exploration in two case studies where a database of the solution space is stored and explored in the post-processing phase.

In the first case study an orthogonal flat reciprocal structure is modeled and optimized for minimal weight. The main design parameter for the form exploration is the engagement length of the planar members which define the geometric variations of the optimization results. The optimization process converges to a reciprocal system with minimum engagement length, which shows the transition of the discrete reciprocal geometry to a more continuous configuration close to a gridshell. This transition implies the behavioral connection between these two types of structural systems which is confirmed numerically by the optimization results. In this regard, it is important to emphasize that while gridshell structure provides more stiffness and strength in comparison to its reciprocal counterpart, it requires long continuous members with limited integrative design potentials. However, a reciprocal system benefits from modular assembly design of relatively short members to span long distances which also accommodates local changes to assembly configuration in the structure. This flexibility adds the potential for an integrative design approach incorporating other design possibilities (e.g. daylighting, acoustics) within the geometry of the structure.

In the second case study a rotational parameter is introduced in the parametric assembly which creates non-orthogonal assemblies in the process of form exploration. In the post-processing step the assembly changes are monitored throughout the optimization process, this process is specifically interesting as it demonstrates how the introduction of a new geometric parameter can accommodate morphological and behavioral changes in the primitive geometry, which in turn can be utilized as a design generator for new assembly logics in a novel reciprocal assembly design approach. The simulation shows that the overall structural capacity of the structure is reduced by introducing the rotational parameter, however, the non-orthogonality of the reciprocal members induced by the rotation parameter change the aperture of reciprocal cells in the structure. Moreover, through the study of the geometric variations in the exploration process we can see how the introduction of new geometric parameters can assist in the ideation of new assembly logics for non-standard reciprocal systems which provide more flexibility for integrative design. Moreover, the proposed methodology can accommodate more accurate connection design and analysis into the process as a performance metric for a better understanding and fabrication of novel assemblies.

The future goal of this research is to integrate physical prototyping and digital tools to develop a performance based method for the computational design and fabrication of reciprocal systems with 2D and 3D elements, which can accommodate multiple performances through the optimization of geometric configurations with the consideration of modular assembly.

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