Multi-objective and multidisciplinary design optimization of large sports building envelopes: a case study.

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Abstract
Currently, in the conceptual envelope design of sports facilities, multiple engineering performance feedbacks (e.g. daylight, energy and structural performance) are expected to assist architectural design decision-making. In general, it is known as Building Performance Optimization in the conceptual architectural design phase. Essentially, it tends to be a Multi-objective and Multidisciplinary Design Optimization problem. Although the potential of Multi-objective Optimization and Multidisciplinary Design Optimization in handling this problem has been demonstrated in different industrial fields, there are still some significant gaps in their current application to the field of building design. The ultimate goal of our research is to find out an effective and efficient Computational Design Optimization approach, for architects, which is suitable for the conceptual design of sports building envelopes. As parts of the final goal, this paper aims to: (1) set up a meaningful benchmark case and method for the comparison of different Multidisciplinary Design Optimization approaches in future research; (2) propose an integrated Computational Design Optimization process to deal with the benchmark case using the benchmark method; and (3) test the overall process through a hypothetical and simplified case study (i.e. a sports hall with a spherical roof). Important aspects of each objective above are highlighted respectively, and thereby bridging the current gaps. Finally, discussion and future research are given.

Keywords: computational design optimization, multi-objective optimization, multidisciplinary design optimization, sports building envelopes, building performance optimization, conceptual architectural design, daylight performance, energy performance, structural performance.
1. Introduction

Computational Design Optimization (CDO) is not a new term for structural engineers. It was first introduced to structural optimization domain in 1990s to encompass all the numerical methods used for the optimum design of engineering systems (Arora [3]). Compared to manual, trial-and-error optimization, the biggest advantage of CDO is that it can be used to explore a larger number of design alternatives and optimize over a larger number of design variables than would otherwise be feasible. In current context, it is also known as Computational Optimization (Koziel and Yang [17]), which consists of three major components: mathematical model, simulator and optimizer, based upon the idea of exploiting the power of numerical simulations and optimizations.

Single-objective Optimization (SOO) is the most basic technique in CDO field. However, to deal with multiple objectives from different disciplines with SOO, a sequential design optimization process is required, within which each objective of a specific discipline is optimized in isolation in a predetermined order, assuming that other designs remain fixed (Figure 1, left). This can cause at least two significant limitations: (1) long design cycle time due to impossibility of concurrent design optimization; and (2) sub-optimal results due to low degree of design freedom in late design phases.

To overcome these limitations, Multi-objective Optimization (MOO) has been paid increasing attention and applied to the building design domain in recent years (Evins [8]). MOO is based on a concurrent design optimization process (Figure 1, left), which allows designers to incorporate multiple conflicting objectives and to specify the trade-offs between them. Instead of obtaining one single optimal solution (by using SOO), a set of non-dominated solutions (Pareto frontier) can be derived by using MOO. However, as a result of increasing complexity of systems, MOO problems may involve different disciplines simultaneously, instead of being constrained in one specific discipline. This may lead to (1) the rapid increase of problem scale and time consumed for solving these problems; and (2) inconsistency between the highly centralized design optimization process and the organizational structure of an interdisciplinary design team.

Whereas, Multidisciplinary Design Optimization (MDO) has been demonstrated as a powerful tool to tackle the above potential challenges, and well documented in many engineering industries, especially in the aerospace industry. MDO has raised a broad attention since the publication of the white papers provided in 1991 and 1998 by the AIAA MDO Technical Committee [2] and Giesing and Barthelemy [16]. It is now widely used in automobile, shipbuilding, mechanical industries etc. Recently, it has broken into the field of building design in the Architecture, Engineering and Construction (AEC) industry. Typical research are found in Flager et al. [9, 11], Flager and Haymaker [10], Geyer [13], Geyer and Beucke [14], Geyer and Rueckert [15], Gerber et al. [12], Lin and Gerber [18], Yang and Bouchlaghem [20] etc.

By using MDO, the potential challenges caused by the increase of interacting disciplines can be solved. The biggest advantage of MDO lies in its abilities of "decomposition" and "coordination",
which supports Disciplinary Autonomy based on a Concurrent Design Optimization Process. MDO approaches are grouped into two categories, i.e. single-level and bi-level frameworks. In general, single-level MDO is usually applied to only small, conceptual level problems, while bi-level MDO is more suitable for the large, complex systems (Brown and Olds [6]). MDO allows designers to "decompose" a complex system into a set of smaller and less complex subsystems, then distribute analysis (analysis autonomy of single-level MDO) and possibly optimization (decision autonomy of multi-level MDO) to subsystems instead of centralizing them, and finally "coordinate" couplings or interactions among subsystems to reach a global optimum. The main challenges lie in how to distribute analysis and/or optimization, and coordinate couplings or interactions among subsystems.

1.1. Current problems in Multi-objective and Multidisciplinary Design Optimization (M-MDO) for the conceptual design of sports building envelopes

Currently, in the conceptual design of sports building envelopes, multiple engineering performance feedbacks (e.g. daylight, energy and structural performance) are expected to assist architectural design decision-making. In general, it is known as Building Performance Optimization in the conceptual architectural design phase. Essentially, it involves multiple objectives from different disciplines, and each discipline may have its own objective to optimize. Thus, it tends to be a Multi-objective and Multidisciplinary Design Optimization (M-MDO) problem, as shown in Figure 1 (right). To tackle M-MDO problems, combining the complementary advantages of MOO and MDO is important, which is also the basic idea of our previous research (Yang et al. [19]). Although the potential of MOO and MDO has been demonstrated in the field of building design, there are still some observed and significant gaps in the current research, thereby most of current design practice is still based on the sequential design optimization process using SOO. These gaps are summarized as follows:

- **Complexity of the dependency relationship between disciplines - coupled or uncoupled**
  In aerospace industry, complex systems are governed by multiple coupled disciplines or made up of coupled components. They usually have very strong and complex dependency between various disciplines. In this case, the advantage of MDO is commonly appreciated in designing an aircraft. However, in AEC industry, multidisciplinary coupled systems are not paid enough attention. The most frequently selected cases have no dependency relationship between disciplines, which are independent or uncoupled. Thus, sufficient case studies are lacking which demonstrate how to deal with the couplings between disciplines to take advantage of MDO.

- **Flexibility of proposed tools and procedure, and their practicability from architects' point of view**
  The tools and procedure of M-MDO in AEC industry should be flexible enough to explore complex geometry and designated for architects (not for internal developers or researchers only), so that they can be readily used by design professionals in practice. However, most proposed tools and procedure in current research are based on some techniques and platforms unfamiliar to architects, and/or have relatively weak ability to deal with complex parametric geometry.

- **Post-processing and interpretation of optimization results**
  Extracting relevant information from optimization results and obtaining sufficient insight into the system of interest are important for reducing the complexity of optimization problems and providing better designs. However, many current research tends to pay less attention on the post-processing and interpretation of optimization results. They usually stop after obtaining the Pareto front.
As a whole, the ultimate goal of our research is to find out an effective and efficient CDO approach, for architects, which is suitable for the conceptual design of sports building envelopes. As parts of the final goal, this paper presents:

- a benchmark case and method for the future comparison of different MDO approaches. The benchmark case contains couplings between disciplines; and the most basic MDO approach - Multidisciplinary Feasible (MDF) - is adopted as the benchmark method (Section 2);
- an integrated CDO process based on Rhino (Mc Neel) and its plug-in Grasshopper and modeFRONTIER (ESTECO) to deal with the benchmark case using the benchmark method. Within the process, MOO is applied to the formerly described MDO approach (Section 3);
- a test on the overall process through a hypothetical and simplified case study - a sports hall with a spherical roof. Post-processing and interpretation of optimization results are highlighted in the case study (Section 4).

Finally, discussion and future research are given in the last section.

2. Benchmark case and method

In order to identify a CDO approach suitable for the conceptual design of sports building envelopes, different MDO approaches can be considered. Therefore, setting up a benchmark method for a specific case is helpful for comparison in future research. Before moving to that, it is worth knowing typical cases in aerospace and AEC industries, respectively.

2.1. Typical cases in aerospace and AEC industries

2.1.1. Notations

- \( z \): design variables, including \( z_{iA}, z_{iB}, z_{iC} \). The subscript \( iA \) represents the design variables which are shared between the different subsystems (global design variables); and the subscripts \( i1, i2, i3 \) denote the design variables which are specific to one subsystem (local design variables).
- \( y \): coupling variables, including \( y_{21}, y_{31}, y_{32}, y_{23}, y_{32}, y_{33} \). These variables are used to link the different subsystems. The double indexation \( y_{21} \) denotes that this coupling variable is transmitted from the 2st subsystem to the 1st subsystem, and so on.
- \( x \): state (or disciplinary) variables, including \( x_{1}, x_{2}, x_{3} \). These variables can vary during the disciplinary analysis in order to find an equilibrium in the state (or disciplinary) equations. They can be defined by either explicit functions (which is the rare case in engineering applications) or implicit functions. The subscripts \( i1, i2, i3 \) denote the state variables that are specific to one subsystem.
- \( f \) and \( f_{1}, f_{2}, f_{3} \): objective functions. \( f \) represents the objective function usually used in aerospace industry, such as a cost criterion or a mass (e.g. gross lift-off weight or payload mass). \( f_{1}, f_{2}, f_{3} \) represent the objective functions chosen from daylight, energy and structure disciplines in the AEC industry (e.g. Spatial Daylight Autonomy, Energy Use Intensity, Total Mass of a Structure).
- \( g \): design inequality constraints. The subscripts \( i1, i2, i3 \) denote the constraints that are specific to one subsystem.

Moreover, in Figure 2 (left) and Figure 3, couplings above the diagonal cells represent feed forward couplings (blue lines), while the ones below represent feed backward couplings (green lines).
2.1.2. MDO problems in aerospace industry

In aerospace industry, different disciplines (e.g. Aerodynamics, Structure and Trajectory) are usually strongly coupled with each other in nature. This means that both feed forward and feed backward couplings commonly exist between disciplines (Figure 2, left), as the example in Adami et al. [1].

Among all MDO approaches, focus is given here to MDF (Balling and Sobieszczanski-Sobieski [5]), which is the most basic single-level MDO approach. In this approach, for given design variables (i.e. $z_{sh}, z_1, z_2, z_3$), multidisciplinary analysis executes in iterations to obtain outputs (i.e. $x_1, x_2, y_{12}, y_{23}, y_{31}$), which are used to evaluate the single objective function (i.e. $f$) and constraints (i.e. $g_3, g_5, g_9$). In this method, "Multidisciplinary Feasibility” of the solution in each design iteration (optimization cycle) is guaranteed, because a fully multidisciplinary analysis is enforced. It should be noted that, in this case, the term "feasibility" does not imply the satisfaction of design constraints but that of individual disciplinary equations and coupling equations.

The main advantage of MDF lies in its simplicity. Unlike other MDO approaches, MDF determines the values of the coupling variables by multidisciplinary analysis, instead of the optimizer which requires disconnection of the couplings, and all governing equations are solved in analysis iterations until the coupling variables converge. Thus, system decomposition is not required in MDF and its implementation is relatively easy. However, the modularity of MDF is poor, each discipline has to wait the previous one to perform its task. Thus, it brings obstacles to parallel computation, and is less compatible with the organizational structure of an interdisciplinary design team. In general, MDF is applicable to the optimization problems in which the different subsystems can be quickly evaluated during multidisciplinary analysis (Balesdent et al. [4]). In fact, considering the time-consuming multidisciplinary analysis in aerospace industry and other disadvantages of MDF, it is not an idea approach. Nevertheless, it is still a good basis or benchmark for developing more advanced MDO approaches to take advantage of system decomposition (which is not the focus of this paper).

2.1.3. MDO problems in AEC industry

Differently than in aerospace, the frameworks commonly used in AEC industry do not usually contain couplings between disciplines. Normally, different disciplines (e.g. Daylight, Energy and Structure)
are entirely uncoupled in many current case studies, which means parallel computation for these independent, single-discipline analysis can be easily implemented (Figure 2, right). When disciplines are uncoupled, both global and local design variables (i.e. \( z_{d1} \), \( z_{d2} \), \( z_{d3} \)) are given respectively to each corresponding discipline, then each discipline can execute their own analysis codes and return the outputs (i.e. \( x_{1} \), \( x_{2} \), \( x_{3} \)) to evaluated multiple objective functions (i.e. \( f_{1} \), \( f_{2} \), \( f_{3} \)) and constraints (i.e. \( g_{1} \), \( g_{2} \), \( g_{3} \)). "Multidisciplinary feasibility" is also guaranteed in this method. Essentially, the framework without interdisciplinary couplings can be considered as the most simplified version of MDF approach. Its complexity level is much lower than that implemented in the aerospace industry, and this level of complexity is enough for some but not all cases in building design.

For an entirely uncoupled MDO framework, advantages in terms of Disciplinary Autonomy are obvious (e.g. allowing parallel computation and being compatible with the organizational structure of an interdisciplinary design team). However, potential couplings between different disciplines (e.g. daylight and energy) are sometimes important for specific cases. Neglecting them may lead to the oversimplification and underestimation of the complexity of AEC problems. This motivates us to rethink about the potential couplings that may exist between AEC disciplines, thereby form a more meaningful and complex benchmark case and benchmark method.

2.2. Benchmark case and method

As an example of cases in which disciplines are coupled, a dependency relationship consisting in between daylight and energy analyzers is explained here following and illustrated in Figure 3. The dependency relationship is represented by the coupling variable \( y_{12} \), which is an output generated by the daylight analyzer and used as an input for the energy analyzer. The value of this coupling variable indicates a quantification of time in which electrical lighting is needed (when daylight is not sufficient). It is required by the energy analyzer to calculate the energy use for electrical lighting. Therefore, in the analysis module of this benchmark case, only one feed forward coupling (i.e. \( y_{12} \)) consists in between daylight and energy analyzers, forming a sequential analysis without iterations (marked in red in Figure 3), and the structure analyzer is independent from others. In the optimization module, MOO will be conducted. Thus, the overall framework illuminates the benchmark method.

![Figure 3: Benchmark case and method](image-url)
3. Proposed process and platforms

3.1. Overall process

In this section, a CDO process that integrates parametric modeling, performance assessment and computational optimization modules is proposed, based on the previous benchmark case and method. The overall process is illustrated by using the Design Structure Matrix (DSM) in Figure 4. DSM is a simple and compact tool to manage complex systems. In this diagram, computational optimization, parametric modeling and performance assessment modules are presented in turn in the diagonal from top-left to bottom-right. Horizontal lines from a cell represent outputs produced by that cell, and vertical lines to a cell represent inputs required by that cell.

- **Computational optimization module:**
  Outputs from this module are values of design variables. In Figure 4, they are differentiated by colors according to the number of disciplines by which they are shared. Inputs to this module are simulation results of each discipline. They are fed to the optimizer to evaluated objective functions and constraints of the optimization problem.

- **Parametric modeling module:**
  The (geometric) parametric modeling is handled by architects. In the optimization process, this module receives values of global/shared design variables given by the optimizer, and generates geometry required by different analyses of engineering disciplines, automatically.

- **Performance assessment modules (marked in grey):**
  A feed forward coupling, marked in blue dash lines, occurs between daylight and energy analyzers, while the structure analyzer is independent from others, according to the benchmark case. Moreover, before running simulations of each discipline, additional processing on geometry is usually required to construct an eligible discipline-specific "Analysis Model". For instance, in daylight and energy simulations, NURBS surfaces are not accepted. Thus, NURBS surfaces should be converted to mesh surfaces in a way that the original geometry can be properly approximated. And, the original geometry is also used as a reference to generate the structural analysis model as well.

![Figure 4: Design Structure Matrix (DSM) of the benchmark method for the benchmark case](image-url)
3.2. Platforms

In the overall process, Rhino/Grasshopper and modeFRONTIER platforms are integrated (Figure 5). Grasshopper is a visual programming language tightly integrated with Rhino 3D modeling tools. It is famous for its ability of handling complex geometries but requires no knowledge of programming or scripting. Thus, it is popular within architects. While, modeFRONTIER is an integration platform for M-MDO used in many engineering industries. It links with third party engineering tools, enables the automation of the design simulation process, and facilitates analytic decision making.

They are integrated through the use of an "Interface", which is a customized Grasshopper plug-in that enables them to communicate with each other and run automatically. In the process, all the simulations are based on the Grasshopper platform by using its plug-ins. For example, "Ladybug and Honeybee" are for the daylight and energy simulations, and "Karamba" is for the structural analysis. Moreover, the modeFRONTIER platform carries out the optimization iteratively by analyzing the simulation outputs and adjusting the values assigned to the input variables. The output variables are used to define objectives and/or constraints of a design problem, and Design of experiments (DOE) assigns initial values to the input variables to start. Within the optimization iterations, smart algorithms (e.g. NSGA-II) play a key role in evaluating improvements of each solution to obtain the "fitness", based on which new values are suggested to the input variables.

4. Case study and results

4.1. Description of the case

To test the proposed overall process and platforms, a hypothetical and simplified case study was conducted. This case is assumed to be an one-story sports hall with a rectangular plan and a spherical roof (Figure 6), located in Guangzhou, in south China. And the windows are constrained to be allocated on the north-facing wall. The reasons for developing such a case lie in: (1) the large volume and span are similar to a typical indoor sports hall (e.g. 40m*70m*25m, without grandstand); and (2) this case raises a challenge of handling non-planar surfaces, as mentioned in Section 3.

Figure 5: The integration of Rhino/Grasshopper and modeFRONTIER platforms
4.2. Design variables
Design variables and ranges of the case are listed in Figure 7 (left). All of them are differentiated by the colors, as mentioned in Section 3, according to the number of disciplines by which they are shared. The total floor area of the hall is constant (2800 m²), while the aspect ratio of the plan is changeable with the change of its dimensions in the X axis. The term "Roof_height" refers to the vertical height difference between the lowest point of the spherical roof to its highest point, which actually represents the convexity of the roof. The roof will maintain a spherical shape while the plan and convexity change. Additionally, the connectivity (i.e. topology) of the spatial structure in this case is varied with the change of the plan automatically, and the maximum grid size is 3 meters.

4.3. Objectives and constraints
Daylight, energy and structure performance should be assessed against their important criteria (Figure 7 right and Figure 8). And these performance criteria from different disciplines are naturally used to construct objectives and constraints of the optimization problem.

To take full advantage of daylight, Spatial Daylight Autonomy (sDA) was chosen to form an objective (i.e. maximizing sDA). It is a metric that describes how much of a space receives sufficient daylight in one year. In LEED v4 specifically, sDA is defined as the percentage of floor area that receives at least 300 lux for at least 50% of the annual operating hours.

Energy Use Intensity (EUI), is one of the most basic way to benchmark a building’s energy efficiency or performance. It is defined as the energy consumption per unit of floor area (kWh/m²) of a building measured over one year, which facilitates direct comparison with other buildings, giving us a general idea of how energy efficient the building is. The objective is to minimize EUI. Moreover, in this case,
the "site EUI" metric was used, which refers to the total on-site energy use only (without accounting for the environmental impacts of energy sources); and the Coefficient of Performance (CoP) of heat pumps was set to 3, assuming that only electricity was used as a secondary energy source.

Minimizing the Total Mass (TM) of a large-span roof structure is frequently selected as an objective of structural performance. In this case, the four corners of the spatial structure worked as supporters, 3 kN/m² dead load and 1.5 kN/m² wind load were applied on the roof. Besides the total mass, a stiffness criterion - Service Limit State (SLS) - was included to form a structural design constraint. It was checked against maximum displacement of the roof to decide feasible solutions.

4.4. Optimization execution and post-processing of results

Before formally running the optimization, a proper optimization algorithm and its settings should be selected, as well as an initial start population. Thus, the original Non-dominated Sorting Genetic Algorithm II (NSGA-II) developed by Deb et al. [7] was selected. The number of generations used was 50, and 10 designs were created by DOE using Latin Hypercube samplings as the initial generation. At this point, the optimization was ready to be run (Figure 9).
After 12.5 hours running on a laptop (Processor: Intel(R) Core(TM) i7-4800MQ CPU @ 2.70GHz; RAM: 16GB), we stopped the process deliberately, considering that the temporary goal of this case is just testing the process, and that a full running (i.e. 500 solutions) may require more than 24 hours. Nevertheless, 245 solutions were obtained at the time we stopped it (each solution took 3 minutes on average), and 81 of them are unfeasible solutions that dissatisfy the design constraint.

Having the data from the run, post-processing and interpretation of the results were highlighted in this case study. One appealing feature of modeFRONTIER consists in its comprehensive data analysis environment, which allows us to do statistical assessment of complex data sets in an effective way and visualize post-processing results in a meaningful manner.

For an M-MDO problem, the most usual outcomes derived from the above data are a scatter 3D chart (Figure 10, left) and a parallel coordinate chart (Figure 11). The scatter 3D chart allows simultaneous visualization of 3 objective values of each solution, by plotting them in a 3D space. The plotted points are differentiated by colors, the black dots represent feasible solutions while the orange ones represent unfeasible solutions, and the Pareto optimal solutions are marked in green, which was gradually approaching to the red corner during the optimization process. The parallel coordinate chart shows the distribution of values of design variables and objectives, and each solution is presented by a polyline. By manipulating the ranges of design variables and objectives in this chart, it allows us to filter out the interesting solutions.

![Figure 10: Scatter 3D chart (left), scatter 2D chart between sDA and EUI (right)](image-url)

![Figure 11: Parallel coordinate chart](image-url)
Moreover, sensitivity analysis can help to identify the most important design variables in respect of objective functions, and thus screen out unimportant ones. In this case, the importance of each design variable was quantified as shown in Figure 7 (right). Specifically, the Plan dimension, Roof_height and Glazing_ratio have dominant effects on sDA and EUI as expected; while the diameter and thickness of the chords and webs have greater influence on the Total Mass of the roof structure than the Plan dimension and Roof_height; the Truss_depth has very low effects on all the three objectives.

In addition, correlation analysis by using a scatter matrix chart (Figure 12) quantifies the linear correlation between design variables, objectives and constraints, which is helpful for us to understand their relationships. It also provides us opportunities to reduce the number of objectives if there is a very strong correlation between them, so as to reduce the dimension of the objective space. In the scatter matrix chart, numbers below the top-left to bottom-right diagonal show the Pearson cross-correlation coefficients, while the corresponding scatter plots are showed above the diagonal. In this case, the correlation coefficient between sDA and EUI is -0.674 (marked in dark blue), which means they have a moderate negative correlation. Although the correlation coefficient is not strong enough to support removing one of the objectives, it is still worth knowing about the conflicting nature between sDA and EUI. By checking the corresponding scatter 2D chart (Figure 10, right), it is confirmed that sDA becomes better while EUI becomes worse, and sub-optimal solutions in terms of sDA and EUI can be Pareto optimal solutions in terms of all three objectives. What is more, the Total Mass is weakly correlated with sDA and EUI, the correlation coefficients are 0.237 and -0.100, respectively.

4. Discussion and future research

In general, to bridge the current gaps in the application of M-MDO to the field of building design, the following aspects are worth more attention:

- Comparison of different MDO approaches applied to the multidisciplinary coupled systems in the conceptual architectural design;
- Flexibility of proposed tools and procedure, and practicability from architects' point of view;
- Post-processing and interpretation of optimization results.
As parts of the final goal (i.e. finding out an effective and efficient CDO approach, for architects, which is suitable for the conceptual design of sports building envelopes), this paper has

- Set up a benchmark case and method for the future comparison of different MDO approaches;
- Proposed an integrated CDO process based on Rhino/Grasshopper and modeFRONTIER;
- Tested the whole process by a hypothetical case study, and conducted some useful data analyses.

Nevertheless, there are still some key techniques to be implemented and tested in future research. For instance, Response Surface Modeling (RSM) and Parallel Computing, which are both valid strategies to tackle heavy simulation processes.

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