

Homework Assignment #12 — Due Thursday, April 13

Textbook problems: Ch. 14: 14.2, 14.4, 14.6, 14.12

14.2 A particle of charge e is moving in nearly uniform nonrelativistic motion. For times near $t = t_0$, its vectorial position can be expanded in a Taylor series with fixed vector coefficients multiplying powers of $(t - t_0)$.

- a) Show that, in an inertial frame where the particle is instantaneously at rest at the origin but has a small acceleration \vec{a} , the Liénard-Wiechert electric field, correct to order $1/c^2$ inclusive, at that instant is $\vec{E} = \vec{E}_v + \vec{E}_a$, where the velocity and acceleration fields are

$$\vec{E}_v = e \frac{\hat{r}}{r^2} + \frac{e}{2c^2 r} [\vec{a} - 3\hat{r}(\hat{r} \cdot \vec{a})]; \quad \vec{E}_a = -\frac{e}{c^2 r} [\vec{a} - \hat{r}(\hat{r} \cdot \vec{a})]$$

and that the total electric field to this order is

$$\vec{E} = e \frac{\hat{r}}{r^2} - \frac{e}{2c^2 r} [\vec{a} + \hat{r}(\hat{r} \cdot \vec{a})]$$

The unit vector \hat{r} points from the origin to the observation point and r is the magnitude of the distance. Comment on the r dependences of the velocity and acceleration fields. Where is the expansion likely to be valid?

- b) What is the result for the instantaneous magnetic induction \vec{B} to the same order? Comment.
- c) Show that the $1/c^2$ term in the electric field has zero divergence and that the curl of the electric field is $\vec{\nabla} \times \vec{E} = e(\hat{r} \times \vec{a})/c^2 r^2$. From Faraday's law, find the magnetic induction \vec{B} at times near $t = 0$. Compare with the familiar elementary expression.

14.4 Using the Liénard-Wiechert fields, discuss the time-averaged power radiated per unit solid angle in nonrelativistic motion of a particle with charge e , moving

- a) along the z axis with instantaneous position $z(t) = a \cos \omega_0 t$.
- b) in a circle of radius R in the x - y plane with constant angular frequency ω_0 .

Sketch the angular distribution of the radiation and determine the total power radiated in each case.

14.6 a) Generalize the circumstances of the collision of Problem 14.5 to nonzero angular momentum (impact parameter) and show that the total energy radiated is given by

$$\Delta W = \frac{4z^2 e^2}{3m^2 c^3} \left(\frac{m}{2}\right)^{1/2} \int_{r_{\min}}^{\infty} \left(\frac{dV}{dr}\right)^2 \left(E - V(r) - \frac{L^2}{2mr^2}\right)^{-1/2} dr$$

where r_{\min} is the closest distance of approach (root of $E - V - L^2/2mr^2$), $L = mbv_0$, where b is the impact parameter, and v_0 is the incident speed ($E = mv_0^2/2$).

- b) Specialize to a repulsive Coulomb potential $V(r) = zZe^2/r$. Show that ΔW can be written in terms of impact parameter as

$$\Delta E = \frac{2zmv_0^5}{Zc^3} \left[-t^{-4} + t^{-5} \left(1 + \frac{t^2}{3} \right) \tan^{-1} t \right]$$

where $t = bmv_0^2/zZe^2$ is the ratio of twice the impact parameter to the distance of closest approach in a head-on collision.

- 14.12 As in Problem 14.4a), a charge e moves in simple harmonic motion along the z axis, $z(t') = a \cos(\omega_0 t')$.

- a) Show that the instantaneous power radiated per unit solid angle is

$$\frac{dP(t')}{d\Omega} = \frac{e^2 c \beta^4}{4\pi a^2} \frac{\sin^2 \theta \cos^2(\omega_0 t')}{(1 + \beta \cos \theta \sin \omega_0 t')^5}$$

where $\beta = a\omega_0/c$.

- b) By performing a time averaging, show that the average power per unit solid angle is

$$\frac{dP}{d\Omega} = \frac{e^2 c \beta^4}{32\pi a^2} \left[\frac{4 + \beta^2 \cos^2 \theta}{(1 - \beta^2 \cos^2 \theta)^{7/2}} \right] \sin^2 \theta$$

- c) Make rough sketches of the angular distribution for nonrelativistic and relativistic motion.