

## Homework Assignment #11 — Due Thursday, April 6

Textbook problems: Ch. 13: 13.1, 13.2, 13.3, 13.9

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- 13.1 If the light particle (electron) in the Coulomb scattering of Section 13.1 is treated classically, scattering through an angle  $\theta$  is correlated uniquely to an incident trajectory of impact parameter  $b$  according to

$$b = \frac{ze^2}{pv} \cot \frac{\theta}{2}$$

where  $p = \gamma mv$  and the differential scattering cross section is  $\frac{d\sigma}{d\Omega} = \frac{b}{\sin \theta} \left| \frac{db}{d\theta} \right|$ .

- a) Express the invariant momentum transfer squared in terms of impact parameter and show that the energy transfer  $T(b)$  is

$$T(b) = \frac{2z^2e^4}{mv^2} \frac{1}{b^2 + b_{\min}^{(c)2}}$$

where  $b_{\min}^{(c)} = ze^2/pv$  and  $T(0) = T_{\max} = 2\gamma^2\beta^2mc^2$ .

- b) Calculate the small transverse impulse  $\Delta p$  given to the (nearly stationary) light particle by the transverse electric field (11.152) of the heavy particle  $q = ze$  as it passes by at large impact parameter  $b$  in a (nearly) straight line path at speed  $v$ . Find the energy transfer  $T \approx (\Delta p)^2/2m$  in terms of  $b$ . Compare with the exact classical result of part a). Comment.
- 13.2 Time-varying electromagnetic fields  $\vec{E}(\vec{x}, t)$  and  $\vec{B}(\vec{x}, t)$  of finite duration act on a charged particle of charge  $e$  and mass  $m$  bound harmonically to the origin with natural frequency  $\omega_0$  and small damping constant  $\Gamma$ . The fields may be caused by a passing charged particle or some other external source. The charge's motion in response to the fields is nonrelativistic and small in amplitude compared to the scale of spatial variation of the fields (dipole approximation). Show that the energy transferred to the oscillator in the limit of very small damping is

$$\Delta E = \frac{\pi e^2}{m} |\vec{E}(\omega_0)|^2$$

where  $\vec{E}(\omega)$  is the symmetric Fourier transform of  $\vec{E}(0, t)$ :

$$\vec{E}(0, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \vec{E}(\omega) e^{-i\omega t} d\omega, \quad \vec{E}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \vec{E}(0, t) e^{i\omega t} dt$$

13.3 The external fields of Problem 13.2 are caused by a charge  $ze$  passing the origin in a straight-line path at speed  $v$  and impact parameter  $b$ . The fields are given by (11.152).

a) Evaluate the Fourier transforms for the perpendicular and parallel components of the electric field at the origin and show that

$$E_{\perp}(\omega) = \frac{ze}{bv} \left(\frac{2}{\pi}\right)^{1/2} \xi K_1(\xi), \quad E_{\parallel}(\omega) = -i \frac{ze}{\gamma bv} \left(\frac{2}{\pi}\right)^{1/2} \xi K_0(\xi)$$

where  $\xi = \omega b/\gamma v$ , and  $K_{\nu}(\xi)$  is the modified Bessel function of the second kind and order  $\nu$ .

b) Using the result of Problem 13.2, write down the energy transfer  $\Delta E$  to a harmonically bound charged particle. From the limiting forms of the modified Bessel functions for small and large argument, show that your result agrees with the appropriate limit of  $T(b)$  in Problem 13.1 on the one hand and the arguments at the end of Section 13.1 on the adiabatic behavior for  $b \gg \gamma v/\omega_0$  on the other.

13.9 Assuming that Plexiglas or Lucite has an index of refraction of 1.50 in the visible region, compute the angle of emission of visible Cherenkov radiation for electrons and protons as a function of their kinetic energies in MeV. Determine how many quanta with wavelengths between 4000 and 6000 Å are emitted per centimeter of path in Lucite by a 1 MeV electron, a 500 MeV proton, and a 5 GeV proton.