Physics 506 **Winter 2006**

Homework Assignment #8 — Due Thursday, March 16

Textbook problems: Ch. 11: 11.13, 11.16, 11.18, 11.27

11.13 An infinitely long straight wire of negligible cross-sectional area is at rest and has a uniform linear charge density q_0 in the inertial frame K'. The frame K' (and the wire) move with a velocity $\vec{v}$ parallel to the direction of the wire with respect to the laboratory frame K.

a) Write down the electric and magnetic fields in cylindrical coordinates in the rest frame of the wire. Using the Lorentz transformation properties of the fields, find the components of the electric and magnetic fields in the laboratory.

b) What are the charge and current densities associated with the wire in its rest frame? In the laboratory?

c) From the laboratory charge and current densities, calculate directly the electric and magnetic fields in the laboratory. Compare with the results of part *a*).

11.16 In the rest frame of a conducting medium the current density satisfies Ohm's law, $\vec{J}' = \sigma \vec{E}'$, where σ is the conductivity and primes denote quantities in the rest frame.

a) Taking into account the possibility of convection current as well as conduction current, show that the covariant generalization of Ohm's law is

$$J^\alpha - \frac{1}{c^2}(U_\beta J^\beta)U^\alpha = \frac{\sigma}{c} F^{\alpha\beta} U_\beta$$

where U^α is the 4-velocity of the medium.

b) Show that if the medium has a velocity $\vec{v} = c\vec{\beta}$ with respect to some inertial frame that the 3-vector current in that frame is

$$\vec{J} = \gamma\sigma[\vec{E} + \vec{\beta} \times \vec{B} - \vec{\beta}(\vec{\beta} \cdot \vec{E})] + \rho\vec{v}$$

where ρ is the charge density observed in that frame.

c) If the medium is uncharged in its rest frame ($\rho' = 0$), what is the charge density and the expression for $\vec{J}$ in the frame of part *b*)? This is the relativistic generalization of the equation $\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B})$ (see p. 320).

11.18 The electric and magnetic fields of a particle of charge q moving in a straight line with speed $v = \beta c$, given by (11.152), become more and more concentrated as $\beta \to 1$, as is indicated in Fig. 11.9. Choose axes so that the charge moves along the z axis in the positive direction, passing the origin at $t = 0$. Let the spatial coordinates of the observation point be (x, y, z) and define the transverse vector $\vec{r}_\perp$, with components x and y. Consider the fields and the source in the limit of $\beta = 1$.

a) Show that the fields can be written as

$$\vec{E} = 2q\frac{\vec{r}_\perp}{r_\perp^2}\delta(ct-z); \qquad \vec{B} = 2q\frac{\hat{v}\times\vec{r}_\perp}{r_\perp^2}\delta(ct-z)$$

where $\hat{v}$ is a unit vector in the direction of the particle's velocity.

b) Show by substitution into the Maxwell equations that these fields are consistent with a 4-vector source density

$$J^\alpha = qcv^\alpha\delta^{(2)}(\vec{r}_\perp)\delta(ct-z)$$

where the 4-vector $v^\alpha = (1, \hat{v})$.

c) Show that the fields of part *a*) are derivable from either of the following 4-vector potentials

$$A^0 = A^z = -2q\delta(ct-z)\ln(\lambda r_\perp); \qquad \vec{A}_\perp = 0$$

or

$$A^0 = 0 = A^z; \qquad \vec{A}_\perp = -2q\Theta(ct-z)\vec{\nabla}_\perp\ln(\lambda r_\perp)$$

where λ is an irrelevant parameter setting the scale of the logarithm. Show that the two potentials differ by a gauge transformation and find the gauge function, χ.

11.27 *a*) A charge density ρ' of zero total charge, but with a dipole moment $\vec{p}$, exists in reference frame K'. There is no current density in K'. The frame K' moves with a velocity $\vec{v} = \vec{\beta}c$ in the frame K. Find the charge and current densities ρ and $\vec{J}$ in the frame K and show that there is a magnetic dipole moment, $\vec{m} = (\vec{p}\times\vec{\beta})/2$, correct to first order in β. What is the electric dipole moment in K to the same order in β?

b) Instead of the charge density, but no current density, in K' consider no charge density, but a current density $\vec{J}'$ that has a magnetic dipole moment $\vec{m}$. Find the charge and current densities in K and show that to first order in β there is an electric dipole moment $\vec{p} = \vec{\beta}\times\vec{m}$ in addition to the magnetic dipole moment.