

Homework Assignment #7 — Due Thursday, March 9

Textbook problems: Ch. 10: 10.14, 10.16, 11.4, 11.5

- 10.14 A rectangular opening with sides of length a and $b \geq a$ defined by $x = \pm(a/2)$, $y = \pm(b/2)$ exists in a flat, perfectly conducting plane sheet filling the x - y plane. A plane wave is normally incident with its polarization vector making an angle β with the long edges of the opening.
- Calculate the diffracted fields and power per unit solid angle with the vector Smythe-Kirchhoff relation (10.109), assuming that the tangential electric field in the opening is the incident unperturbed field.
 - Calculate the corresponding result of the scalar Kirchhoff approximation.
 - For $b = a$, $\beta = 45^\circ$, $ka = 4\pi$, compute the vector and scalar approximations to the diffracted power per unit solid angle as a function of the angle θ for $\phi = 0$. Plot a graph showing a comparison between the two results.
- 10.16 a) Show from (10.125) that the integral of the shadow scattering differential cross section, summed over outgoing polarizations, can be written in the short wavelength limit as

$$\sigma_{\text{sh}} = \int d^2x_{\perp} \int d^2x'_{\perp} \cdot \frac{1}{4\pi^2} \int e^{i(\vec{x}_{\perp} - \vec{x}'_{\perp}) \cdot \vec{k}_{\perp}} d^2k_{\perp}$$

and therefore is equal to the projected area of the scatterer, independent of its detailed shape.

- Apply the optical theorem to the “shadow” amplitude (10.125) to obtain the total cross section under the assumption that in the forward direction the contribution from the illuminated side of the scatterer is negligible in comparison.
- 11.4 A possible clock is shown in the figure. It consists of a flashtube F and a photocell P shielded so that each views only the mirror M , located a distance d away, and mounted rigidly with respect to the flashtube-photocell assembly. The electronic innards of the box are such that when the photocell responds to a light flash from the mirror, the flashtube is triggered with a negligible delay and emits a short flash toward the mirror. The clock thus “ticks” once every $(2d/c)$ seconds when at rest.
- Suppose that the clock moves with a uniform velocity v , perpendicular to the line from PF to M , relative to an observer. Using the second postulate of relativity, show by explicit geometrical or algebraic construction that the observer sees the relativistic time dilatation as the clock moves by.
 - Suppose that the clock moves with a velocity v parallel to the line from PF to M . Verify that here, too, the clock is observed to tick more slowly, by the same time dilatation factor.

11.5 A coordinate system K' moves with a velocity \vec{v} relative to another system K . In K' a particle has a velocity \vec{u}' and an acceleration \vec{a}' . Find the Lorentz transformation law for accelerations, and show that in the system K the components of acceleration parallel and perpendicular to \vec{v} are

$$\vec{a}_{\parallel} = \frac{(1 - v^2/c^2)^{3/2}}{(1 + \vec{v} \cdot \vec{u}'/c^2)^3} \vec{a}'_{\parallel}$$
$$\vec{a}_{\perp} = \frac{(1 - v^2/c^2)}{(1 + \vec{v} \cdot \vec{u}'/c^2)^3} \left(\vec{a}'_{\perp} + \frac{\vec{v}}{c^2} \times (\vec{a}' \times \vec{u}') \right)$$