Physics 506 **Winter 2006**

Homework Assignment #6 — Due Tuesday, February 21

Textbook problems: Ch. 10: 10.2, 10.3, 10.7, 10.10

10.2 Electromagnetic radiation with elliptic polarization, described (in the notation of Section 7.2) by the polarization vector,

$$\vec{\epsilon} = \frac{1}{\sqrt{1+r^2}}(\vec{\epsilon}_+ + re^{i\alpha}\vec{\epsilon}_-)$$

is scattered by a perfectly conducting sphere of radius a. Generalize the amplitude in the scattering cross section (10.71), which applies for $r = 0$ or $r = \infty$, and calculate the cross section for scattering in the long-wavelength limit. Show that

$$\frac{d\sigma}{d\Omega} = k^4 a^6 \left[\frac{5}{8}(1+\cos^2\theta) - \cos\theta - \frac{3}{4}\left(\frac{r}{1+r^2}\right)\sin^2\theta\cos(2\phi - \alpha)\right]$$

Compare with Problem 10.1.

10.3 A solid uniform sphere of radius R and conductivity σ acts as a scatterer of a plane-wave beam of unpolarized radiation of frequency ω, with $\omega R/c \ll 1$. The conductivity is large enough that the skin depth δ is small compared to R.

a) Justify and use a magnetostatic scalar potential to determine the magnetic field around the sphere, assuming the conductivity is infinite. (Remember that $\omega \neq 0$.)

b) Use the technique of Section 8.1 to determine the absorption cross section of the sphere. Show that it varies as $(\omega)^{1/2}$ provided σ is independent of frequency.

10.7 Discuss the scattering of a plane wave of electromagnetic radiation by a nonpermeable, dielectric sphere of radius a and dielectric constant ϵ_r.

a) By finding the fields inside the sphere and matching to the incident plus scattered wave ouside the sphere, determine without any restriction on ka the multipole coefficients in the scattered wave. Define suitable phase shifts for the problem.

b) Consider the long-wavelength limit ($ka \ll 1$) and determine explicitly the differential and total scattering cross sections. Compare your results with those of Section 10.1.B.

c) In the limit $\epsilon_r \to \infty$ compare your results to those for the perfectly conducting sphere.

10.10 The aperture or apertures in a perfectly conducting plane screen can be viewed as the location of effective sources that produce radiation (the diffracted fields). An aperture whose dimensions are small compared with a wavelength acts as a source of dipole radiation with the contributions of other multipoles being negligible.

a) Beginning with (10.101) show that the effective electric and magnetic dipole moments can be expressed in terms of integrals of the tangential electric field in the aperture as follows:

$$\vec{p} = \epsilon \hat{n} \int (\vec{x} \cdot \vec{E}_{\text{tan}})\, da$$

$$\vec{m} = \frac{2}{i\omega\mu} \int (\hat{n} \times \vec{E}_{\text{tan}})\, da$$

where $\vec{E}_{\text{tan}}$ is the *exact* tangential electric field in the aperture, $\hat{n}$ is the normal to the plane screen, directed into the region of interest, and the integration is over the area of the openings.

b) Show that the expression for the magnetic moment can be transformed into

$$\vec{m} = \frac{2}{\mu} \int \vec{x}(\hat{n} \cdot \vec{B})\, da$$

Be careful about possible contributions from the edge of the aperture where some components of the fields are singular if the screen is infinitesimally thick.