Physics 506 **Winter 2006**

Homework Assignment #5 — Due Tuesday, February 14

Textbook problems: Ch. 9: 9.19, 9.22 *a*) and *b*), 9.23
Ch. 10: 10.1

9.19 Consider the excitation of a waveguide in Problem 8.19 from the point of view of multipole moments of the source.

a) For the linear probe antenna calculate the multipole moment components of $\vec{p}$, $\vec{m}$, $Q_{\alpha\beta}$, $Q^M_{\alpha\beta}$ that enter (9.69).

b) Calculate the amplitudes for excitation of the $\mathrm{TE}_{1,0}$ mode and evaluate the power flow. Compare the multipole expansion result with the answer given in Problem 8.19*b*). Discuss the reasons for agreement or disagreement. What about the comparison for excitation of other modes?

9.22 A spherical hole of radius a in a conducting medium can serve as an electromagnetic resonant cavity.

a) Assuming infinite conductivity, determine the transcendental equations for the characteristic frequencies ω_{lm} of the cavity for TE and TM modes.

b) Calculate numerical values for the wavelength λ_{lm} in units of the radius a for the four lowest modes for TE and TM waves.

9.23 The spherical resonant cavity of Problem 9.22 has nonpermeable walls of large, but finite, conductivity. In the approximation that the skin depth δ is small compared to the cavity radius a, show that the Q of the cavity, defined by equation (8.86), is given by

$$Q = \frac{a}{\delta} \qquad \text{for all TE modes}$$

$$Q = \frac{a}{\delta}\left(1 - \frac{l(l+1)}{x_{lm}^2}\right) \qquad \text{for TM modes}$$

where $x_{lm} = (a/c)\omega_{lm}$ for TM modes.

10.1 *a*) Show that for arbitrary initial polarization, the scattering cross section of a perfectly conducting sphere of radius a, summed over outgoing polarizations, is given in the long-wavelength limit by

$$\frac{d\sigma}{d\Omega}(\vec{\epsilon}_0, \hat{n}_0, \hat{n}) = k^4 a^6 \left[\frac{5}{4} - |\vec{\epsilon}_0 \cdot \hat{n}|^2 - \frac{1}{4}|\hat{n} \cdot (\hat{n}_0 \times \vec{\epsilon}_0)|^2 - \hat{n}_0 \cdot \hat{n}\right]$$

where $\hat{n}_0$ and $\hat{n}$ are the directions of the incident and scattered radiations, respectively, while $\vec{\epsilon}_0$ is the (perhaps complex) unit polarization vector of the incident radiation ($\vec{\epsilon}_0{}^* \cdot \vec{\epsilon}_0 = 1$; $\hat{n}_0 \cdot \vec{\epsilon}_0 = 0$).

b) If the incident radiation is linearly polarized, show that the cross section is

$$\frac{d\sigma}{d\Omega}(\vec{\epsilon}_0, \hat{n}_0, \hat{n}) = k^4 a^6 \left[\frac{5}{8}(1 + \cos^2\theta) - \cos\theta - \frac{3}{8}\sin^2\theta\cos 2\phi\right]$$

where $\hat{n} \cdot \hat{n}_0 = \cos\theta$ and the azimuthal angle ϕ is measured from the direction of the linear polarization.

c) What is the ratio of scattered intensities at $\theta = \pi/2$, $\phi = 0$ and $\theta = \pi/2$, $\phi = \pi/2$? Explain physically in terms of the induced multipoles and their radiation patterns.