Physics 506 **Winter 2006**

Homework Assignment #4 — Due Thursday, February 2

Textbook problems: Ch. 9: 9.8, 9.9, 9.16, 9.17

9.8 *a*) Show that a classical oscillating electric dipole $\vec{p}$ with fields given by (9.18) radiates electromagnetic angular momentum to infinity at the rate

$$\frac{d\vec{L}}{dt} = \frac{k^3}{12\pi\epsilon_0}\Im[\vec{p}^* \times \vec{p}]$$

b) What is the ratio of angular momentum radiated to energy radiated? Interpret.

c) For a charge e rotating in the x-y plane at radius a and angular speed ω, show that there is only a z component of radiated angular momentum with magnitude $dL_z/dt = e^2k^3a^2/6\pi\epsilon_0$. What about a charge oscillating along the z axis?

d) What are the results corresponding to parts *a*) and *b*) for magnetic dipole radiation?

9.9 *a*) From the electric dipole fields with general time dependence of Problem 9.6, show that the total power and the total rate of radiation of angular momentum through a sphere at large radius r and time t are

$$P(t) = \frac{1}{6\pi\epsilon_0 c^3}\left(\frac{\partial^2\vec{p}_{\rm ret}}{\partial t^2}\right)^2$$

$$\frac{d\vec{L}_{\rm em}}{dt} = \frac{1}{6\pi\epsilon_0 c^3}\left(\frac{\partial\vec{p}_{\rm ret}}{\partial t} \times \frac{\partial^2\vec{p}_{\rm ret}}{\partial t^2}\right)$$

where the dipole moment $\vec{p}$ is evaluated at the retarded time $t' = t - r/c$.

b) The dipole moment is caused by a particle of mass m and charge e moving nonrelativistically in a fixed central potential $V(r)$. Show that the radiated power and angular momentum for such a particle can be written as

$$P(t) = \frac{\tau}{m}\left(\frac{dV}{dr}\right)^2$$

$$\frac{d\vec{L}_{\rm em}}{dt} = \frac{\tau}{m}\left(\frac{dV}{rdr}\right)\vec{L}$$

where $\tau = e^2/6\pi\epsilon_0 mc^3$ ($= 2e^2/3mc^3$ in Gaussian units) is a characteristic time, $\vec{L}$ is the particle's angular momentum, and the right-hand sides are evaluated at the retarded time. Related these results to those from the Abraham-Lorentz equation for radiation damping [Section 16.2].

c) Suppose the charged particle is an electron in a hydrogen atom. Show that the inverse time defined by the ratio of the rate of angular momentum radiated to the particle's angular momentum is of the order of $\alpha^4 c/a_0$, where $\alpha = e^2/4\pi\epsilon_0\hbar c \approx 1/137$ is the fine structure constant and a_0 is the Bohr radius. How does this inverse time compare to the observed rate of radiation in hydrogen atoms?

d) Related the expressions in parts *a*) and *b*) to those for harmonic time dependence in Problem 9.8.

9.16 A thin linear antenna of length d is excited in such a way that the sinusoidal current makes a full wavelength of oscillation as shown in the figure.

a) Calculate exactly the power radiated per unit solid angle and plot the angular distribution of radiation.

b) Determine the total power radiated and find a numerical value for the radiation resistance.

9.17 Treat the linear antenna of Problem 9.16 by the multipole expansion method.

a) Calculate the multipole moments (electric dipole, magnetic dipole, and electric quadrupole) exactly and in the long-wavelength approximation.

b) Compare the shape of the angular distribution of radiated power for the lowest nonvanishing multipole with the exact distribution of Problem 9.16.

c) Determine the total power radiated for the lowest multipole and the corresponding radiation resistance using both multipole moments from part *a*). Compare with Problem 9.16*b*). Is there a paradox here?