1 Problem 11.8 (part a only)

Substituting $k_0 = \omega/c$ into Jackson's equation 11.29:

$$k_0' = \gamma\left(k_0 - \vec{\beta}\cdot\vec{k}\right)$$
$$\frac{\omega'}{c} = \gamma\left(\frac{\omega}{c} - \beta k\right)$$

Note that since we're given that the light is either parallel or anti-parallel to the velocity of the fluid, $\vec{\beta}\cdot\vec{k} = \beta k$.

Approximating this to first-order, we realize that β is small while $\gamma \approx 1$:

$$\frac{\omega'}{c} \approx \frac{\omega}{c} - \beta k$$
$$\omega - \omega' \approx -c\beta\left(\frac{\omega}{u}\right)$$
$$= -c\beta\omega\frac{\omega}{(c/n)}$$
$$\omega - \omega' = -\beta\omega n \tag{1}$$

Solving for $n(\omega)$:

$$n(\omega) = -\frac{\omega - \omega'}{\beta\omega}$$

Taylor expanding about $\omega = \omega'$:

$$n(\omega) \approx n(\omega') + \frac{\omega - \omega'}{1!}\left.\frac{\partial n}{\partial\omega}\right|_{\omega=\omega'}$$

Substituting in equation (1):

$$n(\omega) \approx n(\omega') + (-\beta\omega n)\left.\frac{\partial n}{\partial\omega}\right|_{\omega=\omega'}$$
$$\approx n(\omega) - \beta\omega n\frac{\partial n(\omega)}{\partial\omega}$$

where we've let $\omega' \approx \omega$.

$$\frac{1}{n(\omega)} \approx \frac{1}{n(\omega)}\left[1 - \beta\omega\frac{\partial n(\omega)}{\partial\omega}\right]^{-1}$$
$$\approx \frac{1}{n(\omega)}\left[1 + \beta\omega\frac{\partial}{\partial\omega}\right] \tag{2}$$

where we've used a first-order binomial series approximation.

$$\begin{aligned}\frac{1}{n^2(\omega)} &\approx \frac{1}{n^2(\omega)}\left[1-\beta\omega\frac{\partial n(\omega)}{\partial\omega}\right]^{-2}\\ &\approx \frac{1}{n^2(\omega)}\left[1+2\beta\omega\frac{\partial n(\omega)}{\partial\omega}\right]\end{aligned} \tag{3}$$

Now, we use the velocity addition formula, noting that the velocity of the fluid is either parallel or anti-parallel to the light:

$$\begin{aligned}u &= \frac{u'+v}{1+u'v/c^2}\\ &= \frac{\frac{c}{n'}+v}{1+\beta/n'} = \frac{c}{n'}\frac{1+\beta n'}{1+\beta/n'}\\ &\approx \frac{c}{n'}\left(1+\beta n'\right)\left(1-\frac{\beta}{n'}\right)\\ &= \frac{c}{n'}\left(1+\beta n'-\frac{\beta}{n'}\right)\\ &\approx \frac{c}{n'}+v-\frac{v}{n'^2}\end{aligned}$$

Because $n' = n(\omega')$ and $\omega \approx \omega'$, we can substitute in equations (2) and (3):

$$\begin{aligned}u &\approx \frac{c}{n(\omega)}\left[1+\beta\omega\frac{\partial n(\omega)}{\partial\omega}\right]+v-\frac{v}{n^2(\omega)}\left[1+2\beta\omega\frac{\partial n(\omega)}{\partial\omega}\right]\\ &= \frac{c}{n(\omega)}+\frac{v\omega}{n(\omega)}\frac{\partial n(\omega)}{\partial\omega}+v-\frac{v}{n^2(\omega)}+2\frac{\beta v\omega}{n^2(\omega)}\frac{\partial n(\omega)}{\partial\omega}\\ &= \frac{c}{n(\omega)}+v\left[1-\frac{1}{n^2(\omega)}+\frac{\omega}{n(\omega)}\frac{\partial n(\omega)}{\partial\omega}+2\frac{\beta\omega}{n^2(\omega)}\frac{\partial n(\omega)}{\partial\omega}\right]\end{aligned}$$

Dropping the last term (because β is small) yields:

$$u \approx \frac{c}{n(\omega)}+v\left[1-\frac{1}{n^2(\omega)}+\frac{\omega}{n(\omega)}\frac{\partial n(\omega)}{\partial\omega}\right]$$

recalling that v may be either parallel (in which case v is positive) or anti-parallel (in which case v is negative) to the light.