

## Homework Assignment #12 — Due Thursday, April 10

Textbook problems: Ch. 14: 14.4, 14.5, 14.8, 14.11

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14.4 Using the Liénard-Wiechert fields, discuss the time-averaged power radiated per unit solid angle in nonrelativistic motion of a particle with charge  $e$ , moving

- a) along the  $z$  axis with instantaneous position  $z(t) = a \cos \omega_0 t$ .
- b) in a circle of radius  $R$  in the  $x$ - $y$  plane with constant angular frequency  $\omega_0$ .

Sketch the angular distribution of the radiation and determine the total power radiated in each case.

14.5 A *nonrelativistic* particle of charge  $ze$ , mass  $m$ , and kinetic energy  $E$  makes a *head-on* collision with a fixed central force field of finite range. The interaction is repulsive and described by a potential  $V(r)$ , which becomes greater than  $E$  at close distances.

- a) Show that the total energy radiated is given by

$$\Delta W = \frac{4}{3} \frac{z^2 e^2}{m^2 c^3} \sqrt{\frac{m}{2}} \int_{r_{\min}}^{\infty} \left| \frac{dV}{dr} \right|^2 \frac{dr}{\sqrt{V(r_{\min}) - V(r)}}$$

where  $r_{\min}$  is the closest distance of approach in the collision.

- b) If the interaction is a Coulomb potential  $V(r) = zZe^2/r$ , show that the total energy radiated is

$$\Delta W = \frac{8}{45} \frac{zmv_0^5}{Zc^3}$$

where  $v_0$  is the velocity of the charge at infinity.

14.8 A swiftly moving particle of charge  $ze$  and mass  $m$  passes a fixed point charge  $Ze$  in an approximately straight-line path at impact parameter  $b$  and nearly constant speed  $v$ . Show that the total energy radiated in the encounter is

$$\Delta W = \frac{\pi z^4 Z^2 e^6}{4m^2 c^4 \beta} \left( \gamma^2 + \frac{1}{3} \right) \frac{1}{b^3}$$

This is the relativistic generalization of the result of Problem 14.7.

14.11 A particle of charge  $ze$  and mass  $m$  moves in external electric and magnetic fields  $\vec{E}$  and  $\vec{B}$ .

- a) Show that the classical relativistic result for the instantaneous energy radiated per unit time can be written

$$P = \frac{2}{3} \frac{z^4 e^4}{m^2 c^3} \gamma^2 [(\vec{E} + \vec{\beta} \times \vec{B})^2 - (\vec{\beta} \cdot \vec{E})^2]$$

where  $\vec{E}$  and  $\vec{B}$  are evaluated at the position of the particle and  $\gamma$  is the particle's instantaneous Lorentz factor.

- b) Show that the expression in part a can be put into the manifestly Lorentz-invariant form

$$P = \frac{2z^4 r_0^2}{3m^2 c} F^{\mu\nu} p_\nu p^\lambda F_{\lambda\mu}$$

where  $r_0 = e^2/mc^2$  is the classical charged particle radius.