Physics 506 **Winter 2008**

Homework Assignment #11 — Due Thursday, April 3

Textbook problems: Ch. 13: 13.1, 13.2, 13.3, 13.5

13.1 If the light particle (electron) in the Coulomb scattering of Section 13.1 is treated classically, scattering through an angle θ is correlated uniquely to an incident trajectory of impact parameter b according to

$$b = \frac{ze^2}{pv}\cot\frac{\theta}{2}$$

where $p = \gamma m v$ and the differential scattering cross section is $\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$.

a) Express the invariant momentum transfer squared in terms of impact parameter and show that the energy transfer $T(b)$ is

$$T(b) = \frac{2z^2e^4}{mv^2}\frac{1}{b^2 + b_{\text{min}}^{(c)\,2}}$$

where $b_{\text{min}}^{(c)} = ze^2/pv$ and $T(0) = T_{\text{max}} = 2\gamma^2\beta^2 mc^2$.

b) Calculate the small transverse impulse Δp given to the (nearly stationary) light particle by the transverse electric field (11.152) of the heavy particle $q = ze$ as it passes by at large impact parameter b in a (nearly) straight line path at speed v. Find the energy transfer $T \approx (\Delta p)^2/2m$ in terms of b. Compare with the exact classical result of part a. Comment.

13.2 Time-varying electromagnetic fields $\vec{E}(\vec{x}, t)$ and $\vec{B}(\vec{x}, t)$ of finite duration act on a charged particle of charge e and mass m bound harmonically to the origin with natural frequency ω_0 and small damping constant Γ. The fields may be caused by a passing charged particle or some other external source. The charge's motion in response to the fields is nonrelativistic and small in amplitude compared to the scale of spatial variation of the fields (dipole approximation). Show that the energy transferred to the oscillator in the limit of very small damping is

$$\Delta E = \frac{\pi e^2}{m}|\vec{E}(\omega_0)|^2$$

where $\vec{E}(\omega)$ is the symmetric Fourier transform of $\vec{E}(0, t)$:

$$\vec{E}(0,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\vec{E}(\omega)e^{-i\omega t}d\omega, \qquad \vec{E}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\vec{E}(0,t)e^{i\omega t}dt$$

13.3 The external fields of Problem 13.2 are caused by a charge ze passing the origin in a straight-line path at speed v and impact parameter b. The fields are given by (11.152).

a) Evaluate the Fourier transforms for the perpendicular and parallel components of the electric field at the origin and show that

$$E_\perp(\omega) = \frac{ze}{bv}\left(\frac{2}{\pi}\right)^{1/2} \xi K_1(\xi), \qquad E_\parallel(\omega) = -i\frac{ze}{\gamma bv}\left(\frac{2}{\pi}\right)^{1/2} \xi K_0(\xi)$$

where $\xi = \omega b/\gamma v$, and $K_\nu(\xi)$ is the modified Bessel function of the second kind and order ν. [See references to tables of Fourier transforms in Section 13.3]

b) Using the result of Problem 13.2, write down the energy transfer ΔE to a harmonically bound charged particle. From the limiting forms of the modified Bessel functions for small and large argument, show that your result agrees with the appropriate limit of $T(b)$ in Problem 13.1 on the one hand and the arguments at the end of Section 13.1 on the adiabatic behavior for $b \gg \gamma v/\omega_0$ on the other.

13.5 Consider the energy loss by close collisions of a fast, but nonrelativistic, heavy particle of charge ze passing through an electronic plasma. Assume that the screened Coulomb interaction $V(r) = ze^2 \exp(-k_D r)/r$, where k_D is the Debye screening parameter, acts between the electrons and the incident particle.

a) Show that the energy transfer in a collision at impact parameter b is given approximately by

$$\Delta E(b) \approx \frac{2(ze^2)^2}{mv^2} k_D^2 K_1^2(k_D b)$$

where m is the electron mass and v is the velocity of the incident particle.

b) Determine the energy loss per unit distance traveled for collisions with impact parameter greater than $b_{\min}$. Assuming $k_D b_{\min} \ll 1$, show that

$$\left(\frac{dE}{dx}\right)_{k_D b<1} \approx \frac{(ze)^2}{v^2}\omega_P^2 \ln\left(\frac{1}{1.47 k_D b_{\min}}\right)$$

where $b_{\min}$ is given by the larger of the classical and quantum minimum impact parameters [(13.16) and above].