Physics 506 **Winter 2008**

Homework Assignment #9 — Due Thursday, March 20

Textbook problems: Ch. 11: 11.27, 11.30
Ch. 12: 12.2, 12.3

11.27 *a*) A charge density ρ' of zero total charge, but with a dipole moment $\vec{p}$, exists in reference frame K'. There is no current density in K'. The frame K' moves with a velocity $\vec{v} = \vec{\beta} c$ in the frame K. Find the charge and current densities ρ and $\vec{J}$ in the frame K and show that there is a magnetic dipole moment, $\vec{m} = (\vec{p} \times \vec{\beta})/2$, correct to first order in β. What is the electric dipole moment in K to the same order in β?

b) Instead of the charge density, but no current density, in K', consider no charge density, but a current density $\vec{J}'$ that has a magnetic dipole moment $\vec{m}$. Find the charge and current densities in K and show that to first order in β there is an electric dipole moment $\vec{p} = \vec{\beta} \times \vec{m}$ in addition to the magnetic dipole moment.

11.30 An isotropic linear material medium, characterized by the constitutive relations (in its rest frame K'), $\vec{D}' = \epsilon \vec{E}'$ and $\mu \vec{H}' = \vec{B}'$, is in uniform translation with velocity $\vec{v}$ in the inertial frame K. By exploiting the fact that $F_{\mu\nu} = (\vec{E}, \vec{B})$ and $G_{\mu\nu} = (\vec{D}, \vec{H})$ transform as second rank 4-tensors under Lorentz transformations, show that the macroscopic fields $\vec{D}$ and $\vec{H}$ are given in terms of $\vec{E}$ and $\vec{B}$ by

$$\vec{D} = \epsilon \vec{E} + \gamma^2 \left(\epsilon - \frac{1}{\mu} \right) [\beta^2 \vec{E}_\perp + \vec{\beta} \times \vec{B}]$$

$$\vec{H} = \frac{1}{\mu} \vec{B} + \gamma^2 \left(\epsilon - \frac{1}{\mu} \right) [-\beta^2 \vec{B}_\perp + \vec{\beta} \times \vec{E}]$$

where $\vec{E}_\perp$ and $\vec{B}_\perp$ are components perpendicular to $\vec{v}$.

12.2 *a*) Show from Hamilton's principle that Lagrangians that differ only by a total time derivative of some function of the coordinates and time are equivalent in the sense that they yield the same Euler-Lagrange equations of motion.

b) Show explicitly that the gauge transformation $A^\alpha \to A^\alpha + \partial^\alpha \Lambda$ of the potentials in the charged-particle Lagrangian (12.12) merely generates another equivalent Lagrangian.

12.3 A particle with mass m and charge e moves in a uniform, static, electric field $\vec{E}_0$.

a) Solve for the velocity and position of the particle as explicit functions of time, assuming that the initial velocity $\vec{v}_0$ was perpendicular to the electric field.

b) Eliminate the time to obtain the trajectory of the particle in space. Discuss the shape of the path for short and long times (define "short" and "long" times).