Physics 506 **Winter 2008**

Homework Assignment #8 — Due Thursday, March 13

Textbook problems: Ch. 11: 11.5, 11.13, 11.14, 11.18

11.5 A coordinate system K' moves with a velocity $\vec{v}$ relative to another system K. In K' a particle has a velocity $\vec{u}\,'$ and an acceleration $\vec{a}\,'$. Find the Lorentz transformation law for accelerations, and show that in the system K the components of acceleration parallel and perpendicular to $\vec{v}$ are

$$\vec{a}_\parallel = \frac{(1-v^2/c^2)^{3/2}}{(1+\vec{v}\cdot\vec{u}\,'/c^2)^3}\vec{a}_\parallel{}'$$
$$\vec{a}_\perp = \frac{(1-v^2/c^2)}{(1+\vec{v}\cdot\vec{u}\,'/c^2)^3}\left(\vec{a}_\perp{}' + \frac{\vec{v}}{c^2}\times(\vec{a}\,'\times\vec{u}\,')\right)$$

11.13 An infinitely long straight wire of negligible cross-sectional area is at rest and has a uniform linear charge density q_0 in the inertial frame K'. The frame K' (and the wire) move with a velocity $\vec{v}$ parallel to the direction of the wire with respect to the laboratory frame K.

a) Write down the electric and magnetic fields in cylindrical coordinates in the rest frame of the wire. Using the Lorentz transformation properties of the fields, find the components of the electric and magnetic fields in the laboratory.

b) What are the charge and current densities associated with the wire in its rest frame? In the laboratory?

c) From the laboratory charge and current densities, calculate directly the electric and magnetic fields in the laboratory. Compare with the results of part a.

11.14 *a*) Express the Lorentz scalars $F^{\alpha\beta}F_{\alpha\beta}$, $\mathcal{F}^{\alpha\beta}F_{\alpha\beta}$ and $\mathcal{F}^{\alpha\beta}\mathcal{F}_{\alpha\beta}$ in terms of $\vec{E}$ and $\vec{B}$. Are there any other invariants quadratic in the field strengths $\vec{E}$ and $\vec{B}$?

b) Is it possible to have an electromagnetic field that apperas as a purely electric field in one inertial frame and as a purely magnetic field in some other inertial frame? What are the criteria imposed on $\vec{E}$ and $\vec{B}$ such that there is an inertial frame in which there is no electric field?

c) For macroscopic media, $\vec{E}$, $\vec{B}$ form the field tensor $F^{\alpha\beta}$ and $\vec{D}$, $\vec{H}$ the tensor $G^{\alpha\beta}$. What further invariants can be formed? What are their explicit expressions in terms of the 3-vector fields?

11.18 The electric and magnetic fields of a particle of charge q moving in a straight line with speed $v = \beta c$, given by (11.152), become more and more concentrated as $\beta \to 1$, as is indicated in Fig. 11.9. Choose axes so that the charge moves along the z axis in the positive direction, passing the origin at $t = 0$. Let the spatial coordinates of the observation point be (x, y, z) and define the transverse vector $\vec{r}_\perp$, with components x and y. Consider the fields and the source in the limit of $\beta = 1$.

a) Show that the fields can be written as

$$\vec{E} = 2q\frac{\vec{r}_\perp}{r_\perp^2}\delta(ct - z); \qquad \vec{B} = 2q\frac{\hat{v} \times \vec{r}_\perp}{r_\perp^2}\delta(ct - z)$$

where $\hat{v}$ is a unit vector in the direction of the particle's velocity.

b) Show by substitution into the Maxwell equations that these fields are consistent with a 4-vector source density

$$J^\alpha = qcv^\alpha \delta^{(2)}(\vec{r}_\perp)\delta(ct - z)$$

where the 4-vector $v^\alpha = (1, \hat{v})$.

c) Show that the fields of part a are derivable from either of the following 4-vector potentials

$$A^0 = A^z = -2q\delta(ct - z)\ln(\lambda r_\perp); \qquad \vec{A}_\perp = 0$$

or

$$A^0 = 0 = A^z; \qquad \vec{A}_\perp = -2q\Theta(ct - z)\vec{\nabla}_\perp \ln(\lambda r_\perp)$$

where λ is an irrelevant parameter setting the scale of the logarithm. Show that the two potentials differ by a gauge transformation and find the gauge function, χ.