Physics 506 **Winter 2008**

Homework Assignment #7 — Due Thursday, March 6

Textbook problems: Ch. 10: 10.12, 10.14, 10.18
Ch. 11: 11.3

10.12 A linearly polarized plane wave of amplitude E_0 and wave number k is incident on a circular opening of radius a in an otherwise perfectly conducting flat screen. The incident wave vector makes an angle α with the normal to the screen. The polarization vector is perpendicular to the plane of incidence.

a) Calculate the diffracted fields an the power per unit solid angle transmitted through the opening, using the vector Smythe-Kirchhoff formula (10.101) with the assumption that the tangential electric field in the opening is the unperturbed incident field.

b) Compare your result in part a with the standard scalar Kirchhoff approximation and with the result in Section 10.9 for the polarization vector in the plane of incidence.

10.14 A rectangular opening with sides of length a and $b \geq a$ defined by $x = \pm(a/2)$, $y = \pm(b/2)$ exists in a flat, perfectly conducting plane sheet filling the x-y plane. A plane wave is normally incident with its polarization vector making an angle β with the long edges of the opening.

a) Calculate the diffracted fields and power per unit solid angle with the vector Smythe-Kirchhoff relation (10.109), assuming that the tangential electric field in the opening is the incident unperturbed field.

b) Calculate the corresponding result of the sclar Kirchhoff approx imation.

c) For $b = a$, $\beta = 45°$, $ka = 4\pi$, compute the vector and scalar approximations to the diffracted power per unit solid angle as a function of the angle θ for $|phi = 0$. Plot a graph showing a comparison between the two results.

10.18 Discuss the diffraction due to a small, circular hole of radius a in a flat, perfectly conducting sheet, assuming that $ka \ll 1$.

a) If the fields near the screen on the incident side are normal $\vec{E}_0 e^{-i\omega t}$ and tangential $\vec{B}_0 e^{-i\omega t}$, show that the diffracted electric field in the Fraunhofer zone is

$$\vec{E} = \frac{e^{ikr-i\omega t}}{3\pi r} k^2 a^3 \left[2c\frac{\vec{k}}{k} \times \vec{B}_0 + \frac{\vec{k}}{k} \times \left(\vec{E}_0 \times \frac{\vec{k}}{k} \right) \right]$$

where $\vec{k}$ is the wave vector in the direction of observation.

b) Determine the angular distribution of the diffracted radiation and show that the total power transmitted through the hole is

$$P = \frac{2}{27\pi Z_0} k^4 a^6 (4c^2 B_0^2 + E_0^2)$$

11.3 Show explicitly hat two successive Lorentz transformations in the same direction are equivalent to a single Lorentz transformation with a velocity

$$v = \frac{v_1 + v_2}{1 + (v_1 v_2 / c^2)}$$

This is an alternative way to derive the parallel-velocity addition law.