

## Homework Assignment #6 — Due Thursday, February 14

Textbook problems: Ch. 10: 10.2, 10.3, 10.8, 10.9a

---

- 10.2 Electromagnetic radiation with elliptic polarization, described (in the notation of Section 7.2) by the polarization vector,

$$\vec{\epsilon} = \frac{1}{\sqrt{1+r^2}}(\vec{\epsilon}_+ + re^{i\alpha}\vec{\epsilon}_-)$$

is scattered by a perfectly conducting sphere of radius  $a$ . Generalize the amplitude in the scattering cross section (10.71), which applies for  $r = 0$  or  $r = \infty$ , and calculate the cross section for scattering in the long-wavelength limit. Show that

$$\frac{d\sigma}{d\Omega} = k^4 a^6 \left[ \frac{5}{8}(1 + \cos^2 \theta) - \cos \theta - \frac{3}{4} \left( \frac{r}{1+r^2} \right) \sin^2 \theta \cos(2\phi - \alpha) \right]$$

Compare with Problem 10.1.

- 10.3 A solid uniform sphere of radius  $R$  and conductivity  $\sigma$  acts as a scatterer of a plane-wave beam of unpolarized radiation of frequency  $\omega$ , with  $\omega R/c \ll 1$ . The conductivity is large enough that the skin depth  $\delta$  is small compared to  $R$ .
- Justify and use a magnetostatic scalar potential to determine the magnetic field around the sphere, assuming the conductivity is infinite. (Remember that  $\omega \neq 0$ .)
  - Use the technique of Section 8.1 to determine the absorption cross section of the sphere. Show that it varies as  $(\omega)^{1/2}$  provided  $\sigma$  is independent of frequency.

- 10.8 Consider the scattering of a plane wave by a nonpermeable sphere of radius  $a$  and very good, but not perfect, conductivity following the spherical multipole field approach of Section 10.4. Assume that  $ka \ll 1$  and that the skin depth  $\delta < a$ .

- Show from the analysis of Section 8.1 that

$$\frac{Z_s}{Z_0} = \frac{k\delta}{2}(1 - i)$$

- In the long-wavelength limit, show that for  $l = 1$  the coefficients  $\alpha_{\pm}(l)$  and  $\beta_{\pm}(l)$  in (10.65) are

$$\alpha_{\pm}(l) \approx -\frac{2i}{3}(ka)^3 \left[ \frac{\left(1 - \frac{\delta}{a}\right) - i\frac{\delta}{a}}{\left(1 + \frac{\delta}{2a}\right) + i\frac{\delta}{2a}} \right]$$

$$\beta_{\pm}(1) \approx \frac{4i}{3}(ka)^3$$

- c) Write out explicitly the differential scattering cross section, correct to *first* order in  $\delta/a$  and lowest order in  $ka$ .
- d) Using (10.61), evaluate the absorption cross section. Show that to first order in  $\delta$  it is  $\sigma_{\text{abs}} \approx 3\pi(k\delta)a^2$ . How different is the value if  $\delta = a$ ?

10.9 In the scattering of light by a gas very near the critical point the scattered light is observed to be “whiter” (i.e., its spectrum is less predominantly peaked toward the blue) than far from the critical point. Show that this can be understood by the fact that the volumes of the density fluctuations become large enough that Rayleigh’s law fails to hold. In particular, consider the lowest order approximation to the scattering by a uniform dielectric sphere of radius  $a$  whose dielectric constant  $\epsilon_r$  differs only slightly from unity.

- a) Show that for  $ka \gg 1$ , the differential cross section is sharply peaked in the forward direction and the total scattering cross section is approximately

$$\sigma \approx \frac{\pi}{2}(ka)^2|\epsilon_r - 1|^2a^2$$

with a  $k^2$ , rather than  $k^4$ , dependence on frequency.