Physics 506 **Winter 2008**

Homework Assignment #5 — Due Thursday, February 7

Textbook problems: Ch. 9: 9.22, 9.23, 9.24
Ch. 10: 10.1

9.22 A spherical hole of radius a in a conducting medium can serve as an electromagnetic resonant cavity.

a) Assuming infinite conductivity, determine the transcendental equations for the characteristic frequencies ω_{lm} of the cavity for TE and TM modes.

b) Calculate numerical values for the wavelength λ_{lm} in units of the radius a for the four lowest modes for TE and TM waves.

c) Calculate explicitly the electric and magnetic fields inside the cavity for the lowest TE and lowest TM mode.

9.23 The spherical resonant cavity of Problem 9.22 has nonpermeable walls of large, but finite, conductivity. In the approximation that the skin depth δ is small compared to the cavity radius a, show that the Q of the cavity, defined by equation (8.86), is given by

$$Q = \frac{a}{\delta} \qquad \text{for all TE modes}$$

$$Q = \frac{a}{\delta}\left(1 - \frac{l(l+1)}{x_{lm}^2}\right) \qquad \text{for TM modes}$$

where $x_{lm} = (a/c)\omega_{lm}$ for TM modes.

9.24 Discuss the normal modes of oscillation of a perfectly conducting solid sphere of radius a in free space.

a) Determine the characteristic equations for the eigenfrequencies for TE and TM modes of oscilation. Show that the roots for ω always have a negative imaginary part, assuming a time dependence of $e^{-i\omega t}$.

b) Calculate the eigenfrequencies for the $l = 1$ and $l = 2$ TE and TM modes. Tabulate the wavelength (defined in terms of the real part of the frequency) in units of the radius a and the decay time (defined as the time taken for the *energy* to fall to e^{-1} of its initial value) in units of the transit time (a/c) for each of the modes.

10.1 *a*) Show that for arbitrary initial polarization, the scattering cross section of a perfectly conducting sphere of radius a, summed over outgoing polarizations, is given in the long-wavelength limit by

$$\frac{d\sigma}{d\Omega}(\vec{\epsilon}_0, \hat{n}_0, \hat{n}) = k^4 a^6 \left[\frac{5}{4} - |\vec{\epsilon}_0 \cdot \hat{n}|^2 - \frac{1}{4}|\hat{n} \cdot (\hat{n}_0 \times \vec{\epsilon}_0)|^2 - \hat{n}_0 \cdot \hat{n}\right]$$

where $\hat{n}_0$ and $\hat{n}$ are the directions of the incident and scattered radiations, respectively, while $\vec{\epsilon}_0$ is the (perhaps complex) unit polarization vector of the incident radiation ($\vec{\epsilon}_0{}^* \cdot \vec{\epsilon}_0 = 1$; $\hat{n}_0 \cdot \vec{\epsilon}_0 = 0$).

b) If the incident radiation is linearly polarized, show that the cross section is

$$\frac{d\sigma}{d\Omega}(\vec{\epsilon}_0, \hat{n}_0, \hat{n}) = k^4 a^6 \left[\frac{5}{8}(1 + \cos^2\theta) - \cos\theta - \frac{3}{8}\sin^2\theta\cos 2\phi\right]$$

where $\hat{n} \cdot \hat{n}_0 = \cos\theta$ and the azimuthal angle ϕ is measured from the direction of the linear polarization.

c) What is the ratio of scattered intensities at $\theta = \pi/2$, $\phi = 0$ and $\theta = \pi/2$, $\phi = \pi/2$? Explain physically in terms of the induced multipoles and their radiation patterns.