Physics 506 **Winter 2008**

Homework Assignment #4 — Due Thursday, January 31

Textbook problems: Ch. 9: 9.6, 9.11, 9.16, 9.17

9.6 *a*) Starting from the general expression (9.2) for $\vec{A}$ and the corresponding expression for Φ, expand both $R = |\vec{x} - \vec{x}\,'|$ and $t' = t - R/c$ to first order in $|\vec{x}\,'|/r$ to obtain the electric dipole potentials for arbitrary time variation

$$\Phi(\vec{x}, t) = \frac{1}{4\pi\epsilon_0}\left[\frac{1}{r^2}\vec{n}\cdot\vec{p}_{\rm ret} + \frac{1}{cr}\vec{n}\cdot\frac{\partial\vec{p}_{\rm ret}}{\partial t}\right]$$
$$\vec{A}(\vec{x}, t) = \frac{\mu_0}{4\pi r}\frac{\partial\vec{p}_{\rm ret}}{\partial t}$$

where $\vec{p}_{\rm ret} = \vec{p}(t' = t - r/c)$ is the dipole moment evaluated at the retarded time measured from the origin.

b) Calculate the dipole electric and magnetic fields directly from these potentials and show that

$$\vec{B}(\vec{x}, t) = \frac{\mu_0}{4\pi}\left[-\frac{1}{cr^2}\vec{n}\times\frac{\partial\vec{p}_{\rm ret}}{\partial t} - \frac{1}{c^2 r}\vec{n}\times\frac{\partial^2\vec{p}_{\rm ret}}{\partial t^2}\right]$$
$$\vec{E}(\vec{x}, t) = \frac{1}{4\pi\epsilon_0}\left\{\left(1 + \frac{r}{c}\frac{\partial}{\partial t}\right)\left[\frac{3\vec{n}(\vec{n}\cdot\vec{p}_{\rm ret}) - \vec{p}_{\rm ret}}{r^3}\right] + \frac{1}{c^2 r}\vec{n}\times\left(\vec{n}\times\frac{\partial^2\vec{p}_{\rm ret}}{\partial t^2}\right)\right\}$$

c) Show explicitly how you can go back and forth between these results and the harmonic fields of (9.18) by the substitutions $-i\omega \leftrightarrow \partial/\partial t$ and $\vec{p}e^{ikr-i\omega t} \leftrightarrow \vec{p}_{\rm ret}(t')$.

9.11 Three charges are located along the z axis, a charge $+2q$ at the origin, and charges $-q$ at $z = \pm a\cos\omega t$. Determine the lowest nonvanishing multipole moments, the angular distribution of radiation, and the total power radiated. Assume that $ka \ll 1$.

9.16 A thin linear antenna of length d is excited in such a way that the sinusoidal current makes a full wavelength of oscillation as shown in the figure.

a) Calculate exactly the power radiated per unit solid angle and plot the angular distribution of radiation.

b) Determine the total power radiated and find a numerical value for the radiation resistance.

9.17 Treat the linear antenna of Problem 9.16 by the multipole expansion method.

a) Calculate the multipole moments (electric dipole, magnetic dipole, and electric quadrupole) exactly and in the long-wavelength approximation.

b) Compare the shape of the angular distribution of radiated power for the lowest nonvanishing multipole with the exact distribution of Problem 9.16.

c) Determine the total power radiated for the lowest multipole and the corresponding radiation resistance using both multipole moments from part a. Compare with Problem 9.16b. Is there a paradox here?