Physics 506 **Winter 2008**

Homework Assignment #3 — Due Thursday, January 24

Textbook problems: Ch. 8: 8.18, 8.20
Ch. 9: 9.1, 9.3

8.18 *a*) From the use of Green's theorem in two dimensions show that the TM and TE modes in a waveguide defined by the boundary-value problems (8.34) and (8.36) are orthogonal in the sense that

$$\int_A E_{z\,\lambda} E_{z\,\mu} da = 0 \qquad \text{for } \lambda \neq \mu$$

for TM modes, and a corresponding relation for H_z for TE modes.

b) Prove that the relations (8.131)–(8.134) form a consistent set of normalization conditions for the fields, including the circumstances when λ is a TM mode and μ is a TE mode.

8.20 An infinitely long rectangular waveguide has a coaxial line terminating in the short side of the guide with the thin central conductor forming a semicircular loop of radius R whose center is a height h above the floor of the guide, as shown in the accompanying cross-sectional view. The half-loop is in the plane $z = 0$ and its radius R is sufficiently small that the current can be taken as having a constant value I_0 everywhere on the loop.

a) Prove that to the extent that the current is constant around the half-loop, the TM modes are not excited. Give a physical explanation of this lack of excitation.

b) Determine the amplitude for the lowest TE mode in the guide and show that its value is independent of the height h.

c) Show that the power radiated in either direction in the lowest TE mode is

$$P = \frac{I_0^2}{16} Z \frac{a}{b} \left(\frac{\pi R}{a} \right)^4$$

where Z is the wave impedance of the TE_{10} mode. Here assume $R \ll a, b$.

9.1 A common textbook example of a radiating system (see Problem 9.2) is a configuration of charges fixed relative to each other but in rotation. The charge density is obviously a function of time, but it is not in the form of (9.1).

a) Show that for rotating charges one alternative is to calculate *real* time-dependent multipole moments using $\rho(\vec{x}, t)$ directly and then compute the multipole moments for a given harmonic frequency with the convention of (9.1) by inspection or Fourier decomposition of the time-dependent moments. Note that care must be taken when calculating $q_{lm}(t)$ to form linear combinations that are real before making the connection.

b) Consider a charge density $\rho(\vec{x}, t)$ that is periodic in time with period $T = 2\pi/\omega_0$. By making a Fourier *series* expansion, show that it can be written as

$$\rho(\vec{x}, t) = \rho_0(\vec{x}\,) + \sum_{n=1}^{\infty} \Re[2\rho_n(\vec{x}\,)e^{-in\omega_0 t}]$$

where

$$\rho_n(\vec{x}\,) = \frac{1}{T}\int_0^T \rho(\vec{x}, t)e^{in\omega_0 t}dt$$

This shows explicitly how to establish connection with (9.1).

c) For a single charge q rotating about the origin in the x-y plane in a circle of radius R at constant angular speed ω_0, calculate the $l = 0$ and $l = 1$ multipole moments by the methods of parts a and b and compare. In method b express the charge density $\rho_n(\vec{x}\,)$ in cylindrical coordinates. Are there higher multipoles, for example, quadrupole? At what frequencies?

9.3 Two halves of a spherical metallic shell of radius R and infinite conductivity are separated by a very small insulating gap. An alternating potential is applied between the two halves of the sphere so that the potentials are $\pm V\cos\omega t$. In the long-wavelength limit, find the radiation fields, the angular distribution of radiated power, and the total radiated power from the sphere.