Physics 506 **Winter 2008**

Homework Assignment #2 — Due Thursday, January 17

Textbook problems: Ch. 8: 8.5, 8.6, 8.7

8.5 A waveguide is constructed so that the cross section of the guide forms a right triangle with sides of length a, a, $\sqrt{2}a$, as shown. The medium inside has $\mu_r = \epsilon_r = 1$.

a) Assuming infinite conductivity for the walls, determine the possible modes of propagation and their cutoff frequencies.

b) For the lowest modes of each type calculate the attenuation constant, assuming that the walls have large, but finite, conductivity. Compare the result with that for a square guide of side a made from the same material.

8.6 A resonant cavity of copper consists of a hollow, right circular cylinder of inner radius R and length L, with flat end faces.

a) Determine the resonant frequencies of the cavity for all types of waves. With $(1/\sqrt{\mu\epsilon}R)$ as a unit of frequency, plot the lowest four resonant frequencies of each type as a function of R/L for $0 < R/L < 2$. Does the same mode have the lowest frequency for all R/L?

b) If $R = 2$ cm, $L = 3$ cm, and the cavity is made of pure copper, what is the numerical value of Q for the lowest resonant mode?

8.7 A resonant cavity consists of the empty space between two perfectly conducting, concentric spherical shells, the smaller having an outer radius a and the larger an inner radius b. As shown in Section 8.9, the azimuthal magnetic field has a radial dependence given by spherical Bessel functions, $j_l(kr)$ and $n_l(kr)$, where $k = \omega/c$.

a) Write down the transcendental equation for the characteristic frequencies of the cavity for arbitrary l.

b) For $l = 1$ use the explicit forms of the spherical Bessel functions to show that the characteristic frequencies are given by

$$\frac{\tan kh}{kh} = \frac{\left(k^2 + \frac{1}{ab}\right)}{k^2 + ab\left(k^2 - \frac{1}{a^2}\right)\left(k^2 - \frac{1}{b^2}\right)}$$

where $h = b - a$.

c) For $h/a \ll 1$, verify that th result of part b yields the frequency found in Section 8.9, and find the first order correction in h/a.