Physics 506 **Winter 2006**

Practice Final

The final will be a 3 hour open book, open notes exam. Do all four problems.

1. Two parallel infinitely long straight wires of negligible cross-sectional area are separated by a distance a and are at rest in an inertial frame $\mathcal{O}'$. Both wires carry a uniform linear charge density q_0 as seen in $\mathcal{O}'$. The frame $\mathcal{O}'$ (and hence each wire) moves with a velocity v parallel to the direction of the wires with respect to the laboratory frame $\mathcal{O}$.

   a) Write down the electric and magnetic fields in the rest frame $\mathcal{O}'$, and calculate the force per unit length between the wires.

   b) What are the electric and magnetic fields in the laboratory frame $\mathcal{O}$?

   c) Compute the force per unit length between the wires in the laboratory frame using the fields (and charge density) measured in the laboratory frame, and show that it is identical to the force seen in the rest frame.

2. A uniform superconducting slab of thickness $2d$ and London penetration depth λ fills space in the region $-d < z < d$. Outside the superconductor, there is a constant magnetic field parallel to the surface

$$\vec{B} = B_0\hat{y} \qquad \text{for } |z| > d$$

   a) What is the magnetic field inside the superconductor? (Note that $\vec{B}$ is continuous, and that the Maxwell equations are of Proca form with $\mu = 1/\lambda$.)

   b) What is the effective current density $\vec{J}$ in the above superconductor?

3. The Lienard-Wiechart potentials for a single moving charged particle can be expressed as

$$\Phi(\vec{x},t) = \left[\frac{e}{R(1-\vec{\beta}\cdot\hat{n})}\right]_{\text{ret}}, \qquad \vec{A}(\vec{x},t) = \left[\frac{e\vec{\beta}}{R(1-\vec{\beta}\cdot\hat{n})}\right]_{\text{ret}}$$

   where the particle's path is given by $\vec{r}(\tau)$. The quantities $\vec{\beta}$ and $\vec{R}$ are given by $\vec{\beta} = \vec{v}/c$ and $\vec{R} = \vec{x} - \vec{r}(\tau)$, with $R = |\vec{R}|$ and $\hat{n} = \vec{R}/R$. The expressions in the square brackets are to be evaluated at the retarded time. Now find the Lienard-Wiechart potentials Φ and $\vec{A}$ of an electric dipole $\vec{p}$ moving with velocity $\vec{\beta}$ in the laboratory frame. Express your results in terms of $\vec{\beta}$, $\hat{n}$, $\vec{p}$, $\dot{\vec{p}}$ and R evaluated at the retarded time. (Note that $\dot{\vec{p}} = d\vec{p}/dt$ is non-vanishing whenever the dipole's orientation is time dependent.) You may wish to consider the dipole $\vec{p} = e\vec{d}$ as a limiting case of a pair of particles with charges $+e$ and $-e$ separated by a distance $\vec{d}$ where $e \to \infty$ and $\vec{d} \to 0$.

4. Two equal and opposite charges $+q$ and $-q$ are connected to the ends of a rigid (uncharged) rod of length d. The rod rotates in the x–y plane with constant angular velocity ω about the z axis which passes through its center.

   a) Compute the instantaneous radiated power $dP(t)/d\Omega$ to leading order in the non-relativistic limit.

   b) Find the total instantaneous radiated power $P(t)$ by integrating $dP(t)/d\Omega$ over the solid angle and show that it is in fact independent of time.