Physics 505 | Electricity and Magnetism | Fall 2003

Prof. G. Raithel

Problem Set 2

Due date: Monday, September 22, 6PM

Note: Problems will be collected and graded. You may bring your homeworks to class or drop them in my mailbox in front of the Physics Department office by 6PM. No late homework will be considered.

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1. Jackson, Problem 1.17

2. Jackson, Problem 1.22

3. Jackson, Problem 2.1

4. Jackson, Problem 2.2

5. **Convergence speed of relaxation methods**. To solve this numerical problem, you may form work groups and use a programming language of your choice (Mathematica should be fine, too). Use 8-byte real numbers in your calculation, if you have a choice.

Consider a two-dimensional square region with three sides on zero potential, and one side on unit potential $V = 1$. Use a square grid of 21x21 potential values, $\{V(i,j), i = 0,1,2...20, j = 0,1,2...20\}$. The grid does include the boundaries. Initialize all internal points to zero.

a) Use the Jacobian iteration of "cross averages", as defined in Eq. 1.80a of the textbook. Consider the potential at the mid point, $V(10,10)$. How many iterations does it take until the corrections to $V(10,10)$ from one iteration to the next drop below 10^{-5} of the current value? What is $V(10,10)$ at that iteration?

b) Repeat the computation using the Gauss-Seidel iteration of "cross averages".

c) The convergence of the Gauss-Seidel iteration can be accelerated using the hyper-relaxation method discussed in class. The plain Gauss-Seidel iteration using "cross averages" operates by the replacement

$$V_{\text{new}}(i,j) = \frac{1}{4}\left(V_{\text{old}}(i+1,j) + V_{\text{old}}(i-1,j) + V_{\text{old}}(i,j+1) + V_{\text{old}}(i,j-1)\right) \quad , \tag{1}$$

where replacements are made "on the fly" (implying that only one array of potential values is needed in the computation). The hyper-relaxation method with hyper-relaxation parameter p operates as follows:

$$\begin{aligned} r1 &= \frac{1}{4}\left(V_{\text{old}}(i+1,j) + V_{\text{old}}(i-1,j) + V_{\text{old}}(i,j+1) + V_{\text{old}}(i,j-1)\right) \\ V_{\text{new}}(i,j) &= V_{\text{old}}(i,j) + p\left(r1 - V_{\text{old}}(i,j)\right) \quad , \end{aligned} \tag{2}$$

where $r1$ is an auxiliary variable. Replacements are made "on the fly". Using the hyper-relaxation method defined in Eq. 2, determine the number of iterations required to achieve the situation explained in part a) for $p = 1, 1.1, 1.2, ..., 2.0$.

d) Discuss all results in view of overall numerical efficiency.