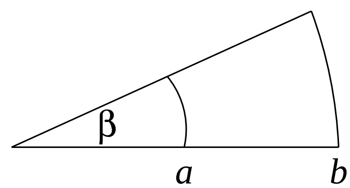
Physics 505 **Fall 2005**

Practice Midterm

The midterm will be a 120 minute open book, open notes exam. Do all three problems.

1. A two-dimensional problem is defined by a semi-circular wedge with $0 \le \phi \le \beta$ and $a \le \rho \le b$.



   a) For the Dirichlet problem, it is possible to expand the Green's function as

$$G(\rho, \phi; \rho', \phi') = \sum_{m=1}^{\infty} g_m(\rho, \rho') \sin\left(\frac{m\pi\phi}{\beta}\right) \sin\left(\frac{m\pi\phi'}{\beta}\right)$$

   Write down the appropriate differential equation that $g_m(\rho, \rho')$ must satisfy.

   b) Solve the Green's function equation for $g_m(\rho, \rho')$ subject to Dirichlet boundary conditions and write down the result for $G(\rho, \phi; \rho', \phi')$.

2. A conducting spherical shell of inner radius a is held at zero potential. The interior of the shell is filled with electric charge of a volume density

$$\rho(\vec{r}) = \rho_0 \left(\frac{a}{r}\right)^2 \sin^2\theta$$

   a) Find the potential everywhere inside the shell.

   b) What is the surface charge density on the inside surface of the shell?

3. A thin disk of radius a lies in the x-y plane with its center at the coordinate origin. The disk is uniformly charged with a surface density σ.

   a) Calculate the multipole moments of the charge distribution. Make sure to indicate which moments are non-vanishing.

   b) Write down the multipole expansion for the potential in explicit form up to the first two non-vanishing terms.