Physics 505 **Fall 2005**

Homework Assignment #9 — Due Thursday, November 17

Textbook problems: Ch. 5: 5.20, 5.22, 5.26
Ch. 6: 6.1

5.20 *a*) Starting from the force equation (5.12) and the fact that a magnetization $\vec{M}$ inside a volume V bounded by a surface S is equivalent to a volume current density $\vec{J}_m = (\vec{\nabla} \times \vec{M})$ and a surface current density $(\vec{M} \times \vec{n}\,)$, show tha in the absence of macroscopic conduction currents the total magnetic force on the body can be written

$$\vec{F} = -\int_V (\vec{\nabla} \cdot \vec{M}) \vec{B}_e \, d^3x + \int_S (\vec{M} \cdot \vec{n}\,) \vec{B}_e \, da$$

where $\vec{B}_e$ is the applied magnetic induction (not including that of the body in question). The force is now expressed in terms of the effective charge densities ρ_M and σ_M. If the distribution of magnetization is now discontinuous, the surface can be at infinity and the force given by just the volume integra.

b) A sphere of radius R with uniform magnetization has its center at the origin of coordinates and its direction of magnetization making spherical angles θ_0, ϕ_0. If the external magnetic field is the same as in Problem 5.11, use the expression of part *a*) to evaluate the components of the force acting on the sphere.

5.22 Show that in general a long, straight bar of uniform cross-sectional area A with uniform lenghwise magnetization M, when placed with its flat end against an infinitely permeable flat surface, adheres with a force given approximately by

$$F \simeq \frac{\mu_0}{2} A M^2$$

Relate your discussion to the electrostatic considerations in Section 1.11.

5.26 A two-wire transmission line consists of a pair of nonpermeable parallel wires of radii a and b separated by a distance $d > a + b$. A current flows down one wire and back the other. It is uniformly distributed over the cross section of each wire. Show that the self-inductance per unit length is

$$L = \frac{\mu_0}{4\pi} \left[1 + 2 \ln \left(\frac{d^2}{ab} \right) \right]$$

6.1 In three dimensions the solution to the wave equation (6.32) for a point source in space and time (a light flash at $t' = 0, \vec{x}\,' = 0$) is a spherical shell disturbance of radius $R = ct$, namely the Green function $G^{(+)}$ (6.44). It may be initially surprising that in one or two dimensions, the disturbance possesses a "wake", even though the source is a "point" in space and time. The solutions for fewer dimensions than three can be found by superposition in the superfluous dimension(s), to eliminate dependence on such variable(s). For example, a flashing line source of uniform amplitude is equivalent to a point source in two dimensions.

a) Starting with the retarded solution to the three-dimensional wave equation (6.47), show that the source $f(\vec{x}\,', t) = \delta(x')\delta(y')\delta(t')$, equivalent to a $t = 0$ point source at the origin in two spatial dimensions, produces a two-dimensional wave

$$\Psi(x, y, t) = \frac{2c\Theta(ct - \rho)}{\sqrt{c^2t^2 - \rho^2}}$$

where $\rho^2 + x^2 + y^2$ and $\Theta(\xi)$ is the unit step function $[\Theta(\xi) = 0\ (1)$ if $\xi < (>)\ 0.]$

b) Show that a "sheet" source, equivalent to a point pulsed source at the origin in one space dimension, produces a one-dimensional wave proportional to

$$\psi(x, t) = 2\pi c\Theta(ct - |x|)$$