Physics 505 **Fall 2005**

Homework Assignment #8 — Due Thursday, November 10

Textbook problems: Ch. 5: 5.10, 5.14, 5.17, 5.19

5.10 A circular current loop of radius a carrying a current I lies in the x-y plane with its center at the origin.

a) Show that the only nonvanishing component of the vector potential is

$$A_\phi(\rho, z) = \frac{\mu_0 I a}{\pi} \int_0^\infty dk\, \cos kz\, I_1(k\rho_<) K_1(k\rho_>)$$

where $\rho_<$ ($\rho_>$) is the smaller (larger) of a and ρ.

b) Show that an alternative expression for A_ϕ is

$$A_\phi(\rho, z) = \frac{\mu_0 I a}{2} \int_0^\infty dk\, e^{-k|z|} J_1(ka) J_1(k\rho)$$

c) Write down integral expressions for the components of magnetic induction, using the expressions of parts *a*) and *b*). Evaluate explicitly the components of $\vec{B}$ on the z axis by performing the necessary integrations.

5.14 A long, hollow, right circular cylinder of inner (outer) radius a (b), and of relative permeability μ_r, is placed in a region of initially uniform magnetic-flux density $\vec{B}_0$ at right angles to the field. Find the flux density at all points in space, and sketch the logarithm of the ratio of the magnitudes of $\vec{B}$ on the cylinder axis to $\vec{B}_0$ as a function of $\log_{10} \mu_r$ for $a^2/b^2 = 0.5$, 0.1. Neglect end effects.

5.17 A current distribution $\vec{J}(\vec{x}\,)$ exists in a medium of unit relative permeability adjacent to a semi-infinite slab of material having relative permeability μ_r and filling the half-space, $z < 0$.

a) Show that for $z > 0$ the magnetic induction can be calculated by replacing the medium of permeability μ_r by an image current distribution, $\vec{J}^*$, with components,

$$\left(\frac{\mu_r - 1}{\mu_r + 1}\right) J_x(x, y, -z), \qquad \left(\frac{\mu_r - 1}{\mu_r + 1}\right) J_y(x, y, -z), \qquad -\left(\frac{\mu_r - 1}{\mu_r + 1}\right) J_z(x, y, -z)$$

b) Show that for $z < 0$ the magnetic induction appears to be due to a current distribution $[2\mu_r/(\mu_r + 1)]\vec{J}$ in a medium of unit relative permeability.

5.19 A magnetically "hard" material is in the shape of a right circular cylinder of length L and radius a. The cylinder has a permanent magnetization M_0, uniform throughout its volume and parallel to its axis.

a) Determine the magnetic field $\vec{H}$ and magnetic induction $\vec{B}$ at all points on the axis of the cylinder, both inside and outside.

b) Plot the ratios $\vec{B}/\mu_0 M_0$ and $\vec{H}/M_0$ on the axis as functions of z for $L/a = 5$.