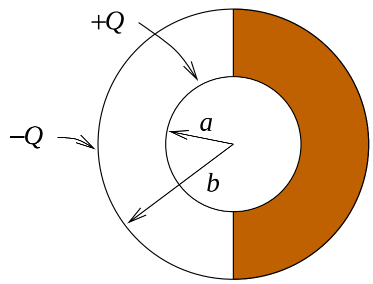
Physics 505 **Fall 2005**

Homework Assignment #7 — Due Thursday, November 3

Textbook problems: Ch. 4: 4.10
Ch. 5: 5.3, 5.6, 5.7

4.10 Two concentric conducting spheres of inner and outer radii a and b, respectively, carry charges $\pm Q$. The empty space between the spheres is half-filled by a hemispherical shell of dielectric (of dielectric constant $\epsilon/\epsilon_0 0$, as shown in the figure.



a) Find the electric field everywhere between the spheres.

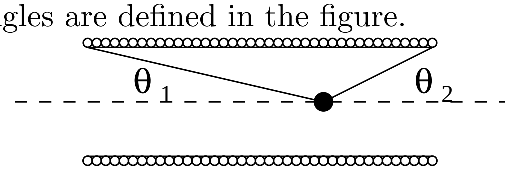
b) Calculate the surface-charge distribution on the inner sphere.

c) Calculate the polarization-charge density induced on the surface of the dielectric at $r = a$.

5.3 A right-circular solenoid of finite length L and radius a has N turns per unit length and carries a current I. Show that the magnetic induction on the cylinder axis in the limit $NL \to \infty$ is

$$B_z = \frac{\mu_0 NI}{2}(\cos\theta_1 + \cos\theta_2)$$

where the angles are defined in the figure.



5.6 A cylindrical conductor of radius a has a hole of radius b bored parallel to, and centered a distance d from, the cylinder axis ($d + b < a$). The current density is uniform throughout the remaining metal of the cylinder and is parallel to the axis. Use Ampère's law and principle of linear superposition to find the magnitude and the direction of the magnetic-flux density in the hole.

5.7 A compact circular coil of radius a, carrying a current I (perhaps N turns, each with current I/N), lies in the x-y plane with its center at the origin.

a) By elementary means [Eq. (5.4)] find the magnetic induction at any point on the z axis

b) An identical coil with the same magnitude and sense of the current is located on the same axis, parallel to, and a distance b above, the first coil. With the coordinate origin relocated at the point midway between the centers of the two coils, determine the magnetic induction on the axis near the origin as an expansion in powers of z, up to z^4 inclusive:

$$B_z = \left(\frac{\mu_0 I a^2}{d^3}\right)\left[1 + \frac{3(b^2 - a^2)z^2}{2d^4} + \frac{15(b^4 - 6b^2a^2 + 2a^4)z^4}{16d^8} + \cdots\right]$$

where $d^2 = a^2 + b^2/4$.

c) Show that, off-axis near the origin, the axial and radial components, correct to second order in the coordinates, take the form

$$B_z = \sigma_0 + \sigma_2\left(z^2 - \frac{\rho^2}{2}\right); \qquad B_\rho = -\sigma_2 z\rho$$

d) For the two coils in part *b*) show that the magnetic induction on the z axis for large $|z|$ is given by the expansion in inverse odd powers of $|z|$ obtained from the small z expansion of part *b*) by the formal substitution $d \to |z|$.

e) If $b = a$, the two coils are known as a pair of Helmholtz coils. For this choice of geometry the second terms in the expansions of parts *b*) and *d*) are absent [$\sigma_2 = 0$ in part *c*)]. The field near the origin is then very uniform. What is the maximum permitted value of $|z|/a$ if the axial field is to be uniform to one part in 10^4, one part in 10^2?